PHYS30121 Introduction to Nuclear and Particle Physics

A Health Warning: Here are some summary notes to cover the nuclear part of the course. These notes ARE NOT comprehensive. They cover some definitions of easily confused quantities, bits of the course that are particularly wordy and some short notes on the more subtle aspects of the course. They need to be read in conjunction with lecture notes, for which they ARE NOT a substitute!

Nuclear Mass: The mass of the nucleus, usually in its ground state unless otherwise stated.

Atomic Mass: The mass of the whole atom, including the mass of the nucleus, all the atomic electrons and the total binding energy of the electrons to the nucleus. Atomic mass is the usual way that masses of different isotopes are quoted. If nuclear masses are required, the atomic mass need to be corrected for the mass of the electrons and the atomic binding energy, although much of the time the latter can be safely neglected. i.e. $m_{\text{nuclear}}(A,Z) = m_{\text{atomic}}(A,Z) - Z.m_{\text{electron}}$.

Binding Energy: The mass-energy associated with the forces that bind a composite system together. As binding forces act and pull together the constituents of the system, their potential energy is reduced and energy is released. The mass of the composite system is therefore lower than the total mass of the separate constituents before they are bound together; the binding energy is the difference between the two.

BE is the energy released when a system of separate items comes together and binds to form a composite object. The energy released in a reaction is just the difference between the mass before and the mass after. For a nucleus, the binding energy is given by:

$$BE(A,Z) = Z.m_{\text{proton}} + N.m_{\text{neutron}} - m_{\text{nuclear}}(A,Z)$$

$$= Z.m_{^1\text{H}} + N.m_{\text{neutron}} - m_{\text{atomic}}(A,Z).$$

All bound systems have binding energy, but for systems bound by the strong force the ratio of the binding energy to overall mass is higher than for electromagnetically bound systems due to the different inherent strengths of the two forces. Some old books might refer to the term *mass deficit* instead of using the words binding energy. This is somewhat old fashioned and be careful not to confuse it with mass defect, which is listed below.

Mass Excess: Another way to express atomic masses, to cut down the number of digits that need quoting:

$$\text{Mass Excess} = m_{\text{atomic}}(A, Z) - A$$

where the two terms on the right-hand side are given in the same units. *Usually you can substitute atomic mass excess for atomic mass in most formulae without worrying.*

Mass Defect: Yet another way to express atomic masses, to cut down the number of digits that need quoting:

$$\text{Mass Defect} = A - m_{\text{atomic}}(A, Z) = -\text{Mass Excess}.$$

This is just minus one times the mass excess, although thankfully the use of mass defect to write down atomic masses is falling out of use. Only mentioned here in case you see it in older textbooks!

Q Value: In a reaction or a decay process, the Q value is the amount of energy liberated. It can be calculated from the difference in the *nuclear* masses before and after the process.

For example, if $W + X \to Y + Z$:

$$Q = m_{\text{nuclear}}(W) + m_{\text{nuclear}}(X) - m_{\text{nuclear}}(Y) - m_{\text{nuclear}}(Z).$$

In such calculations, the nuclear masses can be substituted for atomic masses if the process in question does not destroy or liberate electrons; all the terms involving electron masses should then cancel out if the total number of atomic electrons does not change. If the process does change the overall electron number, e.g. β decay, it is safest to write down the Q value equation in nuclear masses first, then use the definition above to convert it into atomic masses, carefully accounting for the numbers of electrons in the atoms.

If a decay process has a positive Q value, it can happen spontaneously without violating energy conservation. Conversely, if a decay process has a negative Q value, it cannot happen. A reaction process with a positive Q value liberates energy. But a reaction process with a negative Q value requires the reacting nuclei to have sufficient kinetic energy in the CM frame for the reaction to occur; the energy required is called *the threshold energy* and is usually quoted as the positive number, $-Q$.

Average Binding Energy per Nucleon $\langle BE/A \rangle$: is a quantity that expresses how well, on average, the nucleons are bound together to form the nucleus. It is simply the binding energy of the nucleus divided by the total number of nucleons.

As a function of mass for nuclei along the line of beta stability, $\langle BE/A \rangle$, increases rapidly from the deuteron ($\langle BE/A \rangle = 1.11$ MeV/A) up to nuclei around mass $A > 10$. After this the variation in mass is much smoother and on average most nuclei fall in the range with $\langle BE/A \rangle \sim 7 - 8.5$ MeV/A. The tightest bound system is ^{62}Ni with $\langle BE/A \rangle = 8.794$ MeV/A, with lighter and heavier nuclei being less well bound; the binding falls off gradually away from this nucleus along the line of stability with increasing and decreasing mass. Moving away for the line of stability along a chain of isotopes with a constant mass, $\langle BE/A \rangle$ decreases approximately quadratically with Z.

