THIRD YEAR EXAMPLE CLASS SHEET ONE
PHYS30121 Introduction to Nuclear and Particle Physics
Solutions 1: Background and revision

1: Nuclear Physics Units (i)

$$\hbar = 1.055 \times 10^{-34}\ \mathrm{J.s}\ = \frac{1.055 \times 10^{-34}}{1.602 \times 10^{-19} \times 10^{6}}\ \mathrm{MeV.s}\ = 6.58 \times 10^{-22}\ \mathrm{MeV.s}$$

$$c = 2.998 \times 10^{8}\ \mathrm{m.s^{-1}}\ = 2.998 \times 10^{8} \times 10^{15}\ \mathrm{fm.s^{-1}}\ = 2.998 \times 10^{23}\ \mathrm{fm.s^{-1}}$$

$$\Rightarrow \hbar c = 197.3\ \mathrm{MeV.fm}$$

(ii)

$$\frac{e^2}{4\pi\epsilon_0} = \frac{(1.6022 \times 10^{-19})^2}{4\pi \times 8.854 \times 10^{-12}} \frac{\mathrm{C^2}}{\mathrm{C^2N^{-1}m^{-2}}}$$
$$= 2.307 \times 10^{-28}\ \mathrm{Nm^2} = 1.44\ \mathrm{MeV.fm}$$

(iii)

$$\frac{e^2}{4\pi\epsilon_0 \hbar c} = \frac{1.44}{197.3} = \frac{1}{137.0}$$

(iv)

$$\mathrm{Energy} = \frac{e^2}{4\pi\epsilon_0}\frac{1}{r} = \frac{1.44}{5} = 0.288\ \mathrm{MeV}$$

2: Energy of a Charged Sphere

The charge density of the final sphere is:

$$\rho = \frac{Ze}{\frac{4}{3}\pi R^3} = \frac{3Ze}{4\pi R^3}$$

The energy dE to bring up one thin shell of charge with thickness dr, when the radius is r, is:

$$dE = \frac{QdQ}{4\pi\epsilon_0}\frac{1}{r}$$

where Q is the charge of the sphere when it has a radius r and dQ is the charge in the thin shell.

The charge Q is found using the charge density and the volume:

$$Q = \rho\frac{4}{3}\pi r^3$$

And dQ by differentiating the last expression:

$$dQ = 4\pi\rho r^2 dr$$

Combining everything:

$$dE = \frac{1}{4\pi\epsilon_0}\rho\frac{4}{3}\pi r^3\frac{1}{r}4\pi\rho r^2 dr$$

$$= \frac{4\pi}{3\epsilon_0}\rho^2 r^4 dr$$

Integrating:

$$E = \frac{4\pi}{3\epsilon_0}\rho^2\int_0^R r^4 dr$$

$$= \frac{4\pi}{3\epsilon_0}\rho^2\left[\frac{r^5}{5}\right]_0^R$$

$$= \frac{4\pi}{3\epsilon_0}\rho^2\left[\frac{R^5}{5}\right]$$

$$= \frac{4\pi}{3\epsilon_0}\left(\frac{3Ze}{4\pi R^3}\right)^2\frac{R^5}{5}$$

$$= \frac{3}{5}\frac{(Ze)^2}{4\pi\epsilon_0}\frac{1}{R}$$

For a lead nucleus as specificed:

$$E = \frac{3}{5}\frac{(Ze)^2}{4\pi\epsilon_0}\frac{1}{R} = \frac{3}{5}\times\frac{1.44\times 82^2}{17.3} = 335.4\text{ MeV}$$