PHYS20302

<u>ONE HOUR THIRTY MINUTES</u>

A list of constants is enclosed.

UNIVERSITY OF MANCHESTER

Atoms and Nuclei

25th May 2007, 9.45 a.m. - 11.15 a.m.

Answer **<u>ALL</u>** *parts of question 1 and* **<u>TWO</u>** *other questions*

Electronic calculators may be used, provided that they cannot store text.

The numbers are given as a guide to the relative weights of the different parts of each question.

P.T.O.

$\mu_B = 5.8 \cdot 10^{-5}$ eV/T
$hc = 1240$ eV nm
$a_V = 15.9$ MeV
$a_S = 18.3$ MeV
$a_C = 0.72$ MeV
$a_A = 92.8$ MeV
$a_P = 11.5$ MeV

1. (a) Sketch the dependence of the binding energy per nucleon as a function of the mass number A. Label both axes with approximate scales. Indicate the parts of the graph where you expect to find elements that can undergo fusion and that can undergo fission. [5 marks]

(b) Use the angular momentum operator

$$\hat{\ell}_z = i\hbar \frac{\partial}{\partial \phi}$$

and the condition $\Phi(\phi) = \Phi(\phi + 2\pi)$ for the wavefunction $\Phi(\phi)$ to show that $m\hbar$ are the eigenvalues of the z component of the angular momentum with $m = 0, \pm 1, \pm 2..$ and $|m| \leq \ell$. [5 marks]

(c) The only bound state of the deuteron is an s state with total angular momentum $J = 1$. Use this information to explain why bound neutron-neutron and proton-proton systems do not exist. [5 marks]

(d) A bottle of iodine-131 with a half-life of 8 days has a measured activity A of 400 MBq. After how many days will the activity have reached 0.4 MBq ? [5 marks]

(e) With the aid of a diagram, briefly explain the Franck-Hertz experiment and its interpretation. [5 marks]

P.T.O.

2. (a) Describe the difference between the ordinary and the anomalous Zeeman effects. Which is observed for the $^1D_2 -^1 P_1$ transition in cadmium ? [5 marks]

(b) For cadmium, sketch how many spectral lines are observed in the following cases:

(i) without magnetic field;

(ii) with magnetic field in the direction parallel to the observer;

(iii) with magnetic field in the direction transverse to the observer. [4 marks]

(c) Sketch and label the energy levels with and without a magnetic field and indicate the allowed transitions. Briefly explain why certain transitions are not allowed. [5 marks]

(d) An energy splitting of 1.74×10^{-5} eV is observed. Determine the magnetic field strength, B. [5 marks]

(e) In the Stern-Gerlach experiment, a beam of silver atoms is split into two components and the z component of the magnetic moment is measured to be $\mu_z \approx \mu_B$, where μ_B is the Bohr magneton. Use these two observations to derive the value of the gyromagnetic ratio of the electron. [6 marks]

P.T.O.

3. (a) State the total degeneracy of the $n = 3$ level of hydrogen. where n is the primary quantum number, assuming there is no fine structure (FS) or hyperfine structure (HFS). Briefly explain the physical meaning of degeneracy. [4 marks]

(b) Calculate the energy and wavelength corresponding to the $3s - 2p$ spectral line of hydrogen, neglecting FS and HFS. [4 marks]

(c) List the two main effects that contribute to the fine structure of hydrogen. [4 marks]

(d) Sketch the FS splitting of the $3s - 2p$ energy levels in hydrogen. [3 marks]

(e) The FS energy shift is given by

$$E_{\ell s} = \frac{a}{\hbar^2} \langle \hat{\ell} \cdot \hat{s} \rangle$$

with $a = 3 \times 10^{-5}$ eV.

Find $\hat{l} \cdot \hat{s}$ in terms of the eigenvalues of $\hat{j}^2, \hat{l}^2$ and $\hat{s}^2$.

Hence, calculate the size of the FS splitting for the $3s - 2p$ spectral line of hydrogen (in eV). [6 marks]

(f) Muonic hydrogen consists of a proton and a muon. The muon has the same properties as the electron but with a mass $m_\mu = 106$ MeV$/c^2$. Calculate the energy of the $3s - 2p$ spectral line for muonic hydrogen, neglecting FS and HFS. [4 marks]

P.T.O.

