THIRD YEAR EXAMPLE CLASS SHEET THREE
PHYS30121 Introduction to Nuclear and Particle Physics
Solutions 2: Masses, Q Values, Semi-Empirical Mass Formula

1: α Decay

$$Q_\alpha = M_{\mathrm{Pu}} - M_{\mathrm{U}} - M_\alpha = 238.049555 - 234.040947 - 4.002603 = 6.005\times10^{-3}\mathrm{u} = 5.594 \mathrm{\ MeV}$$

2: γ Decay

As with the decay of the neutron done in lectures, if you stick to atomic masses you can show that the β decay Q value in the first step of the decay is given by:

$$Q_{\beta^-} = m_F - m_{Ne^*} = \Delta_F - \Delta_{Ne^*}.$$

This assumes that the mass of the neutrino can be neglected. Ignoring the recoil of the decaying nucleus, the Q value is equal to the maximum β energy. Rearranging to find the mass of the excited state :

$$\Delta_{Ne^*} = \Delta_F - Q_{\beta^-} = -0.017 - 5.390 = -5.407 \mathrm{\ MeV}$$

The Q value of the second equation is equal to the γ-ray energy, again neglecting the recoil:

$$E_\gamma = m_{Ne^*} - m_{Ne} = \Delta_{Ne^*} - \Delta_{Ne}$$

$$\Delta_{Ne} = \Delta_{Ne^*} - E_\gamma = -5.407 - 1.633 = -7.040 \mathrm{\ MeV}$$

Mass of ^{20}Ne is then $\Delta + A = -7.040 + 20 \times 931.494 = 18622.840$ MeV.

3: Semi-Empirical Mass Formula and Fission

(a) Notice that the terms in m_{1H} and m_n will all cancel in working out the relevant Q values. For example, in ^{235}U+1n=^{236}U:

$$Q = m(235, 92) + m_n - m(236, 92)$$

$$= 92m_{1H} + 143m_n - BE(235, 92) + m_n - [92m_{1H} + 144m_n - BE(236, 92)]$$

$$= BE(236, 92) - BE(235, 92)$$

where

$$BE = 15.85A - 18.34A^{2/3} - 0.71\frac{Z(Z-1)}{A^{1/3}} - 23.21\frac{(A-2Z)^2}{A} \pm 12A^{-1/2}$$

So:

$$BE(236, 92) = 3740.6 - 700.4 - 961.9 - 265.9 + 0.8 = 1813.2 \mathrm{\ MeV}$$

$$BE(235, 92) = 3724.8 - 698.4 - 963.2 - 256.9 = 1806.3 \text{ MeV}$$
$$Q = 6.9 \text{ MeV}$$

Also:

$$BE(238, 92) = 3772.3 - 704.3 - 959.2 - 284.4 + 0.8 = 1825.2 \text{ MeV}$$

$$BE(239, 92) = 3788.2 - 706.3 - 957.8 - 293.8 = 1830.3 \text{ MeV}$$
$$Q = 5.1 \text{ MeV}$$

(b) If a nucleus elongates without change in density, the volume term remains constant. The surface term will increase as the surface area increases with elongation. There will also be a change in the Coulomb term, but this is smaller. So 6 MeV is required before fission will occur due to the increase in mass-energy as the nucleus elongates.

Adding a neutron to ^{235}U generates enough energy to overcome this 6 MeV barrier and the nucleus will fission, but in ^{238}U the Q value is not enough.

(c) ^{235}U is easy to fission in normal reactors where neutrons are slow and have thermal energies. ^{238}U can be used in so-called fast reactors where the neutrons have kinetic energies of 2 MeV or more supplying the additional energy needed to overcome the fission barrier.

4: Semi-Empirical Mass Formula and the Line of Stability

$$m(A, Z) = Zm_{1H} + (A - Z)m_n - a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + a_a \frac{(A-2Z)^2}{A} \mp a_p^{-1/2}$$

$$\left[\frac{\partial m}{\partial Z}\right]_A = m_{1H} - m_n + a_c \frac{2Z-1}{A^{1/3}} - 4a_a \frac{(A-2Z)}{A}$$

$$m_{1H} - m_n + a_c \frac{2Z_{min}-1}{A^{1/3}} - 4a_a \frac{(A-2Z_{min})}{A} = 0$$

$$Z_{min}\left[\frac{2a_c}{A^{1/3}} + \frac{8a_a}{A}\right] = m_n - m_{1H} + \frac{a_c}{A^{1/3}} + 4a_a$$

$$Z_{min} = \frac{m_n - m_{^1H} + a_c A^{-1/3} + 4a_a}{[2a_c A^{-1/3} + 8a_a A^{-1}]}$$

With $m_n - m_{^1H} = 0.782$ MeV, $a_c = 0.71$ MeV and $a_a = 23.21$ MeV only one term in the numerator is significant:

$$Z_{min} = \frac{4a_a}{[2a_c A^{-1/3} + 8a_a A^{-1}]}$$

$$Z_{min} = \frac{2Aa_a}{[a_c A^{2/3} + 4a_a]}$$

$$Z_{min} = \frac{2A}{[a_c A^{2/3}/a_a + 4]}$$

$$Z_{min} = \frac{A}{2}\frac{1}{[1 + a_c A^{2/3}/4a_a]}$$

For small A denominator is approximately unity so $Z_{min} = \frac{A}{2}$.
For large A denominator is greater than unity, so $Z_{min} < \frac{A}{2}$ indicating N>Z.

5: Semi-Empirical Mass Formula and Neutron Emission

You will need to find the A value for which the Q value for neutron emission ($A \rightarrow A - 1 + n$) becomes positive and is energetically possible:

$$Q = M(A, Z) - M(A - 1, Z) - m_n$$
$$= BE(A - 1, Z) - BE(A, Z)$$

So calculate this using the semi-empirical mass formula. I get A=30 as the first sodium isotope which can emit neutrons. Notice how the pairing term keeps the even-N systems intact and only the odd-N systems emit neutrons.

A	Volume	Surface	Coulomb	Symmetry	Pairing	Total BE	Q Value
23	364.550	-148.326	-27.463	-1.009	0.000	187.752	
24	380.400	-152.595	-27.076	-3.868	-2.449	194.411	-6.659
25	396.250	-156.805	-26.710	-8.356	0.000	204.380	-9.968
26	412.100	-160.959	-26.363	-14.283	-2.353	208.142	-3.762
27	427.950	-165.060	-26.033	-21.491	0.000	215.366	-7.224
28	443.800	-169.111	-25.720	-29.841	-2.268	216.860	-1.494
29	459.650	-173.114	-25.421	-39.217	0.000	221.899	-5.039
30	475.500	-177.071	-25.135	-49.515	-2.191	221.589	**0.310**
31	491.350	-180.984	-24.862	-60.645	0.000	224.859	-3.270
32	507.200	-184.856	-24.600	-72.531	-2.121	223.092	**1.767**
33	523.050	-188.687	-24.349	-85.103	0.000	224.911	-1.819
34	538.900	-192.480	-24.108	-98.301	-2.058	221.953	**2.958**
35	554.750	-196.236	-23.876	-112.071	0.000	222.567	-0.614