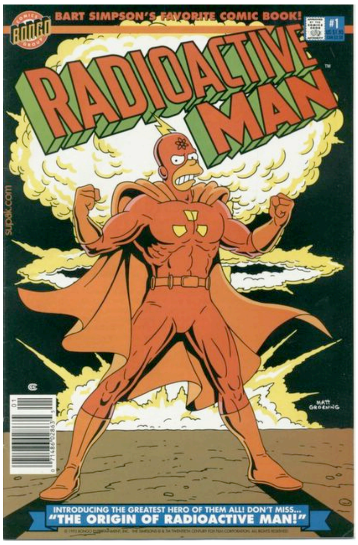
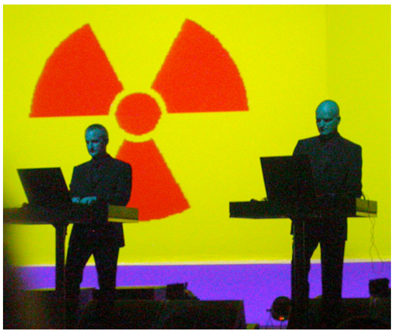
PC30121: Introduction to Nuclear and Particle Physics

Lecture 5: Nuclear Instability and Radioactive Decay





Survey general properties with an emphasis on energetics.

Simple Decay Laws

If a decay process is not forbidden by a conservation law, and a mechanism exists, it will happen!

Probability of decay per unit time λ is a constant.

Most important is that Q>0, which can be estimated from SEMF or measured masses or energies.

For a simple decay of a radioactive nucleus A into a stable nucleus B:

Decay Rate or Activity: $\frac{dN_A}{dt} = -\lambda N_A$

Integrates to: $N_A = N_0 e^{-\lambda t}$

Mean life: $\tau = \frac{1}{\lambda}$

Half life: $t_{1/2} = \frac{ln 2}{\lambda}$

$$N_B = N_0 \left(1 - e^{-\lambda t}\right)$$

Make sure you can derive all the following expressions and quantities from this one assumption.

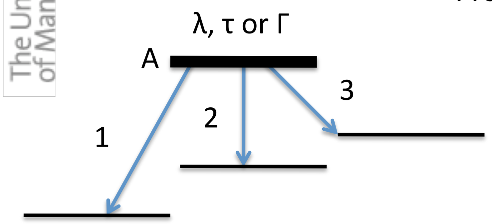
Timescales vary hugely:
e.g. neutron emission $\tau \approx 10^{-21}$s
double beta decay $\tau > 10^{21}$s

Energy-time uncertainty implies that a decaying state has a *width* Γ in energy: $\Gamma = \frac{\hbar}{\tau}$

E.g. Stable state $\tau = \infty$ $\Gamma = 0$
neutron emission $\tau = 10^{-21}$s $\Gamma = 0.1$ MeV

Less Simple Situations

Different decay paths

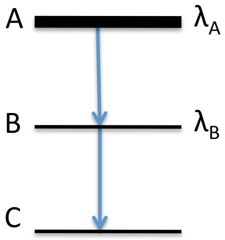


Probabilities add: $\lambda = \lambda_1 + \lambda_2 + \lambda_3$ $\quad \tau = \dfrac{1}{\lambda}$

Branching ratio: $\mathrm{BR} = \dfrac{\lambda_1}{\lambda} = \dfrac{I_1}{I}$

NB: You only EVER measure τ whichever decay process you observe!

Decay chains: for example three isotopes



$$\frac{dN_A}{dt} = -\lambda_A N_A \qquad N_A = N_0 e^{-\lambda_A t}$$

Prove these solutions work

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B \qquad N_B = N_0 \frac{\lambda_A}{\lambda_B - \lambda_A}\left[e^{-\lambda_A t} - e^{-\lambda_B t}\right]$$

Can “easily “ extend to a chain of arbitrary length with Bateman’s equations.

