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Quadrupole Moments

Indicator of non-spherical shapes:

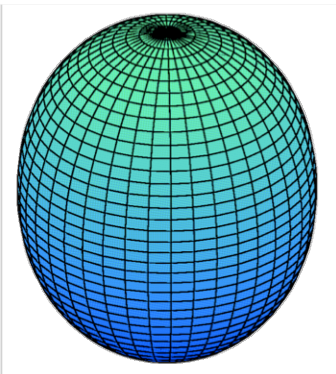
$$Q_0 = \int \rho_{\rm ch} \left[3z^2 - r^2\right] dV$$

Average over the charge distribution:

$$Q_0 = Z \left[3\langle z^2 \rangle - \langle r^2 \rangle\right]$$

Units are area. NB:
$\langle r^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle + \langle z^2 \rangle$

$$\langle z^2 \rangle > \frac{1}{3}\langle r^2 \rangle$$

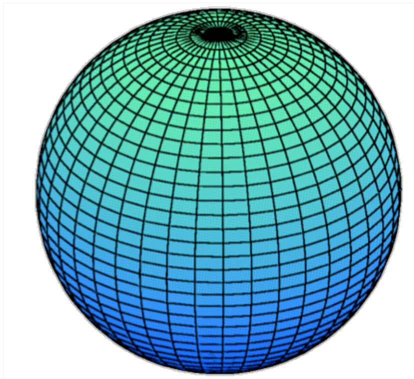


$Q_0 > 0$

PROLATE

$$\langle x^2 \rangle = \langle y^2 \rangle = \langle z^2 \rangle$$

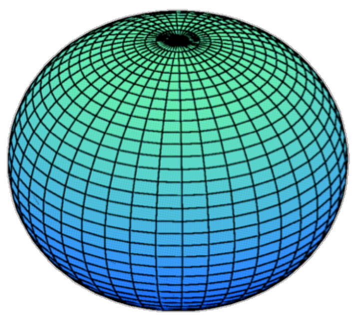
$$3\langle z^2 \rangle = \langle r^2 \rangle$$



