ONE HOUR THIRTY MINUTES

A list of constants is enclosed.

UNIVERSITY OF MANCHESTER

Introduction to Nuclear and Particle Physics

18th January 2011, 2.00 p.m. - 3.30 p.m.

Answer **ALL** parts of question 1, **ONE** from Section A and **ONE** from Section B

Electronic calculators may be used, provided that they cannot store text.

The numbers are given as a guide to the relative weights of the different parts of each question.

P.T.O.

1. (a) Write down an expression that represents the systematic variation of nuclear radius with mass number A for β-stable nuclei and use it to calculate the radius of an ^{27}Al nucleus.

[5 marks]

(b) Give an expression relating nuclear mass $m^{\mathrm{N}}(A, Z)$ to atomic mass $m(A, Z)$.
Show that the Q value for the β^+ decay process $A \rightarrow B + \beta^+ + \nu_e$ can be written as:

$$Q = m_A - m_B - 2m_e$$

where m_A and m_B are the *atomic* masses of A and B and m_e is the mass of an electron. State clearly any approximations that are made.

[8 marks]

(c) Explain qualitatively, with the aid of quark flow diagrams, why the charged pions have a lifetime of $\sim 10^{-8}$ s whereas the lifetime of the neutral pion is $\sim 10^{-16}$ s.

[6 marks]

(d) The particles A and B can be produced in the following strong interaction processes:

$$K^- p \rightarrow \pi^0 A$$
$$K^- p \rightarrow K^+ B.$$

Deduce the charge, baryon, strangeness and charm quantum numbers of the A and B particles, and hence their quark constituents.
The K^- has quark constituents $(s, \bar{u})$.

[6 marks]

P.T.O.

SECTION A

Answer either Question A2 or Question A3

A2. Explain what is meant by *nuclear binding energy.*

[3 marks]

Calculate the binding energy *per nucleon* of a $^{62}_{28}$Ni nucleus using the following atomic mass excesses: $\Delta(^{62}\mathrm{Ni})$ = -66746.1, $\Delta(^{1}\mathrm{H})$=7289.0 and $\Delta(\mathrm{n})$=8071.3 keV.

[5 marks]

Draw a diagram indicating the systematic behaviour of the binding energy per nucleon of β-stable nuclei as a function of mass number A. Carefully annotate the axes in a quantitative fashion. Thus explain how energy is generated when: (a) a single heavy nucleus fissions and (b) two light nuclei fuse.

[10 marks]

Sketch a plot showing the variation of nuclear mass with atomic number Z across a sequence of nuclei with the same *even* mass number A. Use your plot to explain why there are often two β-stable isotopes in an even-A isobar.

[5 marks]

There are only four stable odd-odd nuclei: ^{2}H, ^{6}Li, ^{10}B and ^{14}N. What can be concluded about the effect of pairing in light nuclei?

[2 marks]

P.T.O.

A3. Describe one experimental observation that provides evidence for shell effects in atomic nuclei. [4 marks]

A spin-orbit term with a form proportional to **l.s** is introduced into the single-particle Hamiltonian to reproduce the observed sequence of magic numbers. Show that the expectation of the **l.s** operator can be written as:

$$\langle \mathbf{l}.\mathbf{s} \rangle = \frac{\hbar^2}{2}\left[j(j+1) - l(l+1) - 3/4\right].$$

[6 marks]

In a simple three-dimensional potential well, the first three energy levels have quantum numbers $1s$, $1p$ and $1d$ in order of increasing energy. Explain what is meant by this notation.

[4 marks]

Draw an energy level diagram to show the states produced from these levels when the spin-orbit interaction is introduced, labelling the levels with appropriate quantum numbers.

[5 marks]

Use your answers above to explain why the spin and parity of the ground states of ${}^{3}_{2}\mathrm{He}$ and ${}^{13}_{6}\mathrm{C}$ are $1/2^{+}$ and $1/2^{-}$ respectively.

[6 marks]

P.T.O.

SECTION B

Answer either Question B4 or Question B5

B4. Briefly describe why a particle such as the neutrino was needed to explain the process of β decay.

[7 marks]

Draw a quark flow diagram for the β decay of a free neutron.

[4 marks]

Why was the experimental discovery of the neutrino so difficult, and how was this eventually achieved?