Blackboard notes on two limits: $\lambda_A<\lambda_B$ and $\lambda_A>\lambda_B$

Alpha Decay

Emission of pre-existing nucleons in the form of an α particle.
Rearrangement process to state of lower energy.
Common since the α particle has such a high binding energy.

$$Q = m_{\text{before}} - m_{\text{after}} \qquad BE = Nm_n + Zm_H - M(Z,N)$$

Emission of other light particles usually has negative Q values, so not possible for ground-state decay, for example:

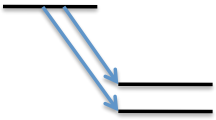
$$^{228}_{90}\text{Th} \rightarrow ^{224}_{88}\text{Ra} + \alpha \quad Q \sim +5.5\ \text{MeV}$$
$$^{228}_{90}\text{Th} \rightarrow ^{227}_{89}\text{Ac} + p \quad Q \sim -6.4\ \text{MeV}$$

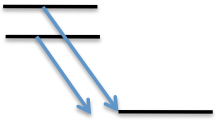
Highly excited states often have enough energy to offset this and weak binding can also alter things. Light particle emission is common at high excitation energy. Proton decay common near the proton drip line.

Q value is shared between the kinetic energy of the α and recoil.

$$T_\alpha = Q\left[\frac{m_R}{m_R + m_\alpha}\right]$$

Prove this!


Badly named α "fine structure"; decay to excited states


Badly named "long-range" α; decay from long-lived excited states

Mass Systematics and Half-Life Puzzles

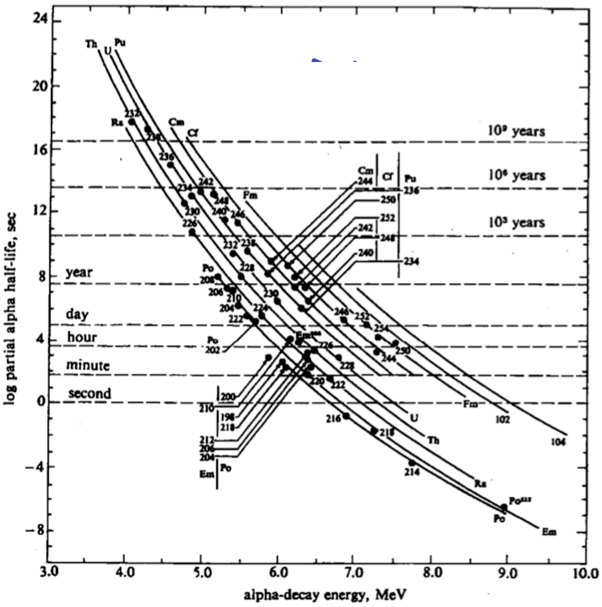
α decay is a way to reduce Coulomb instabilities.

SEMF predicts that nuclei with A>150 will have positive α-decay Q values. Prove this!

But α decay rarely seen in A<200!

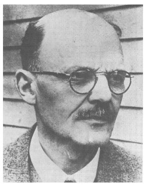
Huge variation in τ for little variation in decay energy, for example:

^{232}Th	1.4x10^{10} years	4 MeV
^{212}Po	10^{-7} seconds	9 MeV



Geiger-Nuttall Rule (1911):

$$ln\ \tau \propto \frac{1}{\sqrt{T_\alpha}}$$


Mechanism: QM Tunnelling

Typical case, ^{228}Th, with α energy of 5.5 MeV.

