Extra problem sheet for Math67201-47201

September 21, 2017

Problem 1. Suppose $f(t) = f_1(t) - f_2(t)$, where f_1, f_2 are increasing functions. Show that f is of bounded variation on any finite interval $[0, T]$.

Problem 2. Let B be a standard Brownian motion. Prove $[B, B]_t = t$ for $t \geq 0$.

Problem 3. Let $B_t, t \geq 0$ be a Brownian motion and $A_t, t \geq 0$ a continuous process of bounded variation. Prove $[B, A]_t = 0$ for $t \geq 0$.

Problem 4. Let $\{B_t, t \geq 0\}$ be a standard Brownian motion. Set $\mathcal{F}_t = \sigma(B_u, 0 \leq u \leq t)$ and $\mathcal{F}_0 = \{\Omega, \emptyset\}$. Prove that $Z_t = B_t^3 - 3\int_0^t B_u du, t \geq 0$ is a martingale w.r.t. $\mathcal{F}_t, t \geq 0$.

(Hint: If $X \sim N(0, \sigma^2)$, $E[X^3] = 0$.)