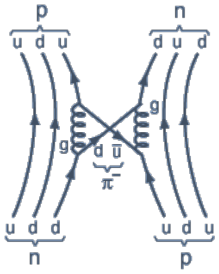
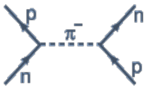
PC30121: Introduction to Nuclear and Particle Physics

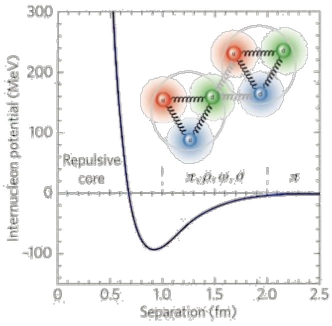
Next Issue: The Inter-Nucleon Potential and Nuclear Forces





Lecture 3: What can you learn about nucleon-nucleon interactions from nuclear structure?

You can find some nice discussions in:
Nuclear and Particle Physics: R.J. Blin-Stoyle (Chapman Hall 1991)
Quantum Mechanics of Atoms, Molecules, Solids, NUCLEI and PARTICLES:
R. Eisberg and R. Resnick (Wiley 1985)
Elements of Nuclear Physics: W.E. Burcham (Longman)
The Properties of Nuclei: G.A. Jones (Oxford 1987)



Nuclear Forces

Information about the forces between two nucleons comes from:
- features of complex nuclei.
- analysis of light, simple nuclei such as the deuteron.
- analysis of nucleon-nucleon scattering experiments.

We'll discover the following properties of the nucleon-nucleon interaction:
- short-ranged <2fm.
  strong and attractive in the range ≈0.5 to 2 fm).
- it has a saturation property.
- it is charge independent (couldn't care if you are a proton or a neutron).
- it has a hard repulsive core at distances less than ≈0.5 fm.
- it is spin dependent.
- it has a tensor component.
- it depends on relative momentum.
- it has an exchange character.

…in other words it is complicated!
But we'll learn a lot in the process…

Force and Potential

A reminder in case you need it; a force can usually be related to a potential function:

$$\underline{F} = -\nabla V(\underline{r})$$

Complex Nuclei

- Nucleons stick together and form nuclei:

  so there has to be a stronger attractive force overcoming the electrostatic repulsion of protons.
- Deviations from pure Coulombic scattering are only seen at distances of closest approach less than around 2 fm:

  so nucleon-nucleon forces have limited range, unlike gravity or electromagnetism.

N-N INTERACTION IS SHORT RANGED AND HAS A STRONGLY ATTRACTIVE COMPONENT.

The force can't be completely attractive; there must be something which overcomes the attraction so that nucleons remain as distinct particles in a nucleus….not melting into a sea of quarks and gluons, at least at low energy.

N-N INTERACTION HAS A HARD REPULSIVE CORE, more on that story later!

Saturation

The potential energy for a unit mass at the surface of a gravitating sphere, mass *M* and radius *R*:

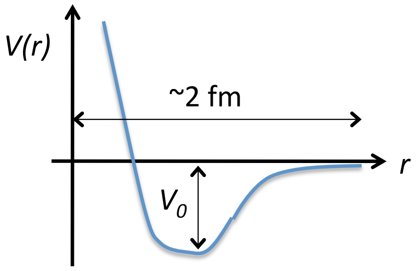
$$V(r) = \frac{GM}{R}$$

Since *M* is proportional to R^3, potential and the BE increase with *M*.

But the constancy of the average BE per nucleon in a nucleus suggests:
(i) the force cannot be long ranged.
(ii) the potential must be very different from *1/r*.

Saturation: each nucleon only interacts strongly with a few nearest neighbours.

The average BE per nucleon is 7-8 MeV. So expect V_0 to be few MeV deep.



