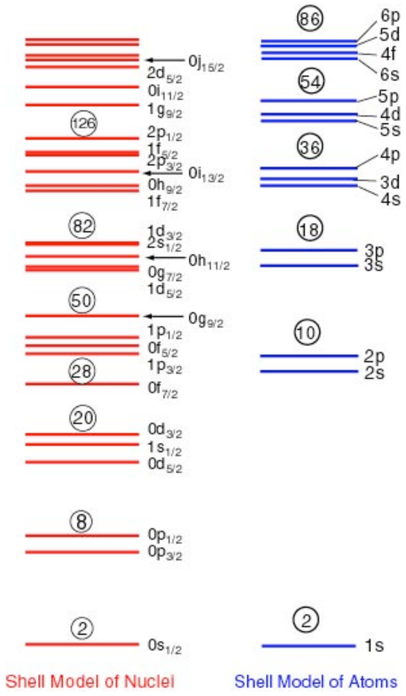
PC30121: Introduction to Nuclear and Particle Physics

Lecture 6: Shell Structure and Single-Particle Models

Theoretical approaches to nuclei:
  liquid-drop model (SEMF)
  ab-initio techniques (the deuteron)
Going to add "mean-field theory".

Shell structure is a common phenomenon in many-body systems:
- nucleons in nuclei
- electrons in atoms
- electrons in conductors (bands!)
- atoms in atomic clusters



Mean-Field Potential

Ab-initio Models: write down the Schrödinger equation for each nucleon including interactions with all other nucleons using nucleon-nucleon interaction.

(Too) Many equations, each one depends on co-ordinates of all the other nucleons.

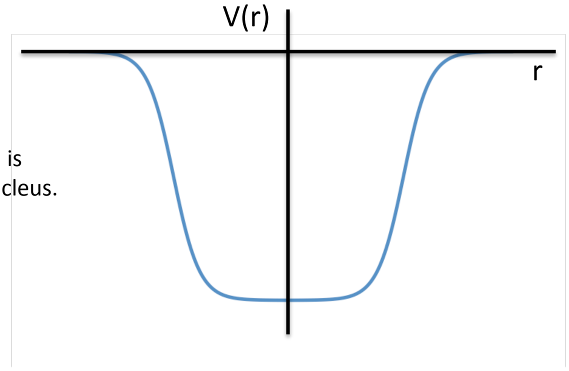
Mean-field Models: Assume that interactions with all other nucleons averages out to some potential that only depends the coordinates of the nucleon in question.

Shape guided by knowledge of an attractive short-ranged force and the variation in nuclear density.

Inside nuclear volume, roughly constant density either side…
….no resultant force and constant potential.

Near surface a nucleon is pulled back into the nucleus.

Add Coulomb potential if dealing with proton.



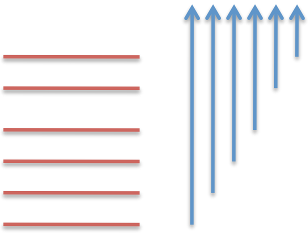
Typical 3D Potential Problem for Fermions

- Write down Schrödinger equation with *V(r)*; 3D partial differential equation.
- Separate into radial and angular equations.
- (Mainly) central field: angular equation is the eigenvalue equation for orbital angular momentum.
- Radial equation contains kinetic, potential and centrifugal terms.
- Solutions will give energy levels in the well.
- To build up whole *nucleus*, add nucleons into levels according to Pauli principle.

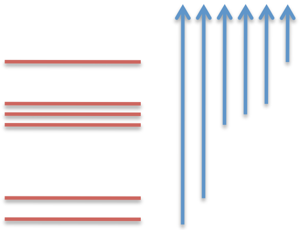
Gaps in any level structure are associated with discontinuities or jumps in properties:

E.g. nucleon separation energy (energy to remove least bound nucleon) as a function of *N* or *Z*

Gradually fill up energy levels with increasing *N* or *Z*



Equally spaced levels. Smooth variation nucleon separation energy with nucleon number.

