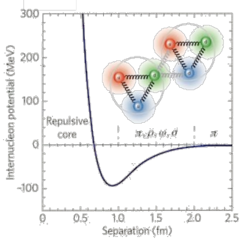
PC30121: Introduction to Nuclear and Particle Physics

The Inter-Nucleon Potential and Nuclear Forces



So far N-N forces:
short-ranged and attractive
hard repulsive core
charge symmetric/independent
depends on orientation of intrinsic spins
is slightly non central…

Lecture 4: What can you learn about nucleon-nucleon interactions from *ELASTIC* N+N scattering?

i.e. nucleon on nucleon
- no loss of total kinetic energy
- no internal excitation of nucleons
- like classical scattering of two billiard balls

Optical Analogies

Scattering of beams of nucleons has parallels in the scattering of light…
…scattering cross sections are somewhat complex to analyze, but can go some way to understanding the basic physics using these optical analogies.

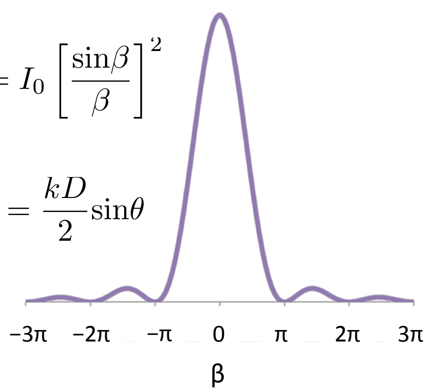
de Broglie waves: $\lambda = \frac{h}{p}$

amplitude & optics
intensity & probability

Single-slit optical diffraction:

$$I = I_0 \left[\frac{\sin\beta}{\beta}\right]^2$$

$$\beta = \frac{kD}{2}\sin\theta$$



First minimum at:

$$\pi = \frac{kD}{2}\sin\theta_{\min}$$

$$\lambda = D\sin\theta_{\min}$$

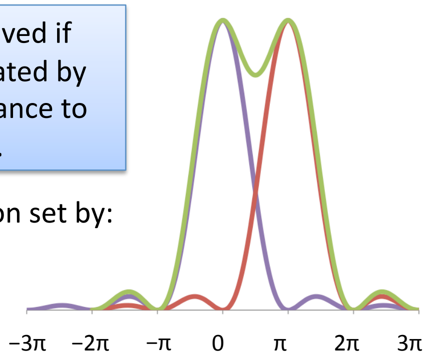
where D is the slit width, θ is angle of observation, k is the wave number and λ is wavelength.

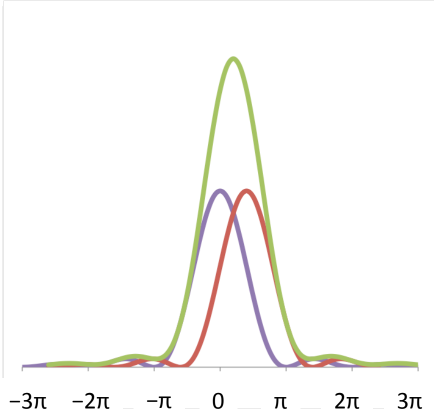
Two slits resolved if patterns separated by more than distance to first min.

Limit of angular resolution set by:

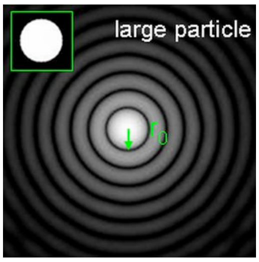
$$\lambda = D\sin\theta_{\min}$$

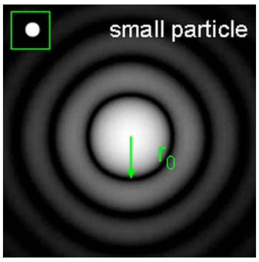
$$\theta_{\min} \sim \frac{\lambda}{D}$$





Same thing with circular apertures:





$$\theta_{\min} \sim 1.22\frac{\lambda}{D}$$

So to see structure with size D you need a wavelength such that:

$$\lambda << D$$

If diffraction occurs then angular minima will occur:

$$\theta_{\min} \sim \frac{\lambda}{D}$$

Example: n + p scattering at low energy (<10 MeV)

$E < m_0c^2$

Classical mechanics ok; use CM i.e. E_{cm}≈10/2 MeV reduced mass≈939/2!