Charge Symmetry and Charge Independence

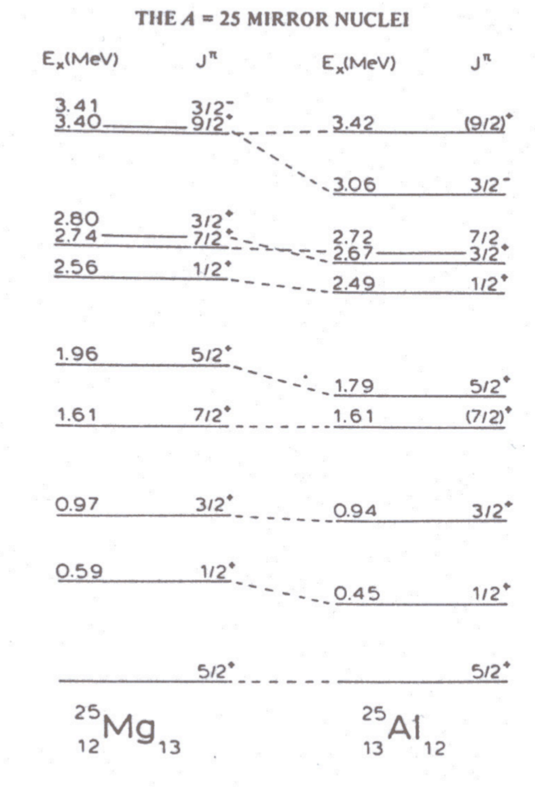
Mirror nuclei: swap all protons for neutrons, and vice versa
for example: $^{25}_{12}\mathrm{Mg}_{13}$ and $^{25}_{13}\mathrm{Al}_{12}$

The mass difference in such a pair is almost all accounted for by the change in Coulomb energy and the proton-neutron mass difference.

All p-p bonds become n-n
All n-n bonds become p-p
All n-p bonds become p-n i.e. stays the same!

Similarity of masses and energy levels suggests n-n forces are very similar to p-p.

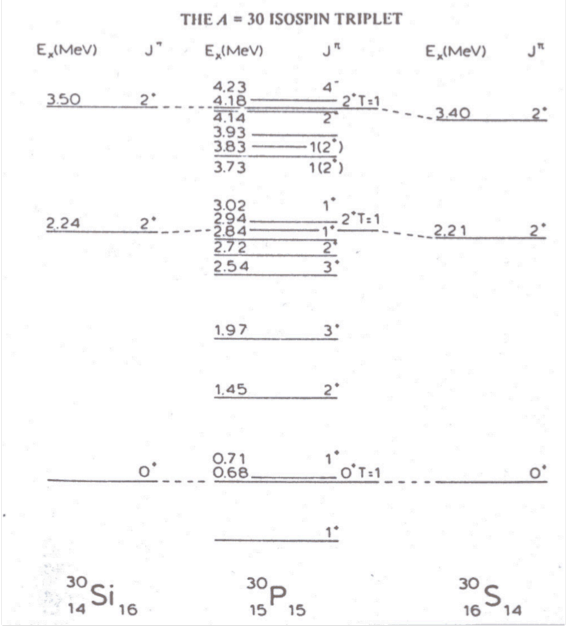
NUCLEAR FORCES ARE <u>CHARGE SYMMETRIC</u>.



Charge Symmetry and Charge Independence

Isobaric multiplets: gradually swap protons for neutrons.

Correct for the change in Coulomb energy and the proton-neutron mass difference. Some of the structures look similar!



^{30}Si: n-n pairs 16x15/2=120
p-p pairs 14x13/2=91
Total like pairs = 211
n-p pairs 14x16=224

^{30}P: n-n pairs 15x14/2=105
p-p pairs 15x14/2=105
Total like pairs = 210
n-p pairs 15x15=225

From Si to P, one like-nucleon pair is lost and one unlike-nucleon pair is created.

Liberation from the Pauli principle generates “extra” states!

Similarity of masses and energy levels suggests n-p forces are very similar to p-p and n-n: **NUCLEAR FORCES ARE <u>CHARGE INDEPENDENT</u>.**

The Deuteron

Key Properties:
- Binding energy 2.224 MeV.
- No excited states.
- Ground-state spin and parity 1^+.

