PC2302

ONE HOUR THIRTY MINUTES

A list of constants is enclosed.

UNIVERSITY OF MANCHESTER

Atoms and Nuclei

21st May 2004, 9.45 a.m. - 11.15 a.m.

Answer **ALL** parts of question 1 and **TWO** other questions

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Electronic calculators may be used, provided that they cannot store text.

The numbers are given as a guide to the relative weights of the different parts of each question.

P.T.O.

Constants:
$\mu_B = 5.8 \cdot 10^{-4}$ eV/T

1. (a) The only bound state of the deuteron is an s state with total angular momentum $J = 1$. Use this information to explain why bound neutron-neutron and proton-proton systems do not exist. [5 marks]

(b) Calculate the reduced mass m_r for positronium and estimate the binding energy of the ground state using the binding energy of hydrogen. [5 marks]

(c) Calculate, in units of $\hbar$, the magnitude of the maximum orbital angular momentum for an $n = 10$ state. [5 marks]

(d) Explain schematically the experiment which demonstrated that the electron spin is $\frac{1}{2}\hbar$. [5 marks]

(e) The α particles in the original Rutherford experiment had a kinetic energy of about 8 MeV. Calculate the radial distance from the centre of the nucleus probed by the α particle ? Compare this result with the nuclear radius of gold ($^{197}_{79}$Au). [5 marks]

2. On Earth, uranium now consists mainly of two isotopes, 99.28% ^{238}U (lifetime $\tau = 6.4 \cdot 10^9$ years) and 0.72% ^{235}U (lifetime $\tau = 1.015 \cdot 10^9$ years).

(a) Calculate the age of the solar system assuming that both uranium isotopes were created in a supernova in equal amounts just before the creation of the solar system.

What is the percentage of ^{238}U that has decayed since the creation of the earth's crust $2.5 \cdot 10^9$ years ago ? [12 marks]

(b) Discuss the neutron cycle in a nuclear fission reactor based on uranium.

In ^{235}U, the thermal fission cross section is 584 barn, while the cross-section for the other (non-fission) absorptive processes is 97 barn. Each fission produces, on average, 2.6 fast neutrons. What is the mean number of fission neutrons produced by ^{235}U per thermal neutron ? [13 marks]

3. (a) Give a short definition of degeneracy. A hydrogen atom is in the 4f state without any external fields. What is the total degeneracy of the state ? [5 marks]

(b) Which two main effects contribute to the fine structure of hydrogen ? Give an order of magnitude estimate of how large the effect of fine structure is compared to the hydrogen energy levels ? [6 marks]

(c) A multi-electron atom with total spin $S = 0$ is put in an external magnetic field of 1.5 Tesla.

Describe the observed effect on this spectrum.

Calculate the energy splitting between two adjacent energy levels. [7 marks]

(d) Explain the origin of all terms in the formula which is used to estimate the energy of K_α lines. Use this formula to calculate the energy of the K_α line of copper (Z=29). [7 marks]

4. (a) The binding energy term of the semi-empirical mass formula can be written as

$$B(A,Z) = a_V A - a_S A^{2/3} - a_C Z(Z-1)A^{-1/3} - a_A(Z - A/2)^2 A^{-1} \pm a_P A^{-1/2}$$

Explain briefly the physical origin of each term, including the A and Z dependence of the first four terms. [10 marks]

(b) For fixed $A = 238$ determine the value of Z which corresponds to the maximum binding energy $B(A,Z)$. [5 marks]

(c) How much energy is released in α decay of $^{226}_{90}$Th ? The binding energy of $^{4}_{2}$He is 28.3 MeV.

You may use the following values:

$$a_V = 15.9 \text{ MeV}$$

$$a_S = 18.3 \text{ MeV}$$

$$a_C = 0.72 \text{ MeV}$$

$$a_A = 92.8 \text{ MeV}$$

$$a_P = 11.5 \text{ MeV}$$

[10 marks]