4. (a) The binding energy term of the semi-empirical mass formula can be written as

$$B(A,Z) = a_V A - a_S A^{2/3} - a_C Z(Z-1)A^{-1/3} - a_A (Z - A/2)^2 A^{-1} \pm a_P A^{-1/2} \ .$$

Explain the physical origin of each term, including the A and Z dependence of the first four terms. [10 marks]

(b) Use the semi-empirical mass formula to show that β^- decay of $^{128}_{52}\text{Te}$ is not allowed. [9 marks]

(b) For fixed $A = 238$ determine the value of Z and the element which corresponds to the maximum binding energy $B(A,Z)$. [6 marks]

END OF EXAMINATION PAPER

PHYSICAL CONSTANTS AND CONVERSION FACTORS

SYMBOL	DESCRIPTION	NUMERICAL VALUE
c	Velocity of light in vacuum	299 792 458 m s^{-1}, exactly
μ_0	Permeability of vacuum	$4\pi \times 10^{-7}$ N A^{-2}, exactly
ϵ_0	Permittivity of vacuum	8.854×10^{-12} C^2 N^{-1} m^{-2}
	where $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$	
h	Planck constant	6.626×10^{-34} J s
$\hbar$	$h/2\pi$	1.055×10^{-34} J s
G	Gravitational constant	6.674×10^{-11} m^3 kg^{-1} s^{-2}
e	Elementary charge	1.602×10^{-19} C
eV	Electronvolt	1.602×10^{-19} J
α	Fine-structure constant, $\frac{e^2}{4\pi\epsilon_0 \hbar c}$	$\frac{1}{137.0}$
m_e	Electron mass	9.109×10^{-31} kg
$m_e c^2$	Electron rest-mass energy	0.511 MeV
μ_B	Bohr magneton, $\frac{e\hbar}{2m_e}$	9.274×10^{-24} J T^{-1}
R_∞	Rydberg energy $\frac{\alpha^2 m_e c^2}{2}$	13.61 eV
a_0	Bohr radius $\frac{1}{\alpha}\frac{\hbar}{m_e c}$	0.5292×10^{-10} m
Å	Angstrom	10^{-10} m
m_p	Proton mass	1.673×10^{-27} kg
$m_p c^2$	Proton rest-mass energy	938.272 MeV
$m_n c^2$	Neutron rest-mass energy	939.566 MeV
μ_N	Nuclear magneton, $\frac{e\hbar}{2m_p}$	5.051×10^{-27} J T^{-1}
fm	Femtometre or fermi	10^{-15} m
b	Barn	10^{-28} m^2
u	Atomic mass unit, $\frac{1}{12} m(^{12}\text{C atom})$	1.661×10^{-27} kg
N_A	Avogadro constant, atoms in gram mol	6.022×10^{23} mol^{-1}
T_t	Triple-point temperature	273.16 K
k	Boltzmann constant	1.381×10^{-23} J K^{-1}
R	Molar gas constant, $N_A k$	8.315 J mol^{-1} K^{-1}
σ	Stefan-Boltzmann constant, $\frac{\pi^2}{60}\frac{k^4}{\hbar^3 c^2}$	5.670×10^{-8} W m^{-2} K^{-4}
M_E	Mass of Earth	5.97×10^{24} kg
R_E	Mean radius of Earth	6.4×10^6 m
g	Standard acceleration of gravity	9.806 65 m s^{-2}, exactly
atm	Standard atmosphere	101 325 Pa, exactly
$M_\odot$	Solar mass	1.989×10^{30} kg
$R_\odot$	Solar radius	6.961×10^8 m
$L_\odot$	Solar luminosity	3.846×10^{26} W
$T_\odot$	Solar effective temperature	5800 K
AU	Astronomical unit, mean Earth-Sun distance	1.496×10^{11} m
pc	Parsec	3.086×10^{16} m
	Year	3.156×10^7 s