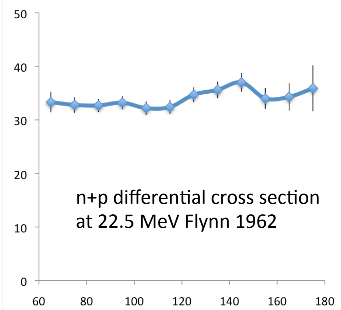
$$p = \sqrt{2mE} = \sqrt{2 \times 469 \times 5} = 68\ \mathrm{MeV/c}$$

Incoming neutron looks like a plane wave with wavelength:

$$\lambda = \frac{h}{p} = \frac{2\pi\hbar}{p} = \frac{2\pi \times 197\ \mathrm{MeV.fm/c}}{68\ \mathrm{MeV/c}} = 18\ \mathrm{fm}$$

…which is much bigger than the radius of a proton, 0.8 fm, so won't reveal any detail about shape or size of the N-N potential, but can give indications of overall strength and range of the interaction.

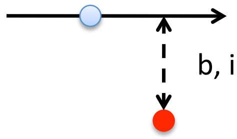
Where is first minimum?
As λ increases with D, the diffraction pattern broadens out, here λ>>D implying no minima exist and diffraction pattern is flat..
The differential cross section for N-N at these energies is ISOTROPIC!



Can understand isotropy in a slightly different way!

Orbital Angular Momentum in Collision

Linear momentum, p



b, impact parameter

Semi-classically,

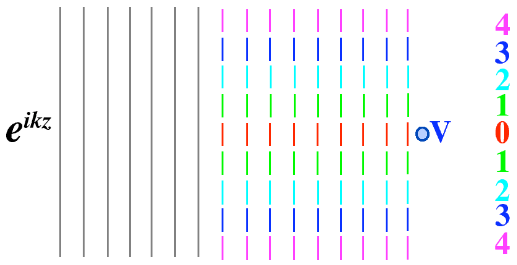
$$\underline{L} = \underline{r} \times \underline{p}$$

$$L = bp = \hbar\sqrt{l(l+1)}$$

$$b = \frac{\hbar}{p}\sqrt{l(l+1)}$$

Example: 10 MeV neutrons: *l=0* S-wave *b≈0* fm
l=1 P-wave *b≈4* fm

Different zones on the in coming plane wave correspond to different L:



Range of N-N force about 2 fm: interaction strong for S wave, small for P wave.

At low energies, only S wave scattering.
S wave in, S wave out.
Wave function Y_{00} has spherical symmetry.
Scattering probability is isotropic.

NB: higher energies, higher p, so smaller b for given l value; more and more l-values come within range!

Spin in Collisions

For n+p scattering at low energy L=0 i.e. S wave:

Triplet state $$S = s_1 + s_2 = \frac{1}{2} + \frac{1}{2} = 1 \qquad M_S = \pm 1, 0$$

Singlet state $$S = s_1 + s_2 = \frac{1}{2} - \frac{1}{2} = 0 \qquad M_S = 0$$

S in these two lines is intrinsic spin, not L=0!

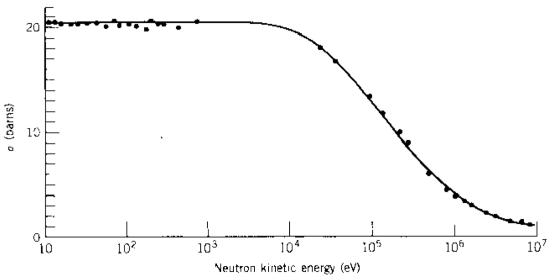
Two possible n+p states: $^3S_1$ or $^1S_0$

For n+n or p+p scattering at low energy L=0 i.e. S wave:

Pauli principle limits this to $^1S_0$.

n+p at low energies < 10MeV

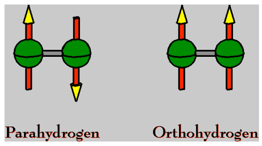
Dominated by S wave scattering.
Constant cross section 20.4 barns.
Falls above energies of 10 keV.