Energy from Nuclei: Up to ^{62}Ni, the fusion of two light nuclei takes nucleons from systems with low $\langle BE/A \rangle$ to a fused system with higher $\langle BE/A \rangle$, so energy must be released in the process. Similarly, if a heavy nucleus splits into two fragments, down to ^{62}Ni, nucleons are taken from a single system with low $\langle BE/A \rangle$ to two systems with higher $\langle BE/A \rangle$. Again energy is released. This explains energy generation is fission and fusion.

Nucleon Separation Energies S_N: The proton/neutron separation energy $S_{p/n}$ is the minimum energy that is required to remove the *least* bound proton/neutron from a nucleus. If a nucleus has an excitation energy greater than $S_{p/n}$, it is energetically possible for it to spontaneously emit nucleons. As neutrons do not have a Coulomb barrier holding them in, neutron emission is usually the most common decay mechanism for nuclear levels above the separation energies. Nucleon emission is a process associated with the strong interaction and generally has a very high probability and occurs very quickly. Given the energy-time uncertainty principle, this implies that a level decaying quickly will have a high degree of uncertainty associated with its excitation energy; levels above the separation energy are said to have a relatively large level width. As the level density increases with excitation energy, a continuum of overlapping levels is found at high excitation energy in nuclei. Below the separation energy, excited states can only decay by electromagnetic (γ), weak (β) decay processes or α particle tunnelling through the Coulomb barrier, all with lower probability, longer lifetimes and therefore narrower widths. Their widths tend to be <1 eV and exist as discrete states,

Properties of Nuclear Forces:
(i) Nuclei exist as bound systems so an *attractive* force must act at short distances 1-2 fm to overcome the Coulomb repulsion of protons.
(ii) Nucleons retain their integrity in nuclei and don't melt, at normal temperatures and pressures, into a sea of more fundamental particles. The nuclear force must therefore become *repulsive at very short distances*, ~ 0.5 fm or less.

(iii) Charge symmetry: the similarity in the masses and excitation energies in mirror nuclei (after correcting for Coulomb-energy and proton-neutron mass differences) suggests that the strong interactions between a pair of protons and a pair of neutrons are the same.
(iv) Charge independence: the similarity in excitation energies of some states in an isobaric multiplet suggests that the interactions between two like nucleons is the same as that between two unlike nucleons.
(v) The lack of a bound L=0, S=0 state in the deuteron, given the L=0, S=1 ground state, suggests that nuclear forces depend on the orientation of the spins of an interacting pair of nucleons.
(vi) Consideration of the dipole and quadrupole moments of the deuteron suggest that the ground state is not pure L=0 and contains a small admixture of L=2. This indicates that nuclear forces are to a degree non-central and has a so-called tensor component.
(vi) The forward-backward asymmetry in neutron-proton scattering indicates that nuclear forces have an exchange character.

Other measurements, including the details of nucleon scattering experiments, confirm these and other features, which include (vii) a spin-orbit coupling (viii) dependence on the relative orbital angular momentum of the nucleons etc....

Exchange Forces: As an alternative to the concept of field, a force can be seen as being mediated by the exchange of so-called virtual particles.

For example, a pair of nucleons can be pictured as interacting by the exchange of virtual mesons. If one nucleon is to emit a meson with rest mass m, an energy of at least mc^2 is needed which violates energy conservation. This is fine as long as the energy is paid back within a time set by the energy-time uncertainty principle $\Delta E \Delta t \approx \hbar$. To remind us that this debt is to be repaid, the emitted particle is referred to as virtual. Substituting the rest mass for ΔE gives $\Delta t \approx \hbar / mc^2$. The impulse felt by the nucleon when emitting a meson is responsible for the force experienced. The second nucleon absorbs the exchanged meson, and again a force is connected to the impulse delivered.

In this time the virtual particle travels a distance $v\Delta t$, so the absolute farthest it can go would be if it travelled at the speed of light. The range of the force is then limited to $R \approx \hbar / mc$. For a range of 2 fm, this corresponds to a mass of around 200 times that of an electron. At this range, pions are a good candidate for the virtual particles, and since they come in charged and uncharged types, they can account for the exchange behaviour seen in neutron-proton scattering. However, the force between two nucleons is rather complicated because at smaller distances, mesons of higher masses can come into play. Multiple pion exchange is also possible. A detailed study can just about reproduce the Lennard-Jones type variation of the potential between two nucleons, although the hard core is difficult.

