ONE HOUR THIRTY MINUTES

A list of constants is enclosed.

UNIVERSITY OF MANCHESTER

Introduction to Nuclear and Particle Physics

20th January 2009, 2.00 p.m. - 3.30 p.m.

Answer **ALL** parts of question 1, **ONE** from Section A and **ONE** from Section B

Electronic calculators may be used, provided that they cannot store text.

The numbers are given as a guide to the relative weights of the different parts of each question.

P.T.O.

1. (a) Explain what is meant by the terms *Q value* and *binding energy*. Use the following mass excesses to calculate the binding energy of ${}^{16}_{8}$O:
$\Delta({}^{1}\mathrm{H}) = 7.289,\ \Delta(\mathrm{n}) = 8.071, \Delta({}^{16}\mathrm{O}) = -4.737\ \mathrm{MeV/c^2}$

[8 marks]

(b) Use the independent particle model to predict the spin-parity of the ground states of: (i) ${}^{16}_{8}$O, (ii) ${}^{5}_{3}$Li and (iii) ${}^{15}_{7}$N.
The order of the first few single-nucleon levels is: $s_{1/2},\ p_{3/2},\ p_{1/2}$.

[5 marks]

(c) Evidence has been claimed for a baryon ($B = +1$) which has the following quantum numbers: $Q = +1, S = +1$. Show that this particle cannot be explained in terms of the simple quark model, and deduce the minimum quark combination that would be needed to accommodate it.

[8 marks]

(d) Estimate the range of the weak interaction given that the masses of the W and Z bosons are about 100 GeV/c^2

[4 marks]

P.T.O.

SECTION A

Answer either Question 2 or Question 3

A2. Explain what is meant by the term *mirror nuclei* and describe briefly what they indicate about the nature of nuclear forces.

[5 marks]

During radioactive decay, ${}^{13}_{7}N_6$ emits positrons with a maximum energy of 1.24 MeV populating the ground state of ${}^{13}_{6}C_7$.

(a) Show that the Q value for this decay can be written as:

$$Q_{\beta^+} = (m_{^{13}N} - m_{^{13}C} - 2m_e)\, c^2$$

where m are the relevant *atomic* masses and m_e is the mass of an electron. Use this expression, and the information given above, to find the mass difference between ${}^{13}N$ and ${}^{13}C$.

[8 marks]

(b) Assuming the electrostatic energy of a *uniformly* charged sphere of radius R and total charge Ze is given by:

$$\frac{3}{5}\frac{1}{4\pi\epsilon_0}\frac{(Ze)^2}{R},$$

show that the atomic mass difference of a mirror pair with $(Z+1, A)$ and (Z, A) can be written as:

$$\delta m = m_{^1H} - m_n + \frac{3}{5}\frac{e^2}{4\pi\epsilon_0}\frac{(2Z+1)}{R}$$

[5 marks]

(c) Using your answers to (a) and (b) above, estimate the radius of these $A = 13$ nuclei, given that $m_n - m_{^1H} = 782$ keV.

[4 marks]

(d) Compare your result to the empirical formula $r = r_0 A^{1/3}$ and comment on the value of r_0 obtained.

[3 marks]

P.T.O.

A3. Explain how the following observations indicate particular aspects of the nucleon-nucleon force:

(i) The binding energy per nucleon is roughly constant in nuclei heavier than $A \sim 16$.

(ii) The scattering of α particles by nuclei deviates from Rutherford scattering when the distance of closest approach is less than about $1.4A^{1/3}$ fm.

(iii) The deuteron has a non-zero quadrupole moment.

(iv) There is a $^3S_1$, but no $^1S_0$, bound state of two nucleons.

(v) Phase shifts in nucleon-nucleon scattering become negative for energies greater than around 300 MeV.

[15 marks]

Sketch the form of the nucleon-nucleon interaction, annotating it in a quantitative fashion.

[4 marks]

If the force between two nucleons is mediated by the exchange of virtual particles, use the Uncertainty Principle to estimate the mass of the particles exchanged by two nucleons in a nucleus.

[6 marks]

P.T.O.