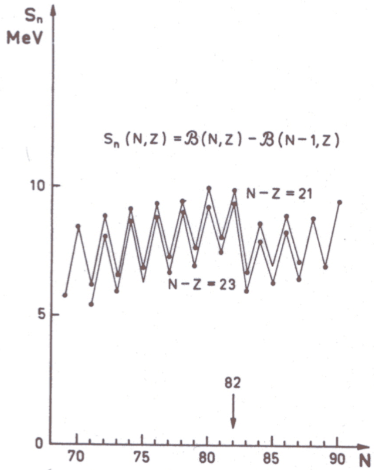


Discontinuities in variation nucleon separation energy with nucleon number.

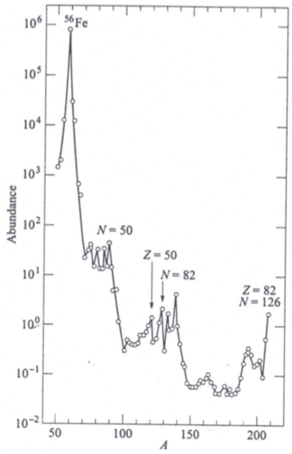
When N or Z corresponds to filling up to a gap, that nucleus is particularly stable wrt neighbours.

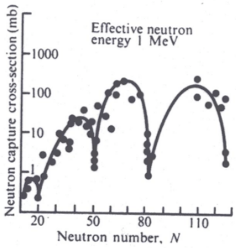
Evidence for “Magic Numbers”

(a) Masses, binding energies and separation energies: show discontinuities at certain nucleon numbers where *magic* numbers are associated with increased binding relative to their neighbours.



(b) Magic numbers show higher abundances compared to neighbouring nuclei.





(c) Reaction rates on nuclei with magic numbers show reluctance to capture neutrons.

(d) First excited states in magic are particularly high (example later).

(e) Effects are exaggerated for both Z and N magic, *doubly magic nuclei* such as ^{208}Pb (Z=82 N=126) and ^{40}Ca (Z=N=20).

(f) Similar numerology inherent in many nuclear properties.

(NEAR STABILITY) MAGIC NUMBERS: 2, 8, 20, 28, 50, 82 AND 126

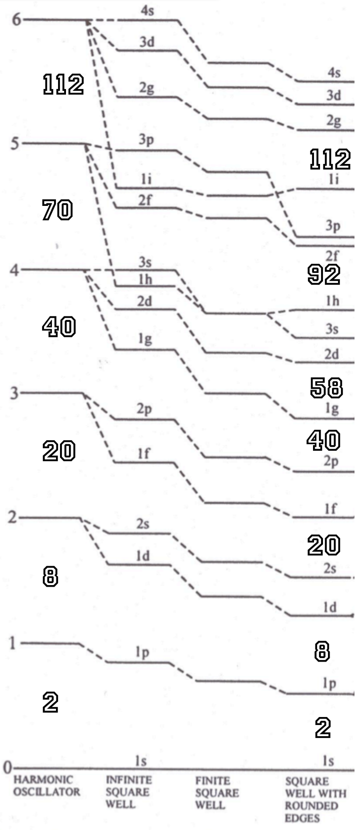
Solutions of the Schrödinger equation

Solutions are not too dependent on the exact form of the potential; neutron scattering suggests that $V_0 \approx 50$ MeV and $r=1.3A^{1/3}$ fm. *Most potentials satisfying this geometry give similar results.*

Quantum numbers from solution are:
n number of nodes in radial wave function except at origin.
l orbital angular momentum.
m_l projection of orbital angular momentum on z axis.
Parity is given by $(-)^l$

Each nucleon also has intrinsic spin $s=½$ *with* $m_s=\pm½$,
Could define a total angular momentum *j* from the vector coupling of orbital angular momentum and intrinsic spin….BUT energies here are only determined by *n* and *l*.