Mean-Field Theory and Shell Models: The nucleus is a difficult many-body quantum mechanical problem. A nucleus has anywhere between 1 and $\sim$300 constituent nucleons, of two types protons and neutrons, that all interact with each other via nuclear forces. The nuclear force acts between each pair of nucleons (two-body forces); but the presence of other nucleons can effect the interaction between a pair (so-called three-body forces and higher). The resulting equations form a complex coupled web due to the presence of two-body and higher forces. In order to simplify this problem, an approximation is often made by introducing a so-called mean-field potential. This is a potential that represents the average effects of all the nucleon-nucleon forces on a particular nucleon, such that its potential energy just depends on its own coordinates, i.e. where it is in the nuclear volume. The mean-field is therefore a one-body potential (as opposed to the full nucleon-nucleon interactions that depend on both the coordinates of the nucleon of interest and the coordinates of all the other nucleons) and this drastically simplifies any calculation.

If the mean field is a good approximation, then the Schrödinger equation for the whole nucleus separates out into Hamiltonians for each individual nucleon; all down to the fact that the potential is a one-body operator. The energy of the whole nucleus is a sum of the nucleon energies, and the nuclear wave function is a product of the nucleon wave function. (Technically this product needs to be anti-symmetrised to obey Pauli's principle and the nucleon wave functions need coupling to give the correct overall angular momentum, details that shouldn't worry us for now). The individual nucleon Hamiltonian can be solved to produce a set of single-particle energy levels. A nucleus is built up by adding nucleons into these levels according to the Pauli principle, gradually filling the orbitals. (for details see lecture notes).

Where a particular nucleus has sufficient nucleons of one particular type to fully fill the single-particle levels up to a gap in the level sequence, the nucleus has a degree of stability with respect to neighbouring nuclei. It is called a magic nucleus. Doubly magic nuclei have both types of nucleon filled up to a gap. The corresponding nucleon numbers are called magic numbers. Experimentally these are found to be 2, 8 , 20, 28, 50, 82 and 126, at least close to the line of stability, where evidence comes from a number of sources (see lecture notes). The mean-field potential should look like the density distribution in a nucleus, given that the nucleon-nucleon force is short ranged, and ends up rather like a smoothed finite square well. But when calculations are done with such a potential, only the first few magic numbers are reproduced. A spin-orbit force needs adding to reproduce the observed magic numbers (see lecture notes).

Nuclear Instability and Decay: If a process is not forbidden by some conservation law and a mechanism exists, it will generally happen with a certain probability λ per second. Generally, the most important conservation law is that of energy and for a decay process to happen spontaneously, $Q > 0$. For a decay from radioactive initial state to a stable final state, a simple exponential decay law holds for the number of nuclei in the radioactive state: $N = N_0 e^{-\lambda t}$. The mean life is given by $\tau = \frac{1}{\lambda}$ and the half life by $\tau_{1/2} = \frac{\ln 2}{\lambda}$. Timescales vary greatly, for example, for neutron emission $\tau \sim 10^{-23}$ s, whereas for double β decay $\tau > 10^{21}$ s. Given that the actual time a particular radioactive state will exist for is uncertain, any decaying state has an uncertainty in energy, often referred to as the width of the state, according to the energy-time uncertainty principle $\Delta E.\Delta t \sim \hbar$. For example, a stable state, $\tau \to \infty$ has a level width $\Gamma \to 0$, but a neutron decaying state with $\tau \sim 10^{-23}$ has a width $\Gamma \sim 100$ keV.

Often a particular excited state can decay in more than one way, each with a particular probability. The individual probabilities add up to give the total decay probability for the state $\lambda = \lambda_1 + \lambda_2 + ...$ and this determines the lifetime of the state, $\tau = \frac{1}{\lambda}$. The relative intensity of the different decay paths is known as the branching ratio $\mathrm{BR} = I_1/I = \lambda_1/\lambda$.

Alpha Decay: This is the emission of a pre-formed ^{4}He nucleus. The Q values are influenced by the increase in the binding energy, by the reduction in atomic number lessening the overall Coulomb repulsion between protons, and by the relatively large binding energy of the α particle. Both conspire to produce large Q values, even when those for the emission of other light particles, such as protons and neutrons, are negative. The energy released is shared between the α particle and the recoiling daughter nucleus according to two-body kinematics, $T_\alpha = Q.m_R/(m_R + m_\alpha)$. Alpha decay usually happens between ground states of the parent and daughter, but sometimes the decay can feed low-lying states in the daughter (so-called α fine structure). Some excited states in the parent may alpha decay, giving higher energy α particles than those from the ground state (so-called long-range α particles).