SECTION B

Answer either Question 4 or Question 5

B4. Give qualitative explanations, using Feynman diagrams, of the quoted lifetimes of the following particles:

(i) $\omega \sim 10^{-23}$ s [5 marks]

(ii) $\pi^0 \sim 10^{-16}$ s [5 marks]

(iii) $\pi^+ \sim 10^{-8}$ s [5 marks]

(iv) $n \sim 900$ s [5 marks]

(v) $e^- > 10^{34}$ s [2 marks]

(vi) $p > 10^{40}$ s [3 marks]

B5. Briefly explain the concepts of *baryon number* and *strangeness*, and explain under which circumstances these quantum numbers may not be conserved.

[10 marks]

Two particles X^- and Y^0 are produced in the strong interaction processes:

$$K^- p \to \pi^+ \pi^0 X^-$$

$$K^- p \to K^+ \pi^- Y^0$$

respectively. Deduce the baryon number, strangeness, charm and bottom quantum numbers of the X^- and Y^0 particles and hence deduce their quark content.

[5 marks]

What isopin partners would you expect the X^- and Y^0 particles to have?

[5 marks]

The two particles are observed to decay via the following reactions:

$$X^- \to n\pi^-$$

$$Y^0 \to \Lambda K^- \pi^+$$

Give crude estimates of their lifetimes. [5 marks]

NB: The K^-, K^+ and Λ particles have strangeness $S = -1, +1$ and -1 respectively.

END OF EXAMINATION PAPER

PHYSICAL CONSTANTS AND CONVERSION FACTORS

SYMBOL	DESCRIPTION	NUMERICAL VALUE
c	Velocity of light in vacuum	299 792 458 m s^{-1}, exactly
μ_0	Permeability of vacuum	$4\pi \times 10^{-7}$ N A^{-2}, exactly
ϵ_0	Permittivity of vacuum where $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$	8.854×10^{-12} C^2 N^{-1} m^{-2}
h	Planck constant	6.626×10^{-34} J s
$\hbar$	$h/2\pi$	1.055×10^{-34} J s
G	Gravitational constant	6.674×10^{-11} m^3 kg^{-1} s^{-2}
e	Elementary charge	1.602×10^{-19} C
eV	Electronvolt	1.602×10^{-19} J
α	Fine-structure constant, $\frac{e^2}{4\pi\epsilon_0\hbar c}$	$\frac{1}{137.0}$
m_e	Electron mass	9.109×10^{-31} kg
$m_e c^2$	Electron rest-mass energy	0.511 MeV
μ_B	Bohr magneton, $\frac{e\hbar}{2m_e}$	9.274×10^{-24} J T^{-1}
R_∞	Rydberg energy $\frac{\alpha^2 m_e c^2}{2}$	13.61 eV
a_0	Bohr radius $\frac{1}{\alpha}\frac{\hbar}{m_e c}$	0.5292×10^{-10} m
Å	Angstrom	10^{-10} m
m_p	Proton mass	1.673×10^{-27} kg
$m_p c^2$	Proton rest-mass energy	938.272 MeV
$m_n c^2$	Neutron rest-mass energy	939.566 MeV
μ_N	Nuclear magneton, $\frac{e\hbar}{2m_p}$	5.051×10^{-27} J T^{-1}
fm	Femtometre or fermi	10^{-15} m
b	Barn	10^{-28} m^2
u	Atomic mass unit, $\frac{1}{12} m(^{12}C \text{ atom})$	1.661×10^{-27} kg
N_A	Avogadro constant, atoms in gram mol	6.022×10^{23} mol^{-1}
T_t	Triple-point temperature	273.16 K
k	Boltzmann constant	1.381×10^{-23} J K^{-1}
R	Molar gas constant, $N_A k$	8.315 J mol^{-1} K^{-1}
σ	Stefan-Boltzmann constant, $\frac{\pi^2}{60}\frac{k^4}{\hbar^3 c^2}$	5.670×10^{-8} W m^{-2} K^{-4}
M_E	Mass of Earth	5.97×10^{24} kg
R_E	Mean radius of Earth	6.4×10^6 m
g	Standard acceleration of gravity	9.806 65 m s^{-2}, exactly
atm	Standard atmosphere	101 325 Pa, exactly
$M_\odot$	Solar mass	1.989×10^{30} kg
$R_\odot$	Solar radius	6.961×10^8 m
$L_\odot$	Solar luminosity	3.846×10^{26} W
$T_\odot$	Solar effective temperature	5800 K
AU	Astronomical unit, mean Earth-Sun distance	1.496×10^{11} m
pc	Parsec	3.086×10^{16} m
	Year	3.156×10^7 s