Combination of scattering in singlet and triplet states, but difficult to separate contributions: $\sigma = \frac{3}{4}\sigma_T + \frac{1}{4}\sigma_S$

Two contributions were sorted out originally by scattering off hydrogen at very low energy.

If E<0.01eV, λ>0.05nm i.e. bigger than a hydrogen molecule!
So neutron scatters coherently off both protons, a bit like Young's slits.



Two types of hydrogen molecule: ortho-hydrogen (proton spins parallel)
para-hydrogen (protons spin antiparallel)
Comparison of coherent scattering from two aligned protons with two anti-aligned protons, allows the $^3S_1$ and $^1S_0$ cross sections to be extracted.
Forces in triplet state are few 10% stronger.

n+p at low energies < 10MeV

To summarise results of many low energy experiments:

- Both singlet and triplet are attractive.
- $^3S_1$ potential is few tens of percent stronger than $^1S_0$.
- Their *effective ranges* are 1.75 and 2.73 fm respectively.
- $^3S_1$ can produce a bound state of the deuteron, at −2.22 MeV.
- But the $^1S_0$ is not strong enough; the 0^+ state is just *unbound* and sits around +77 keV above the top of the potential.
- The relative wavelengths are not small enough to probe the shape of the potential, and many different sorts of potential reproduce these results.

n+p at higher energies

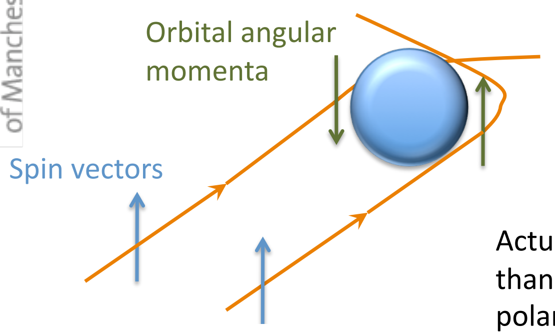
At higher energies, expect:

- smaller λ so should resolve the shape of the potential.
- higher and higher orbital angular momenta contribute¶ e.g. $^1P_1$, $^3P_{0,1,2}$ …
- angular distributions are no longer isotropic, each has a characteristic shape used to disentangle contributions to cross sections due to each L value.
- need to disentangle different J values *(beam polarisation)*.

People tend to talk about phase shifts, which are related to cross sections.

¶ Identify the spectroscopic combinations which are forbidden for p+p and n+n?

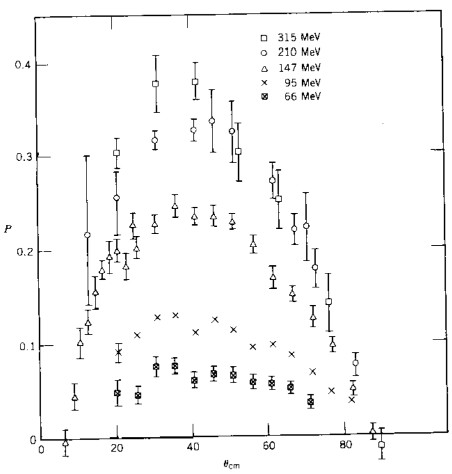
Disentangling J: Beam Polarization



Left-right asymmetries arise whether the total angular momentum transfer is *L+S* or *L-S*.

Actually flipping beam spin up and down is easier than moving spectrometers left and right, and polarisation is measured:

$$P = \frac{N\uparrow - N\downarrow}{N\uparrow + N\downarrow}$$

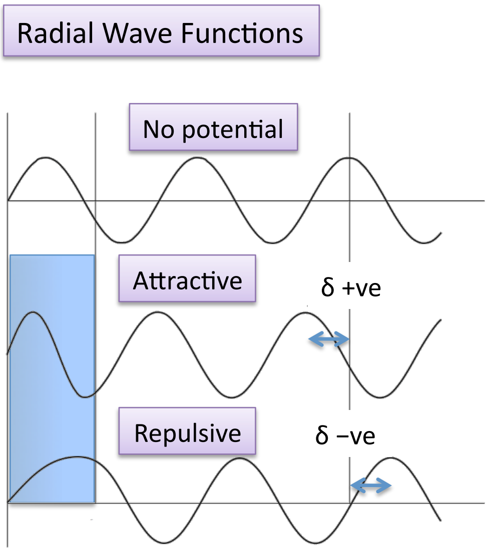


For S wave, at low energies, should be no effect. Grows with energy as higher L contributions grow.