$Q_0 = 0$

SPHERICAL

$$\langle z^2 \rangle < \frac{1}{3}\langle r^2 \rangle$$



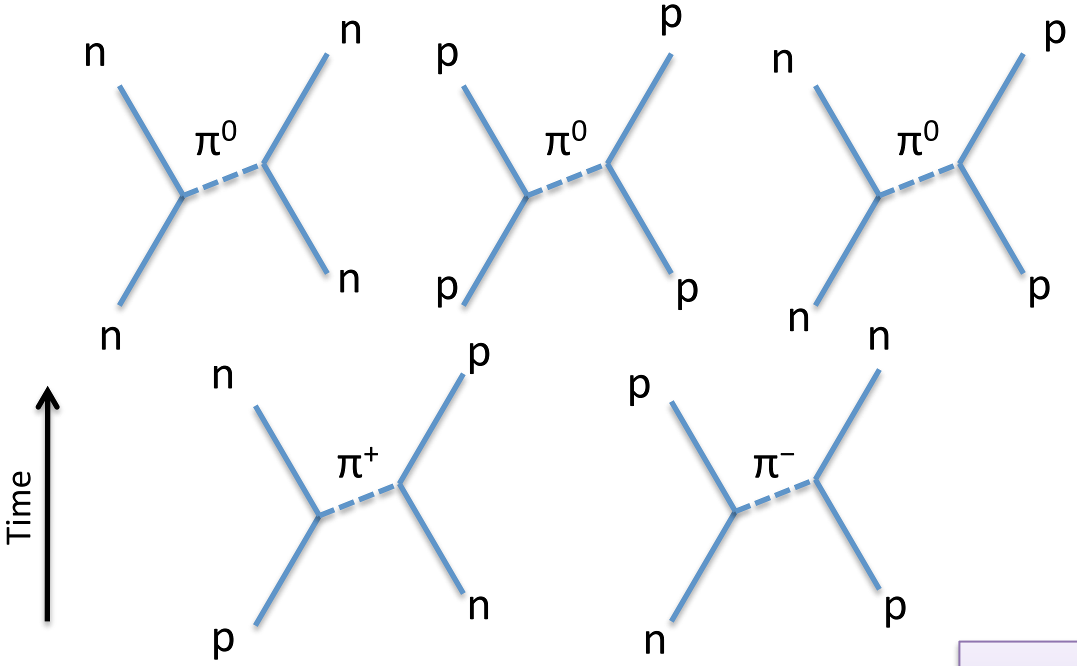
$Q_0 < 0$

OBLATE

Pions

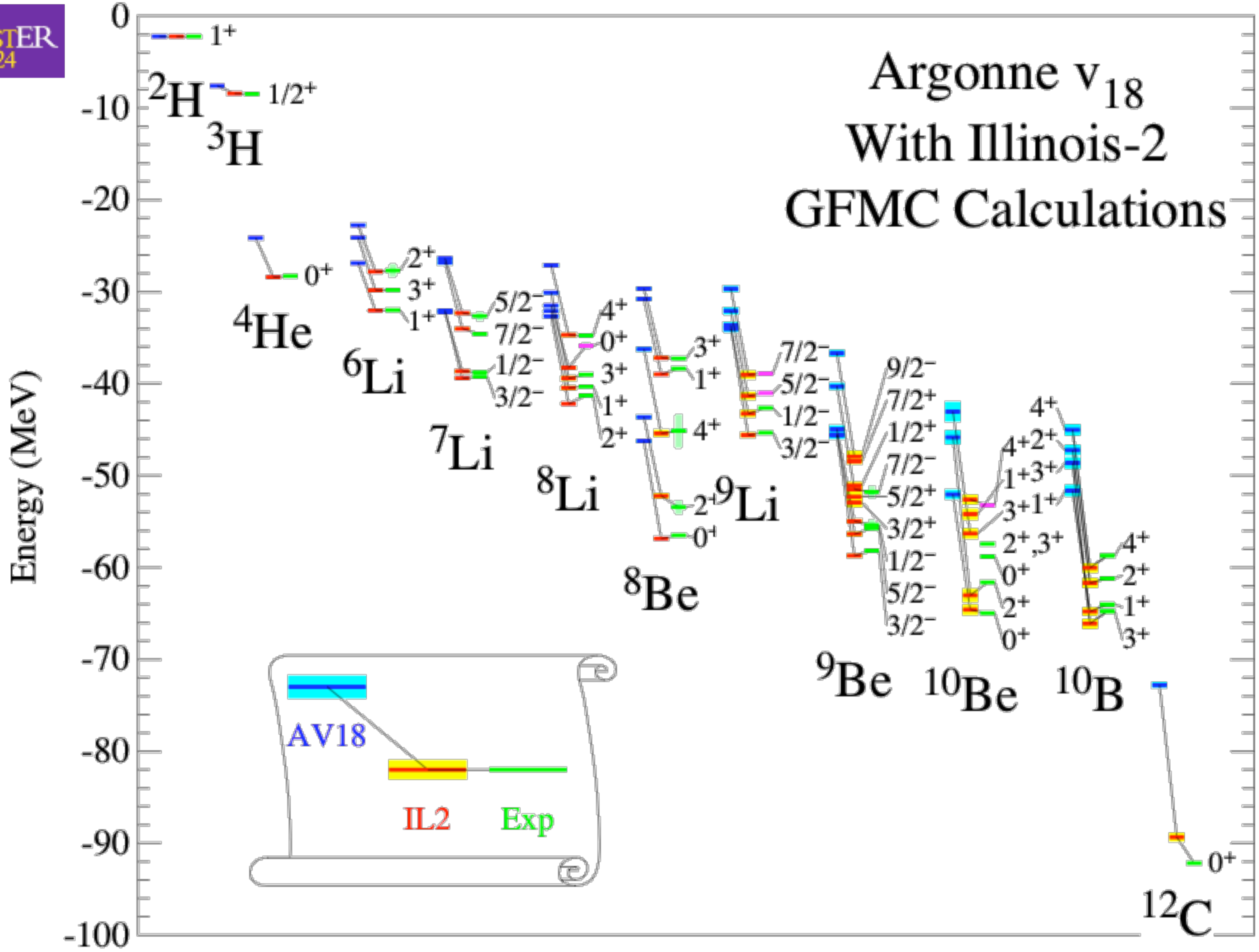
Three types: π^+, π^- and π^0.
All with spin-parity 0^-.
Masses: 139.57, 139.57 and 134.97 MeV/c^2.
Charged ones decay in 2.6×10^{-8}s.
Neutral ones with mean life 0.83×10^{-16}s.