There is a very strong dependence of the lifetime on the Q value of the decay, which is approximately of the form of the Geiger-Nuthall rules $\ln \tau \propto 1/\sqrt{Q}$. The semi-empirical mass formula predicts $Q_\alpha > 0$ for masses larger than $A = 150$, although α decay is rarely seen until $A = 200$. Both of these features arise from the decay mechanism which involves the α particle quantum mechanically tunnelling out of the nucleus through the Coulomb barrier (see lectures notes for details). The presence of the centrifugal barrier for alphas with $L > 0$ explains why the most probable decays are via $L = 0$ corresponding to no spin change between parent and daughter. $L > 0$ decays can happen, as long as the change in the spins of the states involved can coupled to L (angular momentum conservation), but with lower probability.

Beta Decay: This is a generic term for three different decay processes that involve the creation or destruction of electrons via a weak interaction.

In β^- decay, a neutron in the nucleus turns into a proton with the concurrent emission of an electron and a anti-neutrino, increasing the atomic number of the parent:

$$^A_ZX \rightarrow \ ^A_{Z+1}Y + e^- + \bar{\nu}$$

In β^+ decay, a proton in the nucleus turns into a neutron with the concurrent emission of an positron and a neutrino, decreasing the atomic number of the parent.

$$^A_ZX \rightarrow \ ^A_{Z-1}Y + e^+ + \nu$$

In a third process called atomic electron capture (EC), the nucleus swallows an atomic electron, which combines with a proton to form a neutron and a neutrino. This leaves a vacancy in the atomic electron shells and subsequent atomic electron rearrangement produces X-rays. Again, decreasing the atomic number of the parent.

$$^A_ZX + e^- \rightarrow \ ^A_{Z-1}Y + \nu$$

Notice that in all these processes, the mass number A remains the same.

Emitted $\beta^{\pm}$ particles have a continuous energy spectrum up to a limit called the endpoint, which is usually of the order of 1 MeV. This is because the energy liberated in the decay is shared between three final particles. Compare this to alpha decay, where the energy is shared between two final particles and the application of energy and momentum conservation fixes the energy of each (formula given above). In beta decay, there are three unknown energies of the final particles and only two conditions, the conservation laws for energy and momentum. So this is insufficient to fix the final energies to unique values resulting in a continuous spectrum. The continuous nature of the spectrum was used as evidence for the presence of the neutrinos in the decay. For a while people thought that the β particles were pre-formed in the nucleus in a fashion similar to alpha decay. However, the position-momentum uncertainty principle suggests that electrons confined in a space of order 5 fm would have momentum up to 40 MeV, inconsistent with the endpoints of around 1 MeV. Therefore, unlike α particles, the emitted electrons are created in the decay mechanism. Electron capture is different in that the final state has two particles, the recoiling nucleus and the neutrino, so these do have definite energies.

In terms of $atomic$ masses, the Q values of these three processes, for a parent x and daughter y, are given by:

$$Q_{\beta^-} = m_x - m_y$$

$$Q_{\beta+} = m_x - m_y - 2m_e$$

$$Q_{EC} = m_x - m_y - B_{\text{atomic vacancy}}.$$

Make sure you can derive these from the definition of the Q value i.e. the difference in the sum of the NUCLEAR *masses before and after.*

Both EC and β^+ decay increase the Z of the decaying nucleus. By considering the Q values above, $Q_{EC} > Q_{\beta+}$. If $Q_{\beta+} > 0$, both EC and β^+ decay can occur spontaneously. But you can get situations when $Q_{EC} > 0$, but $Q_{\beta+} < 0$, so EC occurs alone. You can also prevent EC from occurring by stripping off all the atomic electrons to form a highly charged ion, then there are no electrons to capture!

The probability of β decay turns out to be sensitive to the Q value according to $\tau \propto 1/Q^5$, reflecting a different underlying mechanism to alpha decay (see S2 lectures on nuclear physics). Beta decay can populate excited states in the daughter nucleus. If any of the populated states lie at energies higher than the neutron separation energy, neutron emission will occur very quickly in a process known as β-delayed neutron emission, reducing the mass number by one unit.

The semi-empirical mass formula for a constant A has a parabolic form in Z. Beta decay transports a nucleus down the parabola, reducing the nuclear masses, until the bottom of the parabola is reached, which corresponds to β-stable nuclei.

