A SUMMARY OF ARGUMENTS FOR THE SEMI-EMPIRICAL MASS FORMULA

The semi-empirical mass formula can be written as:

$$m(A,Z) = Zm_{1H} + (A-Z)m_n - a_v A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}}$$
$$+a_a \frac{(A-2Z)^2}{A} \mp a_p A^{-1/2}.$$

Similarly the binding energy is written:

$$BE(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}}$$
$$-a_a \frac{(A-2Z)^2}{A} \pm a_p A^{-1/2}.$$

Volume Term

Empirically it is found that for medium to heavy nuclei the binding energy per nucleon is roughly constant, between 7 and 8.5 MeV/A. So the binding energy is approximately proportional to the number of nucleons in the nucleus, where the constant a_v is of the order of 8 MeV.

The linear dependence is somewhat surprising; if each nucleon interacted with every other nucleon in the nucleus, there would be $A(A-1)/2$ interacting pairs and the binding energy would scale quadratically with mass number. The linear variation suggests that each nucleon interacts with only its closest neighbours, and not all other nucleons. We'll see that the density of matter inside a nucleus is roughly constant, so each nucleon has roughly the same number of neighbours, and thus contributes about the same amount to the binding energy of the nucleus.

Surface Term

The exception to the arguments above are the nucleons on the surface of the nucleus. They are surrounded by fewer nucleons and so contribute less to the binding energy. Putting $BE = a_v A$ therefore overestimates by giving too much weight to the surface nucleons. This can be corrected by subtracting a term proportional to the number of surface nucleons i.e. something proportional to the surface area of the nucleus. Assuming a sphere, the surface area is $4\pi R^2$.

Taking the statement above that the density of the interior of the nucleus is roughly constant, when adding nucleons the volume of the nucleus has to increase. We'd expect that the volume scales as A so that the nuclear radius scales as $A^{1/3}$. Later on we'll see that $R = r_0 A^{1/3}$.

The surface term needs to be proportional to R^2 and therefore proportional to $A^{2/3}$.

Coulomb Term

Protons inside the nucleus suffer electrostatic repulsion. This acts against the attractive binding of the nuclear forces and a term needs to be added to account for this. The electrostatic energy associated with a uniformly charged sphere is given by:

$$\frac{3}{5}\frac{(Ze)^2}{4\pi\epsilon_0}\frac{1}{R}.$$

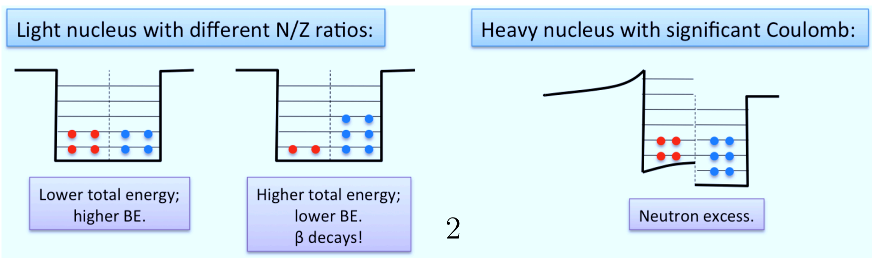
This suggests that a term proportional to $\frac{Z^2}{A^{1/3}}$ should be subtracted from the expression for the binding energy to account for the reduction in binding.

The above expression is deduced by constructing a sphere out of a large number of infinitesimally small charges. However, a nucleus is constructed in pre-built blocks i.e. protons and each proton will repel the other $Z-1$ protons in the nucleus. For this reason, the term is often written with a factor $Z(Z-1)$ rather than just Z^2.

Symmetry Term

For light systems, the stable systems tend to have $N \sim Z$. In order to realistically reproduce the observed stable systems, the binding energy formula must take this into account, otherwise it will suggest light stable nuclei have very large numbers of neutrons. In heavier nuclei, this term should be less important, because the rapid rise in electrostatic repulsion with Z requires an excess of neutrons. This excess reduces the overall charge density in the nucleus, lowering the Coulomb effects. A possible form might be $-a_a\frac{(A-2Z)^2}{A}$ since this gives an advantage if $N = Z$, but this advantage reduces with increasing mass.

The underlying physics can be understood if one considers that the nucleus is a quantal system, so the nucleons will exist in energy levels. As a nucleus is built up, the nucleons of each type fill up these energy levels according to the Pauli principle. The topmost filled state is known as the Fermi level or surface. If there are equal numbers of protons and neutrons, the Fermi levels are the same for both. If there is an excess of one type of nucleon, the one Fermi surface will be much higher in energy than the other. The system can then lower its total energy by beta decay, converting the excess of one type of nucleon to the other. There is a tendency for the two Fermi levels to equalise, although this is mitigated by the Coulomb repulsion of protons. Cartoons of these situations are shown below. A proper analysis of the nucleus as a gas of fermionic particles leads to a term of the form suggested above.



Pairing Term There is a tendency for nucleons of the same type to form pairs. Evidence for this can be found in the numbers of different types of isotope appearing in nature. There are only four stable isotopes with odd numbers of both protons and neutrons. Generally we find that for odd-odd systems, there is an advantage to undergoing beta decay. For example, the odd neutron can convert to a proton and pair up with the existing proton to form an even-even nucleus. It is found empirically that a function of the form $a_p A^{-1/2}$ fits the observed mass dependence of this effect, although there are a number of alternatives. This term is added to the binding energy when dealing with an even-even nucleus, subtracted for odd-odd nuclei, but is zero for all other systems i.e. odd-even or even-odd.