Cecil Powell
Nobel 1950



Exchange of charged pions accounts for "exchange nature" in n-p scattering.

Near equality of pion masses accounts for charge symmetry, and small departure from charge independence



Some "state of the art" nuclear structure calculations using a modern N-N force (Argonne version 18, 1995) and three-body forces.

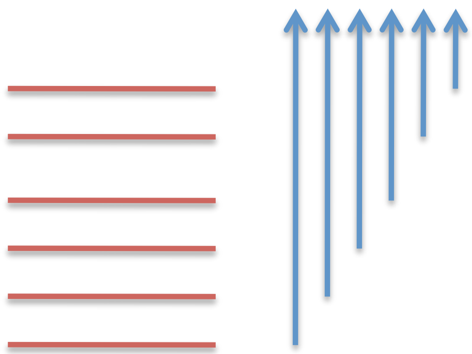
Typical 3D Potential Problem for Fermions

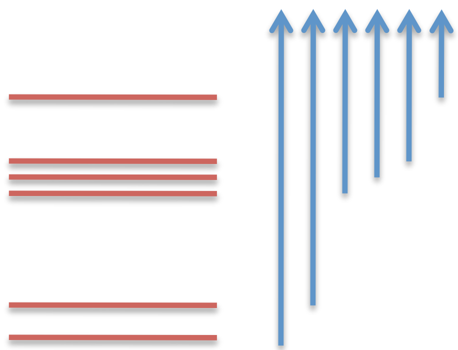
- Write down Schrödinger equation with *V(r)*; 3D partial differential equation.
- Separate into radial and angular equations.
- (Mainly) central field: angular equation is the eigenvalue equation for orbital angular momentum.
- Radial equation contains kinetic, potential and centrifugal terms.
- Solutions will give energy levels in the well.
- To build up whole *nucleus*, add nucleons into levels according to Pauli principle.

Gaps in any level structure are associated with discontinuities or jumps in properties:

E.g. nucleon separation energy (energy to remove least bound nucleon) as a function of *N* or *Z*

Gradually fill up energy levels with increasing *N* or *Z*





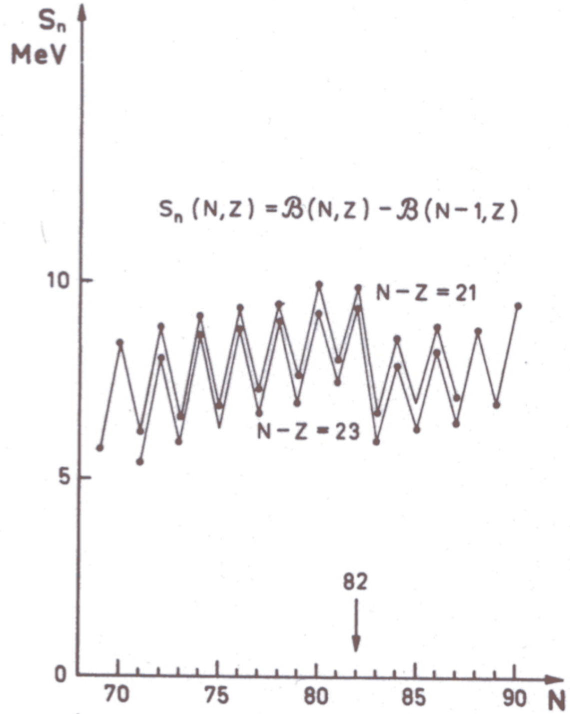
Discontinuities in variation nucleon separation energy with nucleon number.

Equally spaced levels. Smooth variation nucleon separation energy with nucleon number.