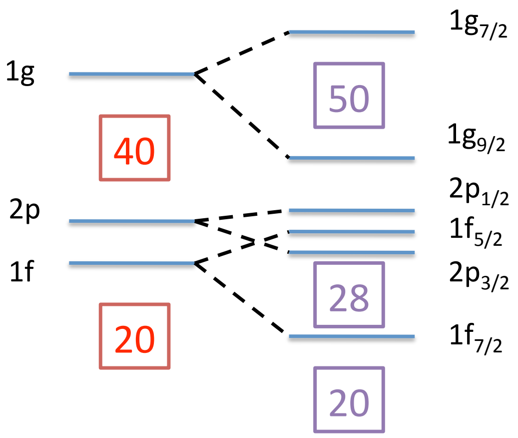
Filling according to Pauli:
each *l* value has $2l+1$ m_l-substates, combined with two intrinsic spin directions, giving a total degeneracy of $2(2l+1)$.



Spin-Orbit Force

If something is introduced which lifts the degeneracy in *j* in the right way, can reproduce the observed magic numbers…

Nobel Prize 1963: Hans Jensen and Maria Mayer



Adding a term of the form:

$$V_{LS} = -U_0 \frac{1}{r}\frac{\partial V}{\partial r}\underline{l}.\underline{s}$$

Does the job since:

$$\underline{l}.\underline{s} = \frac{\hbar^2}{2}\left[j(j+1) - l(l+1) - \frac{3}{4}\right]$$

Similar action to LS coupling in atoms, but atomic version is electromagnetic and NOT from strong force…and is the opposite sign!

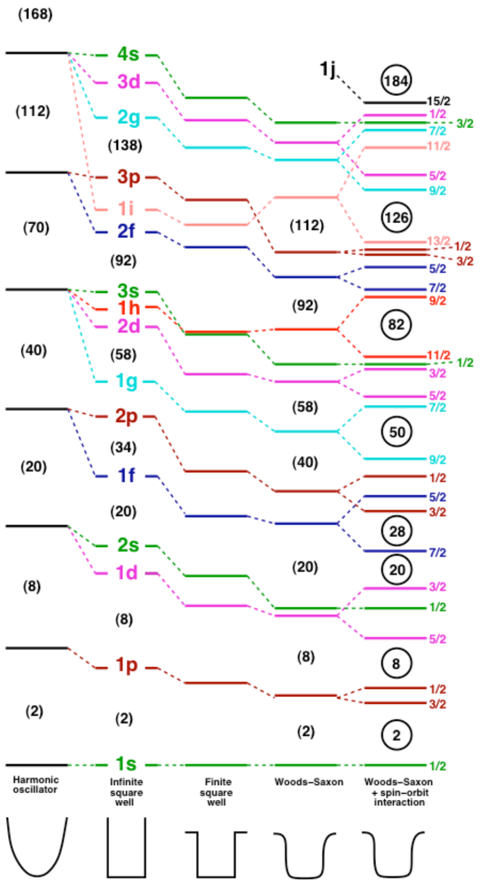
In nuclei, it arises from the L.S. term in the nucleon-nucleon interaction AND complex three-body forces.

Single-Nucleon Levels

Spin-orbit force fixes all the magic numbers seen around stability….BUT don’t have to go far in exotic nuclei before these collapse and new ones form.

Notice general alternation of parities between shells…with the odd high-*j* *intruder* orbital.

“Spuds of pug, dish of pig”
Gets you somewhere to remembering the order.
although this isn’t expected of you!!



Nuclear Spins and Parities

Angular momentum of the whole nucleus is the vector sum of the angular momentum, *j*, of each nucleon.

A full j-orbit, however, will contribute nothing:

Parity of one nucleon in a *j* orbit is $(-)^l$.
Parity of $(2j+1)$ nucleons is $[(-)^l]^{(2j+1)}$ = positive!

Each *j* has $(2j+1)$ m-substates.
If full each substate containing one nucleon.
Then $M=\Sigma m_i = j + (j-1) + (j-2)\ldots + (-j+1) + (-j) = 0$
So can only have a J=0 state.

