THIRD YEAR EXAMPLE CLASS SHEET FIVE
PHYS30121 Introduction to Nuclear and Particle Physics
Problems 3: Forces and the Independent-Particle Model

There are more nuclear physics problems on the webpage for revision purposes.

1: Pauli Exclusion and Ground-State Spin and Parity

Six neutrons in occupy an orbital with total angular momentum $j = 5/2$. What are the m_j values for each neutron? What is the total magnetic quantum number M for this configuration of six neutrons? What is the only total angular momentum J available to this configuration?

Convince yourself that the same argument is true for any particular j orbital, as long as it is completely full of nucleons i.e. it contains $(2j + 1)$ protons or neutrons.

Hence argue that a j orbital that is full to capacity with a particular type of nucleon can give no contribution to the overall nuclear spin.

Use the principle above to predict the spin and parity of the ground states of (i) $^{40}_{20}$Ca (ii) $^{52}_{20}$Ca and (iii) $^{53}_{20}$Ca, using the single-particle levels given in the lectures.

Notice that this argument works well for magic nuclei, when entire nuclear shells are full, and sometimes works for nuclei that correspond to arrangements of full j orbitals. Nuclei corresponding to these systems with one additional nucleon, generally have a ground state *nuclear* spin equal to that of the orbital of the odd nucleon.

2: Pairing and Ground-State Spin and Parity

It was noted when we discussed the semi-empirical mass formula, that nucleons like to form pairs. This is because there is an additional component to the force a nucleon feels in the nucleus that is not accounted for by the mean field in the independent particle model (so-called residual interaction).

This pairing force lowers the energy of two nucleons if they are close together and move in a similar way. It could be maximised if two nucleons were in exactly the same quantum state, i.e. same j, l and m_j. Why doesn't this happen for proton and neutron pairs?

Thinking about classical orbits, explain why pairing favours the formation of nucleon pairs in a particular j orbit if they exist in states with m_j and $-m_j$. What is the total angular momentum of the pair?

This simple argument persists in more formal descriptions of nucleon pairing and fully paired systems favour zero angular momentum.

Use the pairing argument to predict the ground-state spin and parity of ${}^{11}_{6}$C, ${}^{44}_{20}$Ca, ${}^{61}_{28}$Ni and ${}^{73}_{32}$Ge.

Notice that the pairing argument suggests a ground-state spin of 0^+ for all even nuclei (true!). Nuclei corresponding to these systems with one additional nucleon, generally have a ground state *nuclear* spin equal to that of the orbital of the odd nucleon (sometimes true!).

3: Exchange Forces

The ρ and ω mesons and two pion exchange are though to be responsible for mediating nuclear forces at distances shorter than the one-pion exchange region. Use the following masses to estimate their ranges using an uncertainty-principle argument. $m_\rho = 776\ \ \mathrm{MeV/c^2}, m_\omega = 783\ \ \mathrm{MeV/c^2}, m_\pi \sim 137\ \mathrm{MeV/c^2}$

4: Spin-Orbit Splitting

For a spin-orbit interaction of the form:

$$V_{so} = -\frac{\lambda}{\hbar^2}\mathbf{l}.\mathbf{s}$$

where λ is a constant, show that the spin-orbit splitting of the states $j = l + s$ and $j = l - s$ is given by:

$$\frac{\lambda(2l+1)}{2}.$$

Experimentally, the ground state of ${}^{17}_{8}$O has a spin-parity of $5/2^+$. There is an excited state at 5.08 MeV with $J^\pi = 3/2^+$. Interpret the ground and excited state in terms of an independent-particle model and thus find the value of λ.

5: Mirror Nuclei, β Decay, Nuclear Forces and Radii

The maximum kinetic energy of β^+ particles emitted in the decay of ${}^{35}_{18}$Ar to its mirror system, ${}^{35}_{17}$Cl, is measured to be 4.95 MeV. Find the radius of these A=35 nuclei. The proton-neutron mass difference is 1.29 MeV/c^2.

[Hints: Use the measured β^+ endpoint to find the mass difference between ${}^{35}_{18}$Ar and ${}^{35}_{17}$Cl. Since these two are mirror nuclei, their *atomic* mass difference can be attributed to the different numbers of electrons in the atom, the different number of protons and neutrons and the proton-neutron mass difference, and the difference in the Coulomb energies. Use the information to calculate the latter. Then use the energy of a uniformly charged sphere $\frac{3}{5}\frac{(Ze)^2}{4\pi\epsilon_0}\frac{1}{R}$ to find the radius.]

6: Magic Numbers Why are the nuclear magic numbers different from the atomic numbers for Nobel gases? Explain why there are no odd magic numbers?