PC3322

<u>ONE HOUR THIRTY MINUTES</u>

A list of constants is enclosed.

UNIVERSITY OF MANCHESTER

Nuclear Physics

30th May 2006, 2.00 p.m. - 3.30 p.m.

Answer **ALL** parts of question 1 and **TWO** other questions

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Electronic calculators may be used, provided that they cannot store text.

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The numbers are given as a guide to the relative weights of the different parts of each question.

r = 1.2 $A^{1/3}$ fm
1u = $1.67\times10^{-27}$ kg.

**1.** (a) Sketch the nuclear charge density as a function of radius. Label the *skin thickness* and the *root-mean-square radius* on the plot. [5 marks]

(b) The binding energy term of the semi-empirical mass formula can be written as

$$BE(Z,A) = a_{volume}A - a_{surface}A^{2/3} - a_{asymm}\frac{(A-2Z)^2}{A} - a_{Coulomb}\frac{Z^2}{A^{1/3}} + \delta,$$

where all the symbols have their usual meetings. Use this to explain why for even-mass nuclei there may be more than one stable isotope in an isobaric chain.
[5 marks]

(c) Explain why pionic x-rays are a useful probe of the nuclear matter distribution.
[5 marks]

(d) The first four excited states in $^{106}_{46}$Pd are (in keV): 512 ($J^\pi = 2^+$); 1229 ($J^\pi = 4^+$); 1128 ($J^\pi = 2^+$) and 1133 ($J^\pi = 0^+$). What nuclear model best describes these levels?

Briefly explain any deviations from the model predictions. [5 marks]

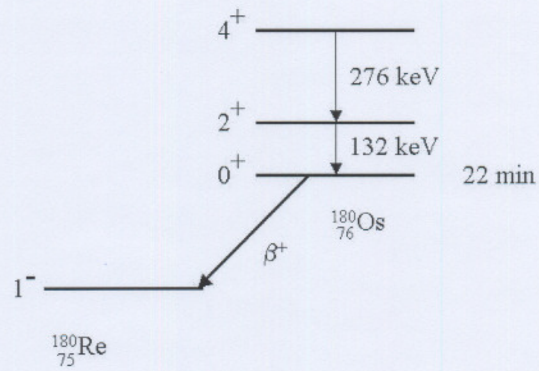
(e) Briefly explain why $\alpha$ decays involving large angular momentum are hindered. Explain why some decays to specific states in the daughter nucleus are not observed. [5 marks]

P.T.O.

**2.** (a) Write down an expression which is commonly used to parametrise collective oscillations of a deformed nuclear surface. Define any symbols you use. [6 marks]

Show that, in a rotational band with constant moment of inertia, $\Im$, the transition energies increase linearly with spin. [6 marks]

(b) The $\beta^+$ decay of $^{180}$Os to $^{180}$Re is shown in the figure below. Also shown are the lowest two excited states in $^{180}$Os.




(i) How is this $\beta$ decay classified? Fully justify your answer. [5 marks]

(ii) Estimate the half-life of the $4^+$ state in $^{180}$Os in terms of the single-particle model. The experimentally measured half-life is $6\times10^{-11}$ s, comment on this result. [5 marks]

(iii) What decay mode can compete with $\beta^+$ decay? What factors influence the branching ratio between these two decay modes? [3 marks]

The Weisskopf single-particle estimates are given as follows:

$$\lambda(E1) = 1.0 \times 10^{14} A^{2/3} E^3 \qquad \lambda(M1) = 5.6 \times 10^{13} E^3$$

$$\lambda(E2) = 7.3 \times 10^{7} A^{4/3} E^5 \qquad \lambda(M2) = 3.5 \times 10^{7} A^{2/3} E^5$$

$$\lambda(E3) = 34 A^2 E^7 \qquad \lambda(M3) = 16 A^{4/3} E^7$$

where $E$ is in MeV and $\lambda$ is in s$^{-1}$.

P.T.O.

**3.** Describe briefly how the *matter* radius of a nucleus can be determined by elastic scattering. [10 marks]

Briefly explain how the detail that can be resolved in elastic-scattering experiments depends on the kinetic energy of the projectile. [3 marks]

Elastic scattering is one method which can be used to determine the charge radius of a nucleus. In this elastic scattering process the magnitudes of the initial and final momenta $P = \hbar K$ of the electron, which is scattered through an angle $\theta$, are equal. Use this information to show that the transferred momentum,

$$q = 2K \sin(\theta/2).$$

[4 marks]

Explain why in these measurements the experimental precision of the measured charge density is poorer in the central region of the nucleus. [4 marks]

The first minimum in the differential cross section for 450-MeV electrons incident on $^{58}$Ni occurs at $q$=0.9 fm$^{-1}$. Estimate the radius of $^{58}$Ni. Compare your answer with that expected from a simple estimate. [4 marks]

P.T.O.

**4.** Give one example why a spin-orbit interaction is necessary to describe the measured properties of nuclei. [2 marks]

Write down the states which are produced when the spin-orbit interaction acts on a $1p$ shell-model state. Write down a general expression for the degeneracy of a shell-model state in terms of the orbital angular momentum, $\mathbf{l}$. [4 marks]

The energy splitting, $\epsilon_j$, introduced by the spin-orbit interaction for a state of spin, $\mathbf{j} = \mathbf{l} + \mathbf{s}$, is given by

$$\epsilon_j = -V_{s.o.}\langle \mathbf{l} \, . \, \mathbf{s} \rangle$$

Use this expression to show that the total spin-orbit energy splitting, $\Delta E$ is given by

$$\Delta E = \frac{V_{s.o}}{2}(2\mathrm{l} + 1)\hbar^2.$$

[7 marks]

Sketch the ordering of the shell-model single-particle levels arising from the 1s, 1p and 1d orbitals. Using your sketch, make predictions for the ground-state spin and parity of $^{13}_{6}C$ and $^{15}_{6}C$. [9 marks]

Using the same diagram for $^{13}_{6}C$, explain the observation of: (i) a low-lying excited state with $J^\pi = \frac{5}{2}^+$, and (ii) a higher-lying excited state with $J^\pi = \frac{7}{2}^-$. [3 marks]

**END OF EXAMINATION PAPER**