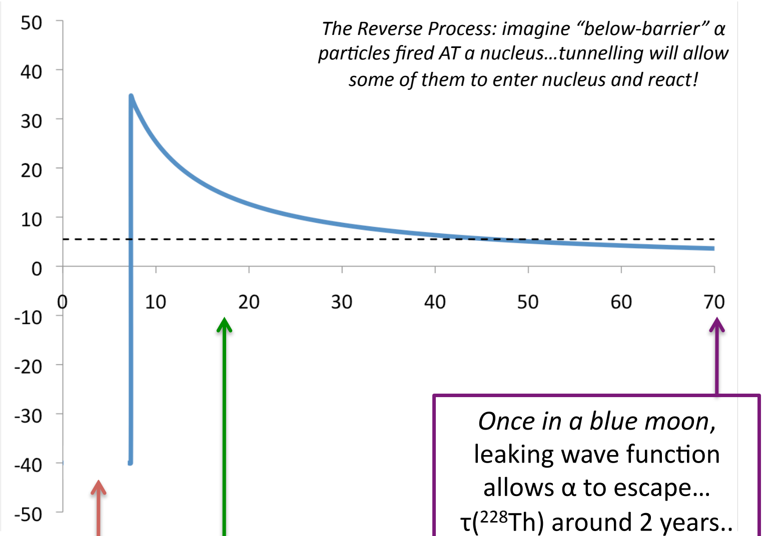
Take well depth ~200 MeV.
Radius $R=r_0A^{1/3}$= 7.3 fm

Coulomb Potential:

$$V_C(r) = \frac{1}{4\pi\epsilon_0}\frac{2Ze^2}{r}$$

α particle is below barrier from:
a=R=7.3 fm to b=50.6 fm

Show this!



The Reverse Process: imagine "below-barrier" α particles fired AT a nucleus…tunnelling will allow some of them to enter nucleus and react!

α in nucleus with approx harmonic radial wave function KE approx. 200+5.5 = 205 MeV
"Hits" barrier around 10^{22} times per second.

Show this!

Classically forbidden region: KE<V
Radial wave function rapidly decays away.

Once in a blue moon, leaking wave function allows α to escape…
τ(^{228}Th) around 2 years..
..so escapes after hitting barrier some 10^{30} times!
Approx. harmonic wave function with longer wavelength than inside.

Show this!

Blackboard notes on barrier penetration

Barrier Penetration Factors

α-decay probability $\lambda = pfP$

Pre-formation factor, *p, probability of finding α* formed in the nucleus.

Frequency of hitting barrier, *f ≈ v/R.*

Probability of transmission through the barrier: $P = e^{-2G}$

$$G = \sqrt{\frac{2\mu}{\hbar^2}} \int_a^b [V(r) - Q]^{1/2}\, dr$$

Notice that this explains the Geiger-Nuttall rule as will give something like: $ln\ \tau \propto \frac{1}{\sqrt{T_\alpha}}$

But some important things left out:

- Difference in nuclear wave functions of parent and daughter.
- Assumed S-wave α particles i.e. L=0 only.
- Assumed spherical nucleus, despite many heavy α emitters being deformed.

Works for other "pre-formed' emissions too:

proton-radioactivity in very neutron-deficient nuclei for example *Nature 381, 25 (1996)*.
double proton decay is a very controversial topic, for example *Nature 439, 279 (2006)*.
rare emission of heavy fragments such as ^{14}C. First observation saw 11 events seen in six months in the decay of ^{223}Ra *Nature 307, 245 (1984)*

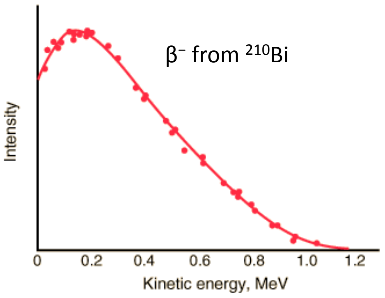
You have enough information to work out why these proton decay only occurs near the dripline and why heavy fragment emission is so rare!

Beta Decay

β⁻ decay: electron emission. β⁺ decay: positron emission.
EC: capture of an electron from an atomic orbital by nucleus.

In all three processes, the nucleon number A is constant, but the proton-neutron ratio changes, with alteration in the overall charge.