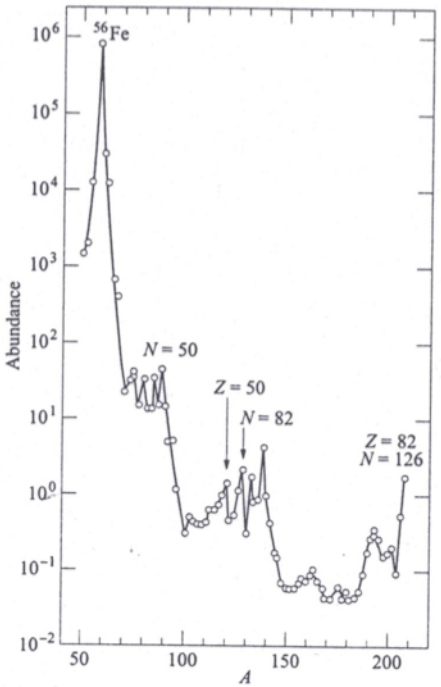
When N or Z corresponds to filling up to a gap, that nucleus is particularly stable wrt neighbours.

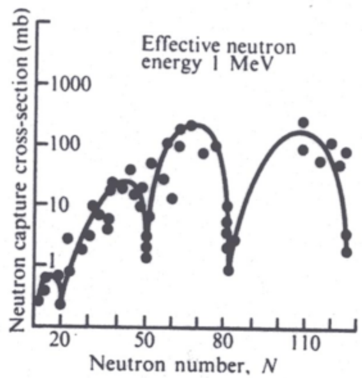
Evidence for “Magic Numbers”

(a) Masses, binding energies and separation energies: show discontinuities at certain nucleon numbers where *magic* numbers are associated with increased binding relative to their neighbours.



(b) Magic numbers show higher abundances compared to neighbouring nuclei.





(c) Reaction rates on nuclei with magic numbers show reluctance to capture neutrons.

(d) First excited states in magic are particularly high (example later).

(e) Effects are exaggerated for both Z and N magic, *doubly magic nuclei* such as ^{208}Pb (Z=82 N=126) and ^{40}Ca (Z=N=20).

(f) Similar numerology inherent in many nuclear properties.

(NEAR STABILITY) MAGIC NUMBERS: 2, 8, 20, 28, 50, 82 AND 126

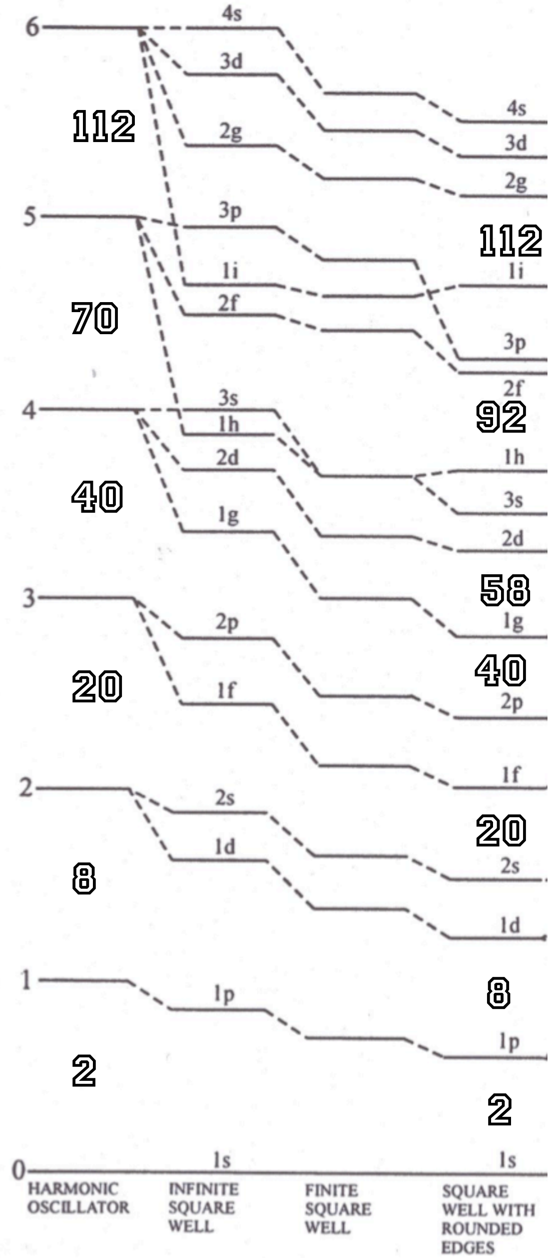
Solutions of the Schrödinger equation

$V_0 \approx$ 50 MeV and $r=1.2A^{1/3}$ fm.
Most wells give similar results.

n number of nodes in radial wave function, except at origin.
l orbital angular momentum.
m_l projection of orbital angular momentum on z axis.
Parity is given by $(-)^l$
Intrinsic spin s=½ with m_s=±½

Energies determined by *n* and *l*
(so far anyway…)

Filling according to Pauli:
each *l* value has $2l+1$ m_l-substates, combined with two intrinsic spin directions, giving a total degeneracy of $2(2l+1)$.