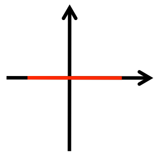
So sensitive to the J value of the state: i.e. how L is aligned relative to S.

See Krane Chapter 4

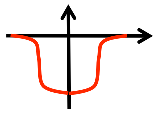
Phase Shifts



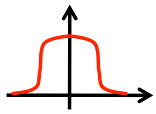
Simple wave: λ =h/p



Projectile falls into well, gaining kinetic energy, $T=E+V_0$. In that region, p is higher and λ smaller.



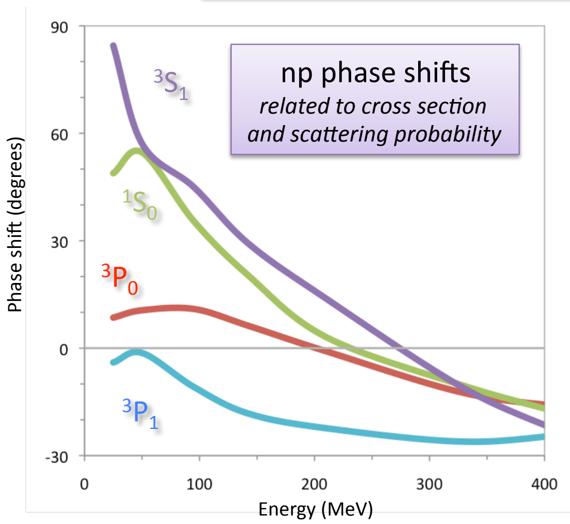
Projectile rolls uphill, losing kinetic energy, $T=E-V_0$. In that region, p is smaller and λ higher.



"It turns out that" the phase shift δ is related to the cross section for scattering.
There is a different phase shift for each spectroscopic state.
Example:

$$\left[\frac{d\sigma}{d\Omega}\right]_{l=0} = \frac{\sin^2\delta_0}{k^2} \qquad \text{Flux} = -\frac{i\hbar}{2m}\left[\psi^*\nabla\psi - \psi\nabla\psi^*\right]$$

Nucleon Scattering at High Energies



Confirmation that strong dependence on the spin orientations i.e. singlet-triplet differences, such as $^1S_0$ compared to $^3S_1$.

Also depends on orientation of orbital and spin angular momenta, such as $^3P_0$ compared to $^3P_1$ and $^3P_2$.
Need to add a term that depends on scalar product L.S i.e. spin-orbit coupling.

NUCLEON-NUCLEON POTENTIAL DEPENDS ON L & S AND THEIR COUPLING VIA SPIN-ORBIT FORCES.

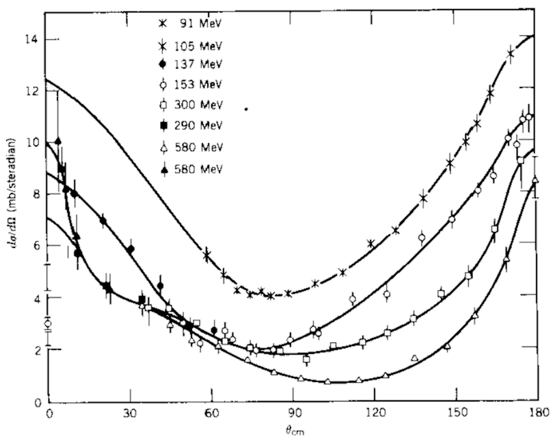
Comparing n-n and p-p tests
Charge Symmetry:
pretty good!
Comparing n-n/p-p with n-p tests
Charge Independence:
<u>DISTINCTLY BROKEN</u>,
but only at the 1-2% level.

Phase shifts positive which then go negative by 300 MeV....confirms suspicions of a repulsion at ~0.5 fm to a generally attractive force.