Emitted β± have a continuous energy spectrum up to some maximum called the *endpoint,* which is usually <1 MeV



β particles are created in the decay. If an electron were confined in the nucleus:

$$\Delta p \sim \frac{\hbar}{\Delta x} \sim \frac{197}{5} \sim 40\ \mathrm{MeV!}$$

Final state must involve more than two particles. Sharing the decay energy (Q value) between two objects leads to mono-energetic radiations.

$$\mathrm{X} \rightarrow \mathrm{Y} + \beta^- + \overline{\nu}$$
$$\mathrm{X} \rightarrow \mathrm{Y} + \beta^+ + \nu$$
$$\mathrm{X} + e^- \rightarrow \mathrm{Y} + \nu$$

EC results in mono-energetic neutrinos and atomic orbital vacancies which themselves decay by X-ray emission.

Q Values and Mass Systematics

Decay Q values are related to *atomic* masses in the following ways:

β⁻ decay: $\mathrm{X} \rightarrow \mathrm{Y} + \beta^- + \overline{\nu}$ $\quad Q_{\beta^-} = m_X - m_Y$

neglects neutrino
neglects DIFFERENCE in atomic BE

β⁺ decay: $\mathrm{X} \rightarrow \mathrm{Y} + \beta^+ + \nu$ $\quad Q_{\beta^+} = m_X - m_Y - 2m_e$

EC: $\mathrm{X} + e^- \rightarrow \mathrm{Y}^* + \nu$ $\quad Q_{\mathrm{EC}} = m_X - m_Y - B_{\mathrm{atomic\ vacancy}}$

NB: Some observations:

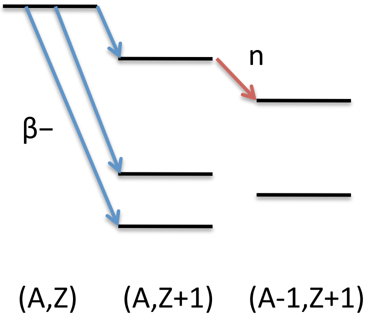
- β^- decay goes from Z to Z+1.
- Although both β^+ and EC go from Z to Z-1, they don't have the same Q value.
- $Q_{\beta+} < Q_{EC}$ can get situations where $Q_{\beta+} < 0$ but $Q_{EC} > 0$, so only EC happens.
- But if β^+ is possible, EC always accompanies it.
- You can make an EC unstable nucleus stable by stripping electrons from it; chemistry DOES effect decay probabilities!

Lifetimes vary greatly from few ms to 10^{16} years and depend strongly on the Q value,

$$\frac{1}{\tau} \propto Q^5$$

β/EC decays can populate excited states:

Population of states above the *nucleon emission threshold* $E^* > S_N$ can lead to so-called *β-delayed nucleon emission.*



SEMF can be re-arranged to give:

$$M(A,Z) = \alpha A - \beta Z + \gamma Z^2 + \frac{\delta}{A^{1/2}}$$

$$\alpha = M_n - a_v + a_s A^{-1/3} + \frac{a_a}{4}$$

$$\beta = a_a + M_n - M_H$$

$$\gamma = \frac{a_a}{A} + \frac{a_c}{A^{1/3}}$$

$$\delta = a_p$$

-11.2 MeV for even-even
0
+11.2 MeV for odd-odd

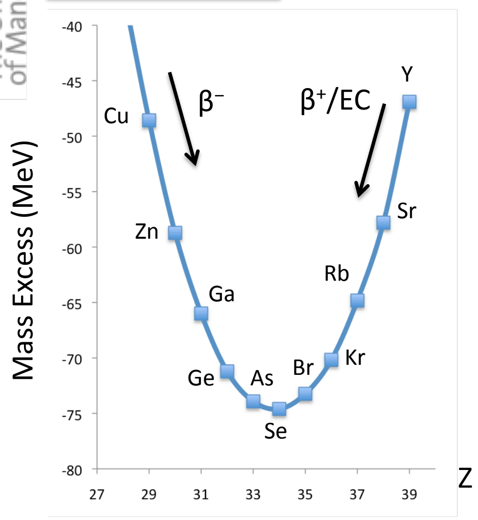
Show this!