Try to understand these in terms of a proton and neutron the interaction between them

Solve the Schrödinger equation:

$$\left[-\frac{\hbar^2}{2m_p}\nabla_p^2-\frac{\hbar^2}{2m_n}\nabla_n^2+V\left(\underline{r}_p,\underline{r}_n\right)\right]\Psi\left(\underline{r}_p,\underline{r}_n\right)=E\Psi\left(\underline{r}_p,\underline{r}_n\right)$$

Just as in classical mechanics, transform to the CM frame and get two equations. One describes the motion of the CM i.e. where the deuteron is in space, and the other describes the internal motion:

$$\left[-\frac{\hbar^2}{2m}\nabla^2+V\left(\underline{r}\right)\right]\psi\left(\underline{r}\right)=E\psi\left(\underline{r}\right)$$

r is the relative separation of the proton and neutron and *m* is the reduced mass: $m=m_pm_n/(m_n+m_p)$

Blackboard notes on Deuteron properties and setting up the Schrodinger equation.

Intuition

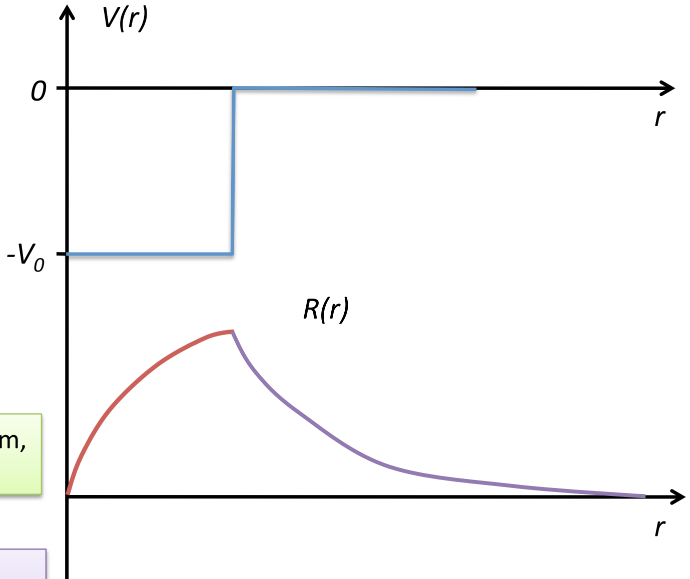
If bound, $E<0$.

So for $r<a$, $V<E$ and V constant: wave function must be harmonic.

Since radial component is $R(r)/r$, need $R(r)$ as a sine wave to stop blow up as r tends to zero.

Low energy states have low momentum, so long wave lengths $\lambda=h/p$.

So for $r>a$, $V>E$ and V constant: classically forbidden region, so wave function decaying exponential.

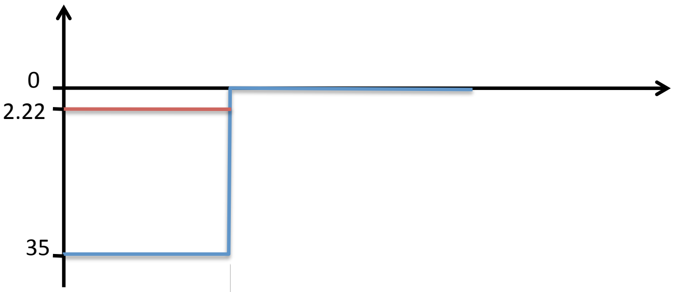


But they need to match properly at the boundary r=a! Both $R(a)$ and $dR(a)/dr$

Blackboard notes on solving Deuteron Schrodinger equation.

So where does that leave us?