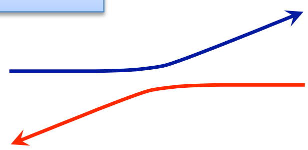
What distance scale corresponds to 300 MeV nucleons?

The University of Manchester

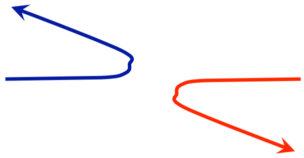
Angular Distributions for n-p



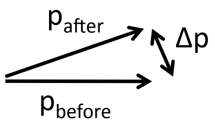
Small angle scatters?



Large angle scatters?



Estimate deflection angles:



$$\sin\theta \sim \frac{\Delta p}{p} = \frac{F\Delta t}{p} = \frac{1}{p}\frac{V_0}{R}\frac{R}{v} = \frac{V_0}{pv} = \frac{V_0}{2T}$$

For a potential with depth V_0 and range R:

$$F = -\frac{dV}{dr} \sim V_0/R$$

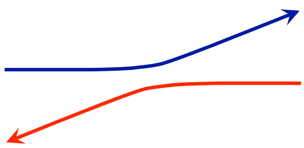
$V_0 \approx$ 35 MeV giving $\theta \approx 10°$ or less!
Large angle scattering picture is wrong!

The University of Manchester

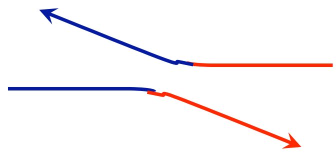
Exchange Character

How can you have a small angle scatter that results in neutron coming backward?
Swap the nucleon identity during collision!

Small angle scatters:



Large angle exchange scatters:



Further reading on N+N elastic scattering:
look at Chapter 17 in Eisberg and Resnick

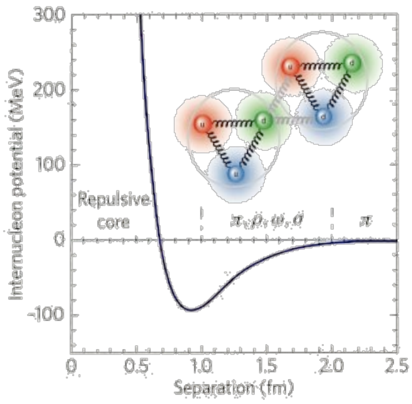
The Nucleon-Nucleon Potential

Summarising the properties of the nucleon-nucleon interaction:

- short-ranged <2fm; it has a *saturation* property.
- it is charge symmetric and approx. charge independent.
- strong and attractive in the range ≈0.5 to 2 fm.
- it has a hard repulsive core at distances less than ≈0.5 fm.
- it is dependent on orientations of intrinsic spin.
- it is slightly non-central; has a *tensor* component.
- it depends on relative angular momenta L &S and coupling.
- it has an exchange character.

Difficult and complex job to sum up in a single potential, but can be done and then used in calculations like the illustration we did for the deuteron!

Shape tends to be similar to a Lennard-Jones intermolecular potential, at least in radial shape!



Exchange Forces

Can understand the np forward/backward scattering AND the slight departure from charge independence by considering a different approach to describing forces.

E.g. ELECTROMAGNETISM: interaction between two charges.

$$F = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r^2}$$

Can get equivalent picture by assuming the exchange of a so-called *virtual photon*:

- energy conservation is violated as the first charge emits a photon with momentum *q*.
- acceptable as long as it is done within time constraints placed by the uncertainty principle, *ΔE.Δt≈ħ*.
- the changes in momentum due to the emission/absorption of such virtual photons generates forces.
- rate of emission/absorption is proportional to electric charge Q.

Uncertainty principle: $qr \sim \hbar$

Can be violated for a time: $t = r/c$

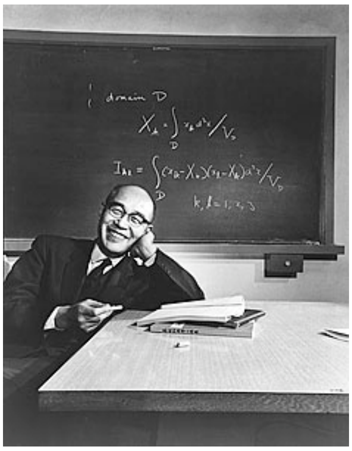
Impulse generated: $\frac{q}{t} = \frac{\hbar c}{r^2}$

Force: $F \propto \frac{Q_1 Q_2}{r^2}$

If propagating e/m fields are quantised as *real photons,* the *virtual photon* model is appealing!