If A is constant as in β/EC decays, the masses form a parabolic function of Z.

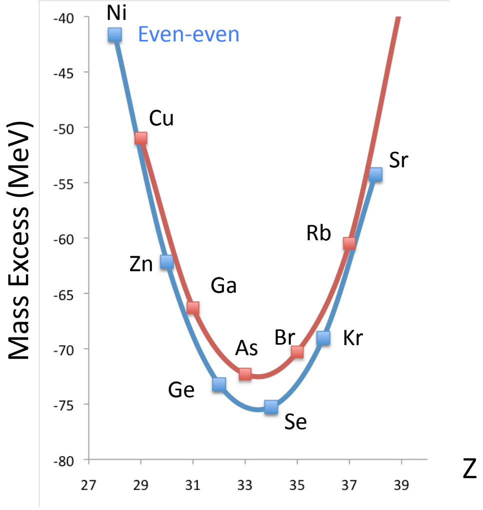
β/EC decays allow nuclei to slide down the parabola to find the most stable nucleus near the bottom.

Mass Parabolas

A=77 Odd Mass



A=76 Even Mass



Only ever one β-stable odd-A nucleus, here ^{77}Se.

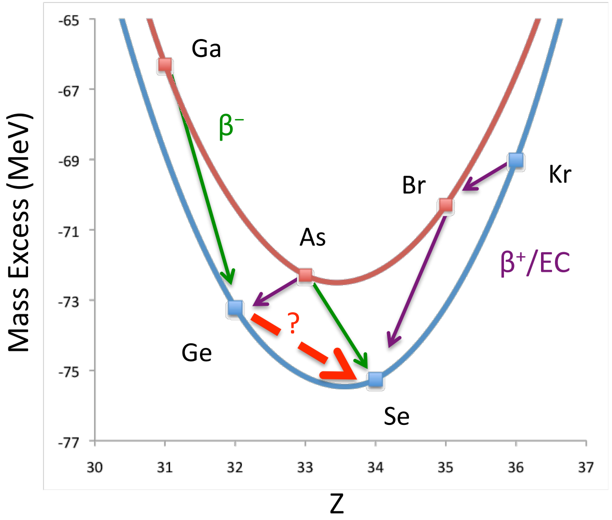
Two parabolas separated by the pairing term. Often several β-stable even-A nuclei, here ^{76}Se and ^{76}Ge.

Odd-odd systems always have at least one more strongly bound even-even nucleus to decay to, so are unstable. Some can decay by β^- *and* β^+/EC e.g. ^{76}As.

Exceptions in light nuclei where ^{2}H, ^{6}Li, ^{10}B and ^{14}N are stable. Increase in asymmetry energy exceeds the decrease in pairing.

Double beta decay:

Example in A=76 Isobars:
Can ^{76}Ge decay to ^{76}Se?



Two-neutrino double beta decay:

$$^{76}\mathrm{Ge} \rightarrow {}^{76}\mathrm{Se} + 2e^- + 2\bar{\nu}$$

essentially two simultaneous normal β decays.
Measured with lifetimes of around 10^{21} years.

Neutrino-less double beta decay:

$$^{76}\mathrm{Ge} \rightarrow {}^{76}\mathrm{Se} + 2e^-$$

occurs if neutrinos are their own antiparticles!
(Probably) not been observed (yet)!
IF it occurs could be the only way to measure the absolute mass of the neutrino.
Lifetimes expected > 10^{25} years!

Decay of Excited States: Low-Lying States

Decay processes can leave the daughter nucleus in an excited state.
Nuclear reactions can also be used to populate excited states.

