PHYS30121 Introduction to Nuclear and Particle Physics
Problems 4: Supplementary Problems

These are a variety of problems. Some are good for revision, some extend the lecture material. I've tried to indicate what sort of problem it is at the beginning. Solutions are available on Teach Web / Blackboard.

See the Teach Web / Blackboard for more problems of the nature found on examination papers; make sure you read the advice note for the older exam papers.

1: Cross Sections

Do this problem if you want to find out more about cross sections.

Cross sections are used in both particle and nuclear physics to describe the rates or probabilities of a particular reaction. They are easiest to understand initially by considering a stationary target foil, containing many target nuclei, and a beam of particles incident at 90° to the foil.

(i) If the density of particles in the beam is n_p and they all travel at the same speed v, what is the flux of particles Φ, i.e. the number crossing a unit area perpendicular to the beam direction per unit time?

(ii) You would expect the rate of a certain reaction to be proportional to the flux. The *cross section* is the constant of proportionality between the rate per target nucleus and the incident flux:

$$\sigma = \text{ event rate per target } /\Phi.$$

Notice that cross section has the units of an area. This is related to classical cross sections; the bigger the target the easier it is to hit it. But in nuclear physics, actual cross sections are often not easy to relate to a geometric area due to the range of the forces involved and the effects of quantum mechanics.

If N targets are exposed to the beam, what is the reaction rate?

(iii) If a beam has a cross sectional area S, how is the beam intensity I, the number of particles per unit time passing a point, related to Φ?

(iv) Write down an alternative form for the reaction rate R in terms of I and show that:

$$R = I\sigma n_t t$$

where n_t is the number of targets per unit volume and t is the target thickness.

How is n_t related to mass density ρ?

NB: $n_t t$ is the number of target atoms in the target per unit area on its surface. ρt is a similar quantity describing the mass per unit area. They are often referred to as areal densities.

Often products are detected at particular angles to the beam direction (θ, ϕ), using a detector subtending a solid angle $d\Omega$. The rate at which products enter the detector is proportional to the incident flux and the number of target atoms, as well as $d\Omega$. In this case the constant of proportionality is called the *differential cross section*, which is often a function of the angles of detection:

$$\mathrm{d}R(\theta, \phi) = \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\theta, \phi) N \Phi \mathrm{d}\Omega.$$

The total cross section discussed above is then:

$$\sigma = \int \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\theta, \phi) \mathrm{d}\Omega.$$

Cross sections are usually quoted using a unit called the barn, where 1b$=10^{-28}$m^2 $=10^{-24}$cm^2, reflecting a typical size of the first cross sections that were measured. More often cross sections are of the order of mb or smaller!

(v) A detector with a solid angle of 2.8 msr is placed at an angle where the differential cross section is 10 mbsr^{-1}. If a beam of α particles, constituting an electrical current of 10 nA is incident on a ^{27}Al target of areal density 100μgcm^{-2}, how many products enter the detector per unit time? If the detector has an efficiency of 25%, what is the event rate?

2: Nuclear Radii

For revision on nuclear radii.

(i) Estimate what electron energy is needed to resolve structure inside a ^{197}Au nucleus? How much higher would the energy need to be to see sub-nucleon structure?

(ii) Find an expression for the distance of closest approach of a light particle with charge $Z_1 e$ approaching a heavy target head on with charge $Z_2 e$ with energy E, assuming only Coulomb interaction between them.

(iii) Estimate the energy at which elastic scattering cross section for α particles on a gold foil ($Z = 79$) begins to deviate from the Rutherford scattering formula

(iv) Calculate the Bohr radius of the lowest atomic orbit for an electron and a muon in a ^{27}Al atom. Compare these radii to the size of the nucleus. [Bohr radius $= \frac{4\pi\epsilon_0\hbar^2}{Zme^2}$.]
Electron mass is 0.510998918(44) MeV/c^2 and muon mass is 105.658369(9) MeV/c^2.

3: Binary Reaction Q Values

To learn a bit more about how nuclear masses are extracted from reaction measurements.

The following expression was quoted in lectures for the Q value of the binary reaction A(a,b)B:

$$Q = T_b\left(1+\frac{m_b}{m_B}\right) - T_a\left(1-\frac{m_a}{m_B}\right) - \frac{2\ \cos\theta}{m_B}\sqrt{T_aT_bm_am_b}$$

where T are the kinetic energy of the relevant particles, θ is the *scattering* angle i.e. the angle between the beam direction and the trajectory of b, and m are the relevant masses.

(i) Using classical expressions for kinetic energy and linear momentum, prove that this expression arises from the application of energy and momentum conservation.

[Hint: Use energy conservation to write the Q value in terms of kinetic energies. Use momentum conservation in the beam direction and perpendicular to it to find the components of $\mathbf{p_B}$ along those directions. Use them to eliminate p_B^2 from the energy expression.]

(ii) The neutron stripping reaction ^{208}Pb(d,p)^{209}Pb is performed using a deuteron beam with an energy of 8 MeV. Protons with an energy of 9.704 MeV are detected at an angle of $15°$.

Given the atomic masses below, what is the Q value of the reaction? Use your result to infer the mass of ^{208}Pb. Atomic masses: ^{209}Pb: 194622.719 MeV/c^2, ^{2}H 1875.6127550 MeV/c^2 and ^{1}H 938.27200 MeV/c^2.

4: Daughter Recoil in Decay Processes *An exercise to revise different decay processes and learn a bit about how recoil influences them.* In a decay process, a stationary parent emits a particle with mass m and kinetic energy T, causing the daughter† nucleus with mass M to recoil.