[5 marks]

Explain the concept of neutrino type.

[6 marks]

What phenomenon associated with neutrinos would lead to the non-conservation of lepton flavour, and what properties would the different types of neutrino need to have for this to occur?

[3 marks]

P.T.O.

B5. Show how the simple 3-flavour quark model can be used to explain the J=1, L=0 meson nonet. You do not need to identify the names of particles in this nonet.

[6 marks]

The ρ^-, ρ^0, ρ^+, ω^0 and ϕ^0 mesons are the non-strange members of this nonet. Deduce the quark content of each of these mesons.

[7 marks]

Draw quark flow diagrams for the most likely decay channel for the ρ^0 and ϕ^0 mesons.

[8 marks]

Give an estimate of the lifetime and natural width of the ρ^0.

[4 marks]

The following particle masses may be useful:
$\rho \approx$780 MeV/c^2, $\omega \approx$780 MeV/c^2, $\phi \approx$1020 MeV/c^2, $\pi \approx$140 MeV/c^2, $K \approx$490 MeV/c^2.

END OF EXAMINATION PAPER

PHYSICAL CONSTANTS AND CONVERSION FACTORS

SYMBOL	DESCRIPTION	NUMERICAL VALUE
c	Velocity of light in vacuum	$299\,792\,458$ m s^{-1}, exactly
μ_0	Permeability of vacuum	$4\pi \times 10^{-7}$ N A^{-2}, exactly
ϵ_0	Permittivity of vacuum where $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$	8.854×10^{-12} C^2 N^{-1} m^{-2}
h	Planck constant	6.626×10^{-34} J s
$\hbar$	$h/2\pi$	1.055×10^{-34} J s
G	Gravitational constant	6.674×10^{-11} m^3 kg^{-1} s^{-2}
e	Elementary charge	1.602×10^{-19} C
eV	Electronvolt	1.602×10^{-19} J
α	Fine-structure constant, $\frac{e^2}{4\pi\epsilon_0 \hbar c}$	$\frac{1}{137.0}$
m_e	Electron mass	9.109×10^{-31} kg
$m_e c^2$	Electron rest-mass energy	0.511 MeV
μ_B	Bohr magneton, $\frac{e\hbar}{2m_e}$	9.274×10^{-24} J T^{-1}
R_∞	Rydberg energy $\frac{\alpha^2 m_e c^2}{2}$	13.61 eV
a_0	Bohr radius $\frac{1}{\alpha}\frac{\hbar}{m_e c}$	0.5292×10^{-10} m
Å	Angstrom	10^{-10} m
m_p	Proton mass	1.673×10^{-27} kg
$m_p c^2$	Proton rest-mass energy	938.272 MeV
$m_n c^2$	Neutron rest-mass energy	939.565 MeV
μ_N	Nuclear magneton, $\frac{e\hbar}{2m_p}$	5.051×10^{-27} J T^{-1}
fm	Femtometre or fermi	10^{-15} m
b	Barn	10^{-28} m^2
u	Atomic mass unit, $\frac{1}{12} m(^{12}\text{C atom})$	1.661×10^{-27} kg
N_A	Avogadro constant, atoms in gram mol	6.022×10^{23} mol^{-1}
T_t	Triple-point temperature	273.16 K, exactly
k	Boltzmann constant	1.381×10^{-23} J K^{-1}
R	Molar gas constant, $N_A k$	8.314 J mol^{-1} K^{-1}
σ	Stefan-Boltzmann constant, $\frac{\pi^2}{60}\frac{k^4}{\hbar^3 c^2}$	5.670×10^{-8} W m^{-2} K^{-4}
M_E	Mass of Earth	5.97×10^{24} kg
R_E	Mean radius of Earth	6.4×10^6 m
g	Standard acceleration of gravity	$9.806\,65$ m s^{-2}, exactly
atm	Standard atmosphere	101 325 Pa, exactly
$M_\odot$	Solar mass	1.989×10^{30} kg
$R_\odot$	Solar radius	6.96×10^8 m
$L_\odot$	Solar luminosity	3.84×10^{26} W
$T_\odot$	Solar effective temperature	5.8×10^3 K
AU	Astronomical unit, mean Earth-Sun distance	1.496×10^{11} m
pc	Parsec	3.086×10^{16} m
	Year	3.156×10^7 s