Yukawa Theory

In 1935, Hideki Yukawa postulated that the nuclear force was mediated by massive force field quanta.



1949 Nobel Prize
Living Human Treasure

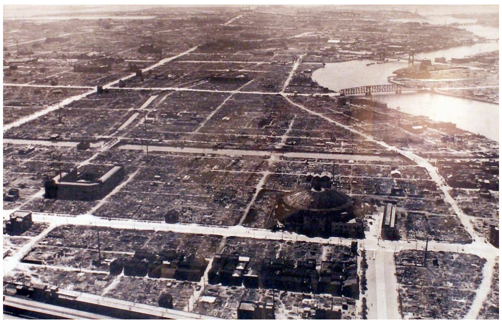
The fact that their mass is non-zero limits the range of the force. Easy to use a simple uncertainty argument:

$$\Delta E \Delta t \sim \hbar \qquad \Delta t \sim \frac{\hbar}{mc^2} \qquad R \sim v\Delta t \leq \frac{\hbar}{mc}$$

For a range of around 1 fm, the quanta require a mass of around 100 MeV.

The muon, discovered in 1936 with mass 106 MeV, was initially thought to be a candidate. But was subsequently found not to interact via strong forces.

Charged pions $\pi^{\pm}$ were discovered by Powell in cosmic rays in 1949, followed by π^0 in 1950, which were more plausible field quanta.

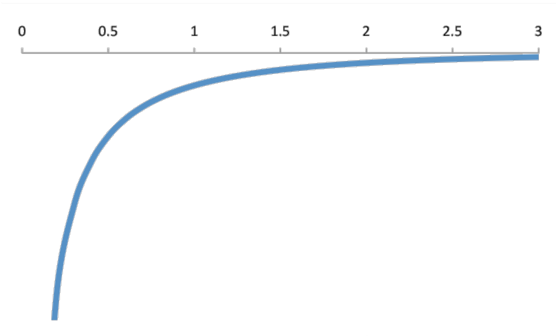


Klein-Gordan Equation

Blackboard notes

Yukawa Potential

Rough scale for g by taking parameters of the deuteron: V_0=35 MeV and R=2 fm.



$$V(r) = \frac{g^2}{4\pi r} e^{-r/R} = \frac{g^2}{4\pi} \frac{0.3679}{2 \text{ fm}} = 35 \text{ MeV}$$

$$\frac{g^2}{4\pi} = 190 \text{ MeV.fm}$$

~~$\alpha = \frac{e^2}{4\pi\hbar c} = \frac{1}{137}$~~

Making a unit-less quantity, to compare with the fine structure constant:

$$\alpha_s = \frac{g^2}{4\pi\hbar c} \sim 1 \qquad \alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} = \frac{1}{137}$$

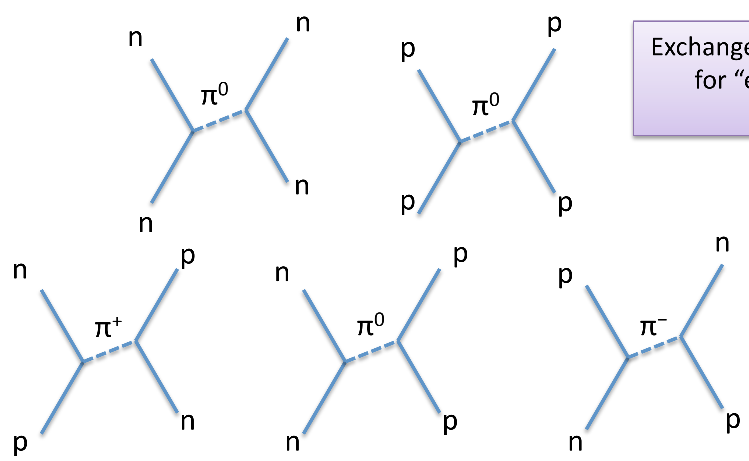
…the coupling constant for strong forces is “strong” compared to e/m!