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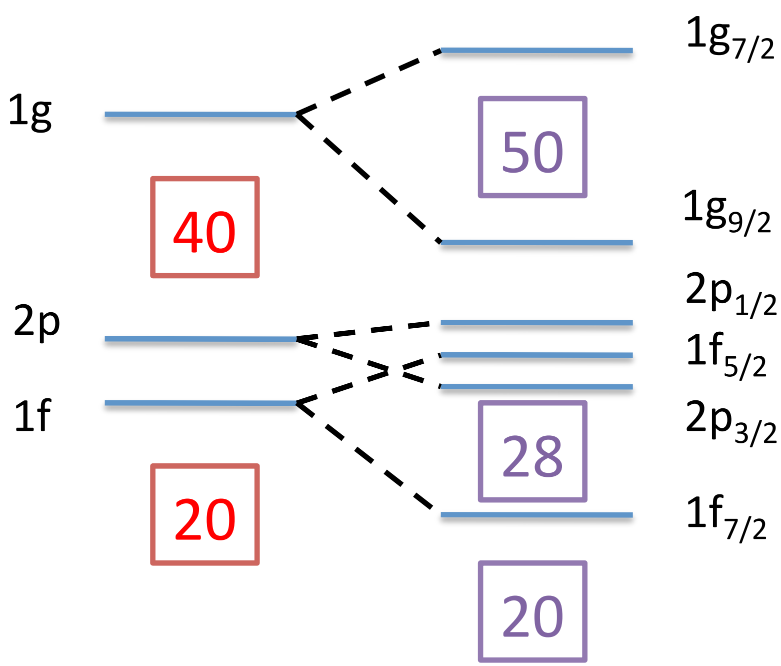
Spin-Orbit Force

If something is introduced which lifts the degeneracy in *j* in the right way, can reproduce the observed magic numbers…

Nobel Prize 1963: Hans Jensen and Maria Mayer



Adding a term of the form:

$$V_{LS} = -U_0 \frac{1}{r}\frac{\partial V}{\partial r} \underline{l}.\underline{s}$$

Does the job since:

$$\underline{l}.\underline{s} = \frac{\hbar^2}{2}\left[j(j+1) - l(l+1) - \frac{3}{4}\right]$$

Similar action to LS coupling in atoms, but atomic version is electromagnetic and NOT from strong force…and is the opposite sign!

