PC312(1)

ONE HOUR FORTY MINUTES

A list of constants is enclosed.

UNIVERSITY OF MANCHESTER

Particles, Nuclei and Cosmology

30th January 1997, 2.00 p.m. - 3.40 p.m.

Answer **TWO** questions

Electronic calculators may be used, provided that they cannot store text.

The numbers indicate the relative weights of the different parts of each question and do **NOT** represent a marking scheme.

P.T.O.

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1. The cross-sections for the low-energy elastic scattering of photons, pions and neutrinos from protons are of order 1 μb, 10mb and $10^{-8}\mu$b respectively. Explain the difference in terms of the fundamental coupling constants involved. Give an example of a decay process for each of these interactions.

[10 marks]

The three leptons, e, μ and τ may be assumed to have the same weak interaction coupling strength. The τ-lepton (mass 1.78 GeV/c^2) and the muon (mass 0.105 GeV/c^2) have decay modes and branching ratios as follows:

$$\mu \rightarrow e + \nu + \overline{\nu} \qquad (100\%)$$

$$\tau \rightarrow e + \nu + \overline{\nu} \qquad (17\%).$$

Given that the mean lifetime of the muon is 2.2 μs, use dimensional arguments to make an estimate of the mean lifetime for the τ-lepton.

[10 marks]

If τ-leptons with momentum 5 GeV/c are produced in an e^+e^- collider, what will be their mean flight path before decay?

[5 marks]

2. Explain briefly how the Rutherford model of the atom accounts for the results of his experiments on the scattering of α-particles by nuclei. What are the limitations of the model, and under what circumstances were deviations observed? What modifications to the model would be needed if high energy electrons were used instead of α-particles?

[10 marks]

Electrons are scattered by a spherically symmetric nucleus with density ρ(R) through an angle θ. The scattering amplitude is changed from that for a point nucleus by a factor $F(q^2)$, where

$$F(q^2) = \frac{1}{Ze} \int_o^R \rho(R) \frac{\sin qR}{qR} 4\pi R^2 dR$$

and q is the magnitude of the momentum transfer in the scattering.

Show that, for small values of qR,

$$F(q^2) = 1 - \frac{q^2 < R^2 >}{6} + ...,$$

where $< R^2 >$ is the mean square radius of the nucleus.

[8 marks]

In the elastic scattering of 200 MeV electrons through 21° by a gold foil, it is found that the differential cross-section is only 10% of that expected for a point nucleus. Estimate the r.m.s. radius of the gold nucleus, assuming that it is spherically symmetric. What other property of the gold nucleus would you measure to check this assumption?

[7 marks]

[The Rutherford cross-section

$$\frac{d\sigma}{d\Omega} = \left(\frac{zZe^2}{4\pi\epsilon_0 T}\right)^2 \frac{1}{\sin^4 \frac{\theta}{2}} .$$

In natural units 1 m = 5.07×10^{15} GeV^{-1}.]

3. Show in graphical form how the standard cosmological model of the big-bang accounts for the development of the universe during its first million years. Sketch the graph to give the dependence of approximate temperature and equilibrium energy of the universe as a function of time. Indicate on your graph the significant physics processes that occur at certain times.

[12 marks]

Estimate the temperature and mean energy of a radiation dominated universe at an elapsed time of 1 second, and give examples of the reactions taking place at this time. Show that the neutron/proton ratio has reached a value of approximately 0.36 at this time, and explain why this ratio becomes frozen at around 0.14 by $t = 100$ s. Explain how ^4He is formed and estimate the ratio of ^4He to ^1H nuclei. Why can only minute quantities of other light nuclei be produced in primordial nucleosynthesis?

[13 marks]

[The temperature of a radiation-dominated universe can be assumed to be given by $\frac{1.5 \times 10^{10}}{t^{\frac{1}{2}}}$ K, and $(m_n - m_p)c^2 = 1.294$ MeV.]