Tutorial 1

Question 1. An inverted pendulum can be described by the following set of differential equations

$$M\ddot{y} + ml\ddot{\theta} - u(t) = 0 \tag{1}$$

$$ml\ddot{y} + ml^2\ddot{\theta} - mlg\theta = 0 \tag{2}$$

where M is the mass of the cart, and m is the mass of the ball over the pendulum with $m << M$, y is the horizontal position of the cart and θ is the angle of the pendulum, u is the control input. Write the state space equation of the this system.

Question 2. A system is described by the following differential equation

$$\dot{x} = \begin{bmatrix} -1 & 0 \\ 2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \tag{3}$$

$$\tag{4}$$

Determine $\Phi(s)$ and $\Phi(t)$ of the system.

Question 3. Obtain the transfer function of the following state space system

$$\dot{x} = \begin{bmatrix} -1 & 0 \\ 2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \tag{5}$$

$$y = [1 \quad 0]x \tag{6}$$

Question 4. Write the state space equations in the controller and observer canonical forms for the following systems described by the transfer functions

$$G_1(s) = \frac{s+1}{s^2+5s+5}, \quad G_2 = \frac{s+1}{4s^2+4s+1} \tag{7}$$

Question 5. Determine the controllability and observability of the state space systems described in Question 3.

Question 6. Determine the controllability and observability of the state space systems

$$\dot{x} = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t) \tag{8}$$

$$y = [1 \quad 0]x \tag{9}$$

If the system is not controllable or observable, can you explain why?