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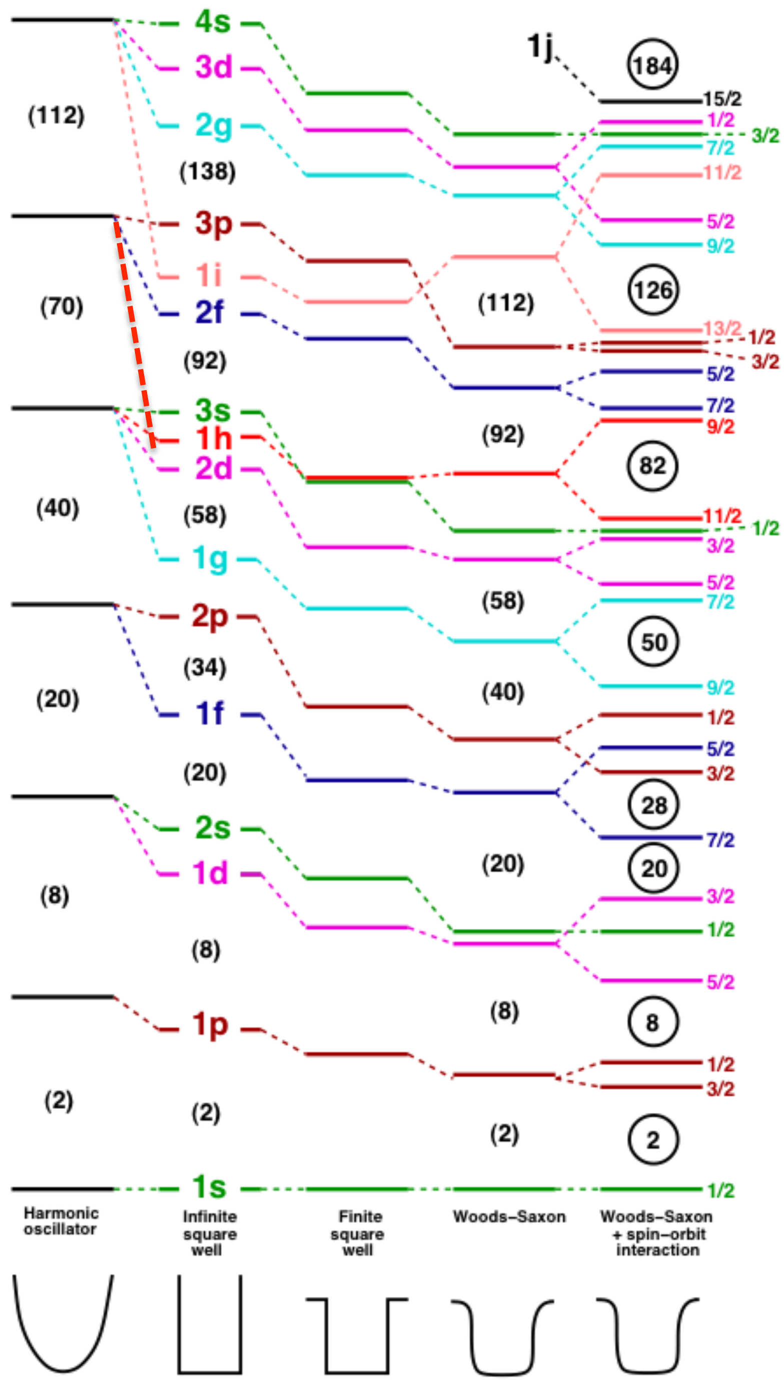
Single-Nucleon Levels

Notice general alternation of parities between shells…with the odd high-*j* *intruder* orbital.

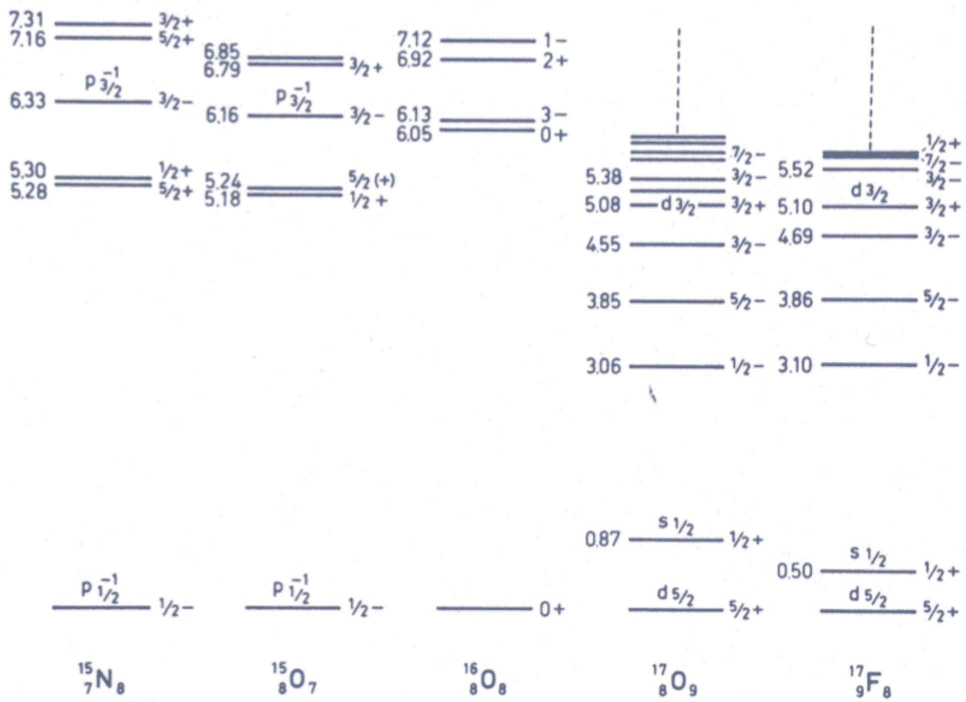
“Spuds of pug, dish of pig”

Gets you somewhere to remembering the order, although this isn’t expected!



