Tutorial 3

Question 13. Consider a dynamic system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \quad (19)$$

$$y = [1 \;\; 0]x \quad (20)$$

Design a state feedback controller such that the system output tracks a constant r.

Question 14. Consider a first order dynamic system

$$\dot{x} = x + u \quad (21)$$

The control input is designed as

$$u = -kx \quad (22)$$

such that the system is stale. Evaluate the performance index

$$J = \int_0^\infty x^2 dt \tag{23}$$

with $x(0) = 2$, and hence obtain an optimal value of k such that J is minimum.

Question 15. Repeat the optimal control design in Question 14, with the control performance index

$$J = \int_0^\infty [x^2 + ru^2] dt \tag{24}$$

where r is a constant.

Question 16. consider a dynamic system described by

$$\dot{x} \;\; = \;\; \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \tag{25}$$

The initial value is given as $x(0) = [1,\ 1]^T$. With the feedback control in the form of

$$u = -kx_1 - kx_2 \tag{26}$$

obtain the relation between the performance index given by

$$J = \int_0^\infty x^T x dt \tag{27}$$

and the controller gain k.

Question 17. Re-design the optimal control gain k in Question 16 using the optimal control index

$$J = \int_0^\infty (x^T x + u^2) dt \tag{28}$$

Question 18. Determine the roots of the closed-loop control systems obtained in Questions 15, 16 and 17.