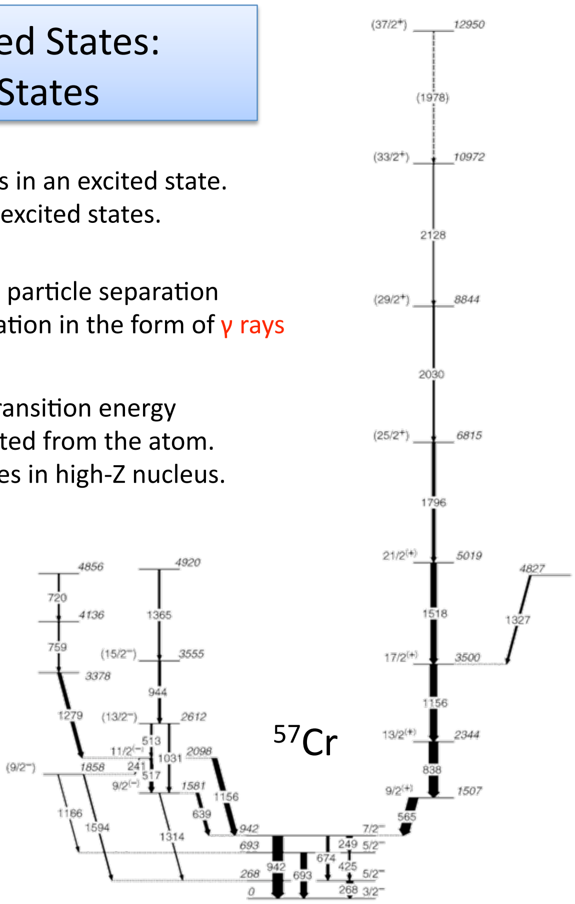
Low-lying states, where the excitation energy is less than particle separation energies, usually decay by emitting electromagnetic radiation in the form of γ rays during a transition to a state of lower energy.

A common alternative is *internal conversion* where the transition energy is directly transferred to an atomic electron which is ejected from the atom. Particularly important for large spin or low energy changes in high-Z nucleus.

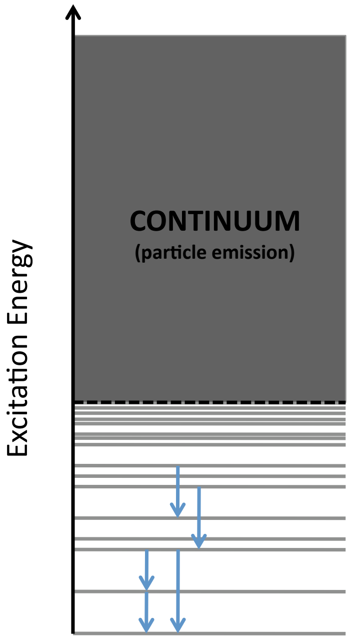
Much less common is *internal pair creation* and,
if all else fails, *two-photon emission* can occur.

Lifetimes depend sensitively on spin changes and transition energies, but are generally in range ns to fs. Level widths are then < 1 eV…discrete states.

Occasionally see usually long-lived excited states, especially where spin change is large or energy low, referred to as *isomers*. E.g. naturally occurring isomer at 77keV ^{180}Ta* 10^{15}yrs, 1^{+} g.s. 8.15 hours.



Decay of Excited States: High-Lying States



CONTINUUM
(particle emission)

Neutron separation energy

Discrete states with Γ< 1eV decay by gamma-ray emission.

Level densities increase approximately exponentially with excitation energy.

Above the nucleon separation energy, nucleon emission becomes possible. Strong interaction process with high λ, short lifetimes (~10^{-22}s) large widths.

Neutron emission is often most important as unhindered by a Coulomb barrier.

Increasing level widths and level density, leads to a continuum where there are very many overlapping states.

Key Ideas … Lecture FIVE

Nuclear Instability and Radioactive Decay:

- Q>0 processes are energetically allowed.
- Exponential decay: simple and more complex forms.
- Alpha decay: Coulomb barrier and tunnelling.
- Beta decay: different forms, energetics and mass parabolas.
- Decay of excited states at low and high energy.