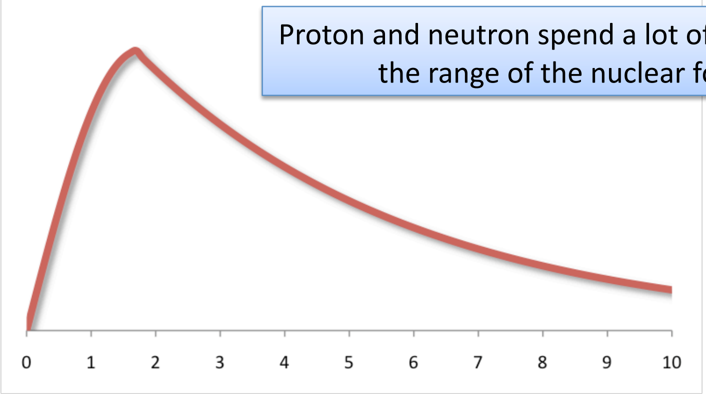
If a is something like the range of nuclear forces 1 to 2 fm, say 1.7 fm…
Then V_0 is 35 MeV.
But E is only 2.22 MeV !



Average kinetic energy is comparable to the depth of the potential.

Weakly bound!

Proton and neutron spend a lot of time outside the range of the nuclear forces!



No room for excited states! Addition of the centrifugal term makes well even shallower and narrower…levels with *l>0* move above *E=0* and are unbound!

So far, so good…

Crude square-well potential does give a “reasonable” value for the well depth
and gives some understanding of the weak binding….
what about other measured properties?

Ground-state spin-parity is 1^+.

Well…*l=0*…and parity of a spherical harmonic is *$(-)^l$*.

The total angular momentum must come from intrinsic spin then?
Two spin-1/2 nucleons….couple together.

Vector addition: $\underline{S} = \underline{s}_1 + \underline{s}_2$ i.e. $S = s_1 + s_2,\ s_1 + s_2 - 1, \ldots |s_1 - s_2|$
S=1 has three substates M_s=+1, 0 and -1 so called a *triplet* state.
S=0 has one substates M_s=0 so called a *singlet* state.

Often use spectroscopic notation: $^{2S+1}L_J$
e.g. triplet $^3S_1$
and singlet $^1S_0$

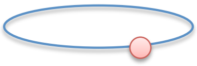
Ground state must be $^3S_1$ …. but no $^1S_0$? Must lie higher in energy and be unbound!

NUCLEON-NUCLEON INTERACTION MUST DEPEND ON THE SPIN ORIENTATION!

Electromagnetic Properties

Blackboard notes

Magnetic Dipole Moments



Electron orbiting: current around a loop…has a dipole moment

$$\mu_l = IA = -\frac{e}{2\pi r/v} = -\frac{evr}{2} = -\frac{e}{2m}L = -\frac{e\hbar}{2m}l = g_l\mu_B l$$

NB: quantised *L* and g_l=-1

Bohr magneton: $\mu_B = \frac{e\hbar}{2m_e} = 9.274... \times 10^{-24}\ \mathrm{JT}^{-1}$

Electron also has spin, so there is an intrinsic magnetic moment also, for which quantum electrodynamics predicts g_s=-2.0023192… $\mu_s = g_s\mu_B s = \frac{1}{2}g_s\mu_B s$

ALL elementary particles (except neutrino) have an analogous intrinsic magnetic moment, and if charged can generate an orbital magnetic moment!

Protons: $\mu_p = \frac{1}{2}g_p\mu_N$ $\quad g_p = +5.5856...$ $\quad \mu_p = +2.7928\mu_N$ $\quad g_l = 1.0$

Neutrons: $\mu_n = \frac{1}{2}g_n\mu_N$ $\quad g_n = -3.8262...$ $\quad \mu_n = -1.9131\mu_N$ $\quad g_l = 0.0$

Nuclear magneton: $\mu_N = \frac{e\hbar}{2m_p} = 5.05078... \times 10^{-27}\ \mathrm{JT}^{-1}$

Deuteron Magnetic Moment

No orbital contributions, intrinsic spins are parallel in the triplet ground state:

$$\mu = \mu_n + \mu_p = 0.8797\mu_N$$

Experiment says NO! $\mu = 0.85743823(3)\mu_N$

Are you sure there isn't a small orbital component?