So expect doubly magic nuclei to have ground-state spin-parities of 0^+.
And similarly for nuclei composed of full *j* orbits.
Examples: ^{48}Ca *(Z=20, N=28)* 0^+ *ground state,* ^{208}Pb *(Z=82, N=126)* 0^+ *ground state*
^{12}C *(Z=6, N=6) has full* $1p_{3/2}$ *level and* 0^+ *ground state.*

Adding one nucleon gives the whole nucleus the spin-parity of the orbital it goes into.
Examples: ^{49}Ca *(Z=20, N=29) odd neutron in* $2p_{3/2}$ *so ground state has* $3/2^-$,
^{209}Bi *(Z=83, N=126) odd proton in* $1h_{9/2}$ *so ground state has* $9/2^-$,
^{13}C *(Z=6, N=3) odd neutron in* $1p_{1/2}$ *level and* $1/2^-$ *ground state.*

Residual Interactions and Pairing

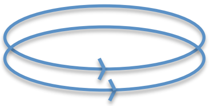
Independent-particle model: each nucleon moving in orbits characterised by (*njl*) not influenced by the motion of any other….a crude approximation.

There is a residual interaction, so far not accounted for, that acts between nucleons and introduces scattering and correlations.

Expect such a major part of this to be residual interaction to be attractive and short ranged; so attraction is maximized if nucleons are close to each other.

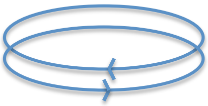
Pauli prevents this:

Two orbits, same *m*



But allows this:

Two orbits, ±*m*
i.e. total M=0, J=0



Short-ranged part of the residual interaction favour the formation of pairs of nucleons.

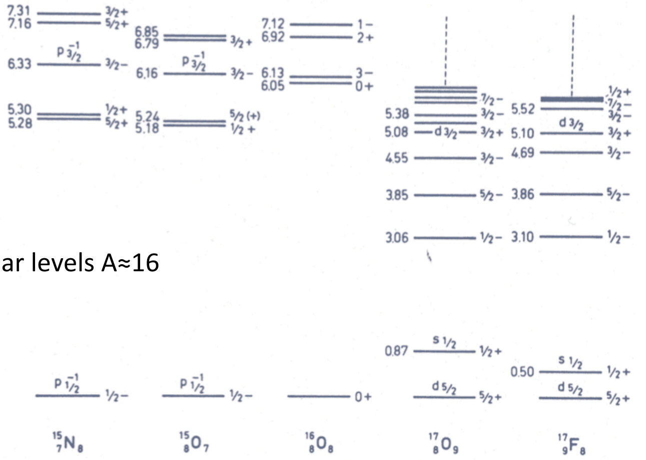
So all even-even nuclei will have ground state with 0^+. TRUE!

Similar to superconductivity (electron pairs) and superfluidity (pairs of He atoms).

So then all odd nuclei will have ground state with spin-parity is often determined by *njl* of the odd nucleon. MANY CASES where OK, but SOMETIMES WRONG!
(usually for very good reasons…deformed nuclei, other parts of residual interaction etc).

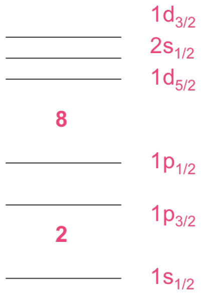
Excited States in Nuclei

Some nuclear excited states can be understood in terms of promoting nucleons into higher-lying single-particle orbits:



Nuclear levels A≈16

Single-particle levels Z/N≈8



Other nuclear excited states cannot be simply understood in terms of promoting nucleons, and require other ideas. For example, surface vibrations of a liquid drop or rotations of a deformed nucleus…..but this is now the stuff of PHYS30322!

Key Ideas … Lecture SIX

Single-particle models:

- Discontinuities in nuclear properties as evidence for shell structure.
- Mean-field potentials.
- Magic numbers and spin-orbit force.
- Residual interactions and pairing.
- Spin-parities of nuclear ground states.
- Simple ideas about excited states.