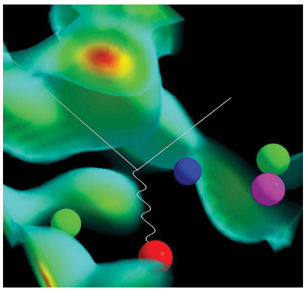
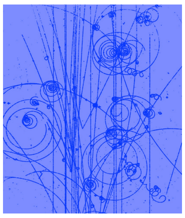
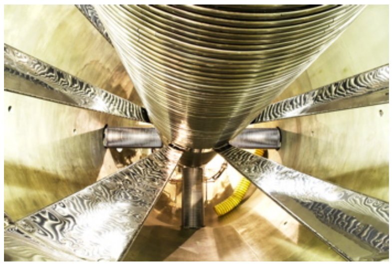
MANCHESTER 1824

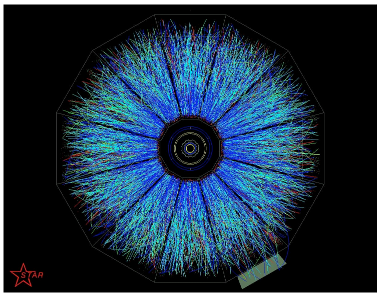
The University of Manchester

# PC30121: Introduction to Nuclear and Particle Physics









Lecture Two: Masses, Abundances, Binding Energies and the Liquid Drop Model

MANCHESTER 1824

The University of Manchester

## Measuring Masses