Quadrupole Moments

Indicator of non-spherical shapes:

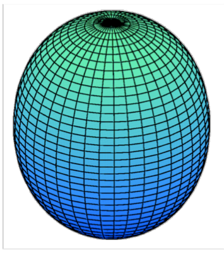
$$Q_0 = \int \rho_{\text{ch}} \left[3z^2 - r^2\right] dV$$

Average over the charge distribution:

$$Q_0 = Z\left[3\langle z^2\rangle - \langle r^2\rangle\right]$$

Units are area. NB:
$\langle r^2\rangle = \langle x^2\rangle + \langle y^2\rangle + \langle z^2\rangle$

$$\langle z^2\rangle > \frac{1}{3}\langle r^2\rangle$$

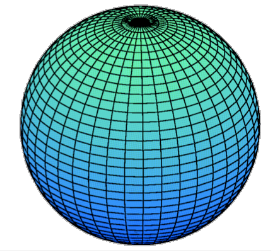


$Q_0 > 0$

PROLATE

$$\langle x^2\rangle = \langle y^2\rangle = \langle z^2\rangle$$

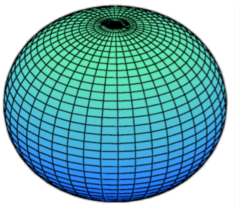
$$3\langle z^2\rangle = \langle r^2\rangle$$



$Q_0 = 0$

SPHERICAL

$$\langle z^2\rangle < \frac{1}{3}\langle r^2\rangle$$



$Q_0 < 0$

OBLATE

In Quantum Mechanics

For completeness sake…

Replace ρ with ψ*ψ in the integral

Then some weirdness….the measured Q is related to Q_0 for J>0 by:

$$Q = \frac{2J-1}{2(J+1)}Q_0$$

For J=1/2 Q in above formula.

For J=0, Q vanishes identically since no axis can be defined.

…but not important now.

Deuteron Quadrupole Moment

Our model suggests *Q=0* since spherically symmetric wave function…

Experiment says NO!
$Q_0 = +0.00286(2)$ barns = $+0.286(2)$ fm^2

Wave function can't be pure $^3S_1$

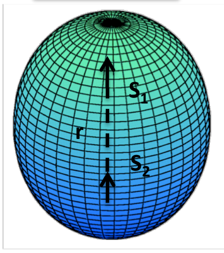
Possible couplings with J=1: $^1P_1$ (S=0 L=1 J=1) or $^3P_1$ (S=1 L=1 J=1) or $^3D_1$ (S=1 L=2 J=1)

Possible couplings with $J=1^+$: $^3D_1$ (S=1 L=2 J=1)

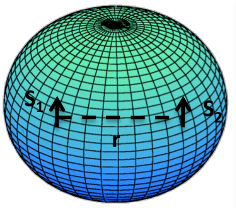
Wave function: $\psi = \alpha\, ^3S_1 + \beta\, ^3D_1$ and $\beta \approx 0.08$ gives right Q_0.

N-N INTERACTION MUST HAVE A NON-CENTRAL COMPONENT!

Favoured



Unfavoured



Add another term in N-N potential:

$$V_T = f_T(r)\left[\frac{3\,(\underline{s}_1.\underline{r})\,(\underline{s}_2.\underline{r})}{r^2} - \underline{s}_1.\underline{s}_2\right]$$

so-called *tensor force*.

Also fixes magnetic moment by introducing orbital contribution.

Key Ideas … Lecture THREE

Nucleon-nucleon interaction:

- Short-ranged and attractive.
- Hard repulsive core.
- Charge independent.
- Depends on spin orientation.
- Has a non-central component.

↕ Complex nuclei.

↕ Deuteron.

Ideas on the way:

- Saturation.
- Mirror and isobaric systems.
- Intuitive ideas about solutions of Schrödinger equation.
- Magnetic dipole moments.
- Electric quadrupole moments.

Not the end of the story though!

You'd better read about how these electromagnetic moments are measured!
Introductory Nuclear Physics:
K.S. Krane (Wiley 1987)