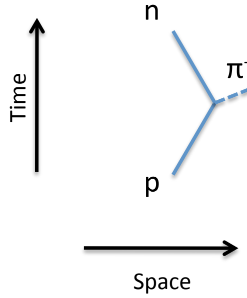
Pions

Three types: π^+, π^- and π^0.
All with spin-parity 0^-.
Masses: 139.57, 139.57 and 134.97 MeV/c^2.
Charged ones decay in 2.6x10^{-8}s.
Neutral ones with mean life 0.83x10^{-16}s.

Cecil Powell
Nobel 1950

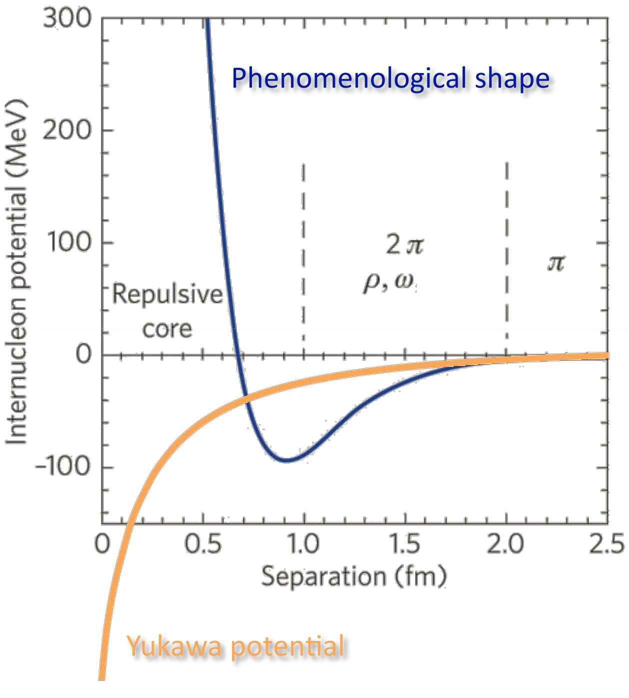
Exchange of charged pions accounts for “exchange nature” in n-p scattering.





Near equality of pion masses accounts for charge symmetry, and small departure from charge independence

Hang on, can't be the whole story!



A proper treatment of the one-pion exchange yields a OPE Potential which has many of the right spatial-spin properties: short range, spin dependence, tensor component etc.

But doesn't only works for distances > 1fm!!

Need to invoke exchange of other mesons such as ρ (770 MeV/c^2) and ω (783 MeV/c^2), and multiple π exchange.

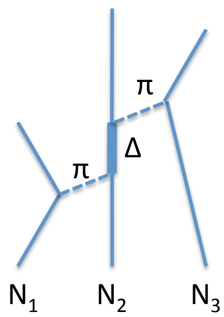
Calculations do quite well… although the repulsive core is still difficult… see Nature 445 (2007) 156

Pions and Nucleons

Backing up suspicions of pions as exchanged quanta between nucleons, beams of pions were found to interact strongly with nucleons.

The cross section for π+N scattering increases greatly at a CM energy of 1232 MeV. This corresponds to the production of the Δ resonance, an excited state of a nucleon where all three quark spins are aligned to give a total spin-parity of $3/2^+$.

This starts to give a mechanism for "three-body forces" in many-nucleon systems:



…and many more complex diagrams!

Key Ideas … Lecture FOUR

The properties of the nucleon-nucleon interaction:

- short-ranged <2fm (strong and attractive in the range ≈0.5 to 2 fm).
- it has a saturation property.
- it is charge symmetric and approx. charge independent.
- it has a hard repulsive core at distances less than ≈0.5 fm.
- it is dependent on orientations of intrinsic spin.
- it has a tensor component and is slightly non-central.
- it depends on relative momentum via spin-orbit coupling.
- it has an exchange character.

Ideas on the way:

- Analogies in scattering and optics
- Exchange mechanism for "fundamental" interactions.
- Klein-Gordan equation.
- Pions and Yukawa potential.