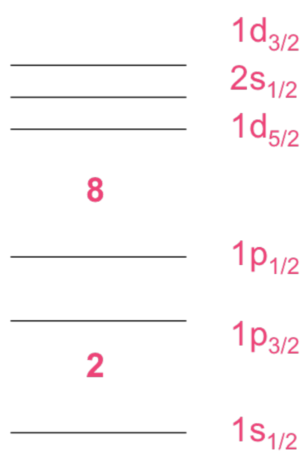
Excited States in Nuclei

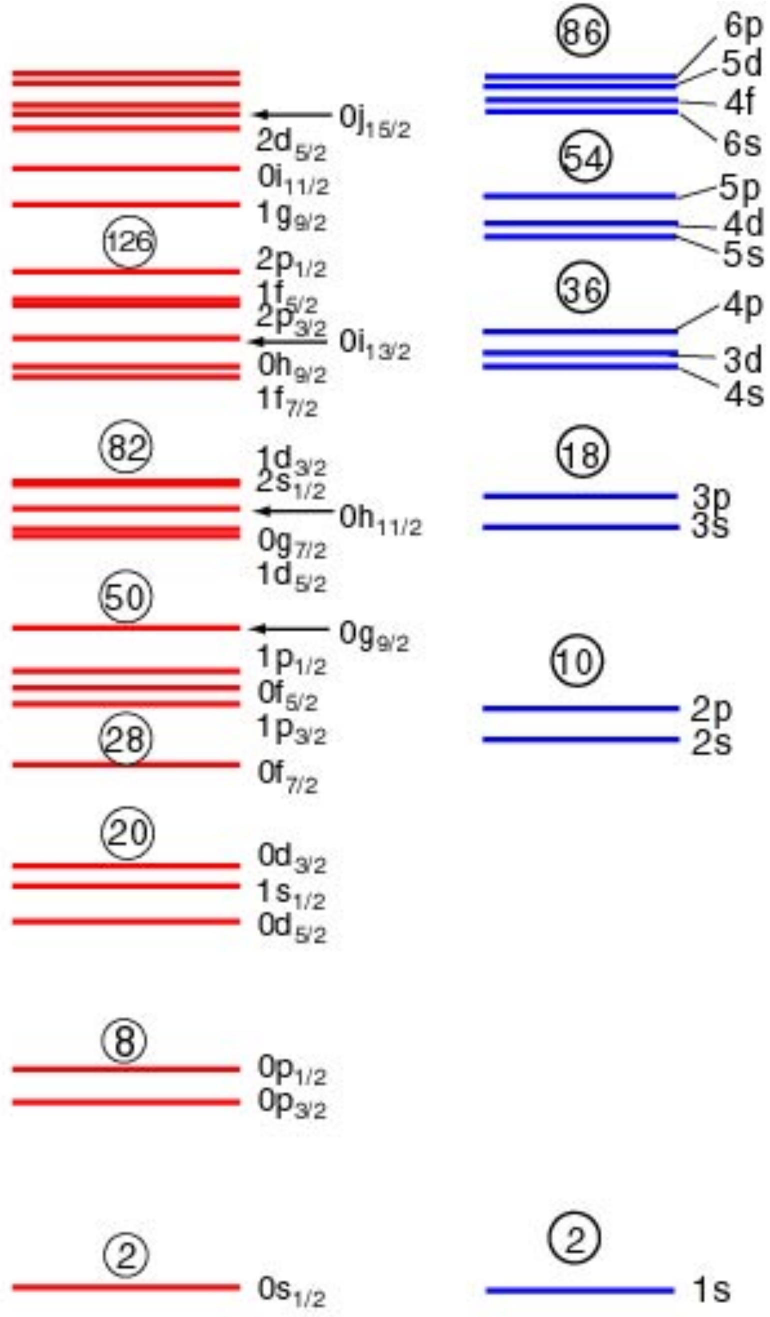
Some nuclear excited states can be understood in terms of promoting nucleons into higher-lying single-particle orbits:



Nuclear levels A≈16

Single-particle levels Z/N≈8





Shell Model of Nuclei

Shell Model of Atoms