Prove the following the expressions for the recoil energies and calculate the recoil energies in typical cases suggested. Think carefully in each case whether relativity is needed:

(i) 1 MeV γ ray from an $A = 100$ nucleus; $T_R = E_\gamma^2/2Mc^2$

(ii) 1 MeV β particle from an $A = 100$ nucleus; max $T_R = (T_\beta^2{+}2T_\beta m_e c^2)/2Mc^2$

(iii) 5 MeV α particle from an $A = 200$ nucleus; $T_R = T_\alpha m_\alpha/M$

Why have I chosen different typical numbers in the last case?

NB: The Q value in a decay is shared between the recoil and the emitted particle. For some cases, Q is roughly equal to the energy of the emitted particle and the recoil energy can be neglected to some approximation. In the following questions, explicitly state when you are neglecting the recoil energy.

5: β Decay

A revision problem on β decay. Show that ^{7}Be only decays by electron capture. What would be the minimum ^{7}Be mass that would allow β^+ decay? The mass excesses of ^{7}Be and ^{7}Li are 15770.034 and 14908.14 keV/c^2 respectively. The electron mass is 0.510998910 MeV/c^2

6: Production of Sources *Problems 6 and 7 are to revise and extend your previous knowledge of radioactive decay laws.*

A radioactive nucleus A is produced at a rate R nuclei per second in a nuclear reaction. It decays with a probability λ per second.

(a) Write down a differential equation describing the rate of change of the number of nuclei of isotope A.

(b) Show that $N(t) = \frac{R}{\lambda}(1 - e^{-\lambda t})$ is a solution to this equation.

(c) Show that the activity of isotope A is a constant, and equal to the production rate, at times long compared to the half life.

(d) If the accelerator is costly to run, how long would you irradiate for to get the best value for money i.e. maximum activity per buck.

7: Dating

Natural uranium consists mainly of two isotopes: 99.3% ^{238}U and 0.7% ^{235}U. Their half-lives are 4.47$\times 10^9$ and 7.04$\times 10^8$ years respectively. How long is it since they existed in equal abundance?

8: Alpha-Decay Q Values

As promised in the lectures, revision of SEMF with α decay.

The binding energy of the alpha particle is 28.3 MeV. Use the semi-empirical mass formula for an even-even nucleus:

$$B(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}}$$

$$-a_a \frac{(A-2Z)^2}{A} + a_p A^{-1/2}$$

to show that the Q value for α decay can be estimated as:

$$Q_\alpha = 28.3 - 4a_v + \frac{8}{3}a_s A^{-1/3} - 4a_c A^{-1/3}\left[\frac{Z(Z-1)}{3A} - \frac{2Z-1}{2}\right] - 4a_a \frac{(A-2Z)^2}{A^2} + \frac{4a_p}{A^{3/2}}.$$

[Hint: Estimate the change in the binding energy between parent and daughter as $dB = \frac{\partial B}{\partial A}dA + \frac{\partial B}{\partial Z}dZ$ with $dA = 4$ and $dZ = 2$.]

Assuming that: $Z >> 1$ so $(Z-1)$ terms can be replaced by Z; $Z = 0.41A$ for heavy systems on the line of β stability; and using the SEMF parameters ($a_v = 15.85,\ a_s = 18.34,\ a_c = 0.71,\ a_a = 23.21,\ a_p = 12$ MeV), show that:

$$Q_\alpha = -38.108 + 48.91A^{-1/3} + 1.005A^{2/3} + \frac{48}{A^{3/2}}$$

Hence show that α decay is energetically possible if $A > 154$.

9: Excited States

Problems 10-13 revise and extend material on single-particle models.

The low-lying states in $^{91}_{41}$Nb have the following excitation energies and spin-parities:

$$0.00 \text{ MeV } 9/2^+,\ 0.104 \text{ MeV } 1/2^-,\ 1.187 \text{ MeV } 5/2^-,\ 1.313 \text{ MeV } 3/2^-,$$

$$1.581 \text{ MeV } 7/2^+.$$

Using the independent-particle model, suggest shell model configurations which describe these states. [Hint: For each state, consider which single-particle orbit the odd proton must be in to get the correct spin-parity.]

10: Energies and Masses

Estimate the energy gap between the $1p_{1/2}$ and $1d_{5/2}$ neutron orbitals for $A \sim 16$ given that the total binding energy of an ^{15}O atom is 111.9556 MeV, of ^{16}O is 127.6193 MeV and ^{17}O is 131.7627 MeV. [Hint: ^{17}O might be thought of as ^{16}O plus a neutron, and ^{15}O as ^{16}O minus a neutron]

11: Odd-Odd Nuclei The following odd-odd nuclei have the spin-parities shown: ${}^{12}_{5}$B (1^+), ${}^{16}_{7}$N (2^-), ${}^{34}_{15}$P (1^+), ${}^{28}_{13}$Al (3^+)

In these case, which single-particle levels are occupied by the odd proton and odd neutron? Couple the angular momenta of these single-particle levels and find the possible values of the nuclear spins. Comparing these possible values to the experimental measurements, is there any preferential tendency in the coupling of the angular momenta of the odd particles? [Hint: Draw a simple vector diagram to show how the $\mathbf{j}$ vectors couple, then add to show how the $\mathbf{j}$ is made up of $\mathbf{l}$ and $\mathbf{s}$. You should find something to remind you of the case of the deuteron.]

12: More on Spin-Orbit Splitting

A particular nl orbital has an energy of E_0 before spin-orbit coupling is considered. For the spin-orbit force used in the question above, show that the average energy of the spin-orbit pair of states, $j = l \pm s$, weighted by their occupancy is independent of strength, λ, and equal to E_0. Use your answers from the question above as starting point.