For odd-A, there is one parabola and therefore usually beta decays from either side of the parabola will ultimately feed into one single nucleus near the minimum mass. This explains why most odd-A isobars have only one β-stable species.

For even-A systems, the pairing term offsets a parabola corresponding to odd-odd nuclei from a lower parabola of even-even nuclei. Beta decays between neighbouring isobars correspond to hopping between these two parabolas. At the bottom of the odd-odd parabola, the most stable odd-odd nucleus is still above the bottom of the even-even parabola, and therefore usually has positive Q values to decay, either by β^- or β^+/EC, to the even-even parabola, feeding two different even-even nuclei. Given that normal β decay will only change Z by one unit, these two even-even nuclei have nowhere to decay two, even though one of them may have a greater mass than the other. Therefore, there are often two β-stable nuclei in an even-even isobar.

In lighter nuclei, pairing is often not well developed and the two parabolas are not well separated. Due to the lack of separation of the two parabolas, the odd-odd nucleus may well be low enough in mass for it to be impossible to decay to the even-even neighbours. It is therefore only in the lightest masses that a small number of stable odd-odd systems exist.

There is a process known as double-beta decay that can allow one of the two "stable" even-even isobars to decay into the other. One type of double beta decay has been observed, that essentially is two normal β^- decays happening simultaneously. The chances of two decays happening at the same time is small, and so the process has a small probability and a very long half life, so long that it is very difficult to observe and unless you make special efforts the nuclei appear stable. A more mysterious process called neutrinoless double beta decay has been postulated to occur, where two electrons are emitted, but no neutrinos. This has not yet been observed (well there is one claim but it is not widely accepted) but may be the key to measuring the mass of the neutrino.

Gamma Decay: Both alpha and beta decay, as well as nuclear reactions, can populate excited states in nuclei. Below the nucleon separation energies, the only option for decay is via electromagnetic transitions to lower-lying states, with a cascade of γ rays ultimately leading to the ground state. The transition probability for gamma decay depends on the transition energy and the spin changes between the initial and final levels, as well as the nuclear structure of the two states. Lifetimes generally fall in the range of ns to fs, although sometimes large spin changes, small transition energies or big differences in nuclear structure can conspire to give unusually large lifetimes, producing metastable states or nuclear isomers. The definition of an isomer is rather loose, but anything longer than a few 10s of nanoseconds might be referred to as an isomer. Some isomers have half-lives of tens of years! The more typical lifetimes, ns to fs, correspond to level widths less than 1eV, so below the separation energies excited states appear as discrete levels. (See above for a discussion of the continuum above S_n.)

Nuclear Sizes: One important method to measure nuclear sizes is to elastically scatter particles from the nuclei of interest. The measured angular distributions of the outgoing particles are compared to predictions assuming a point-like nucleus and from the comparison the nuclear size can be determined, as well as the charge/matter distribution within the nucleus.

Projectiles that only interact via the Coulomb force are used to determine the charge distributions and electron scattering has been a particularly important method. Projectiles such as neutrons can be used to determine the matter distribution as they interact with both protons and neutrons via the strong force only. Projectiles that interact with both interactions, such as alpha particles, are more complicated.

The details of the reaction process need analysing carefully using quantum mechanics, but there are strong similarities with optics and diffraction. Diffraction limits the resolution of an optical system; the wavelength must be shorter than the distance scale to be resolved. In quantum mechanics, $\lambda = \hbar/p$, so essentially particles higher than a particular momentum or energy are required. For example, to resolve scales of 1 fm, momentum greater than $\hbar/1\,\mathrm{fm} = 197\,\mathrm{MeV/c}$

are required; for electrons, this corresponds to an energy of $E = \sqrt{c^2p^2 + m^2c^4} \sim cp = 197\ \mathrm{MeV/c^2}$. (NB: Electrons at this energy are relativistic, a modification to Rutherford scattering is needed, so-called Mott scattering).

The results of many experiments suggest that the charge and mass distribution in a nucleus follows a so-called Woods-Saxon form:

$$\rho = \rho_0 \left[1 + \exp\left(\frac{r - R}{a}\right)\right]^{-1}$$

where R is the radius at which the density falls to half its central value ρ_0 and a is the diffuseness. The density changes from 90% of ρ_0 to 10% in a distance $4.4a$. For the charge density in β-stable systems $R = 1.18A^{1/3} - 0.48$ fm and $a = 0.55$ fm. For the matter distribution, the results are consistent with $R = 1.2A^{1/3}$ fm, which is often taken as a good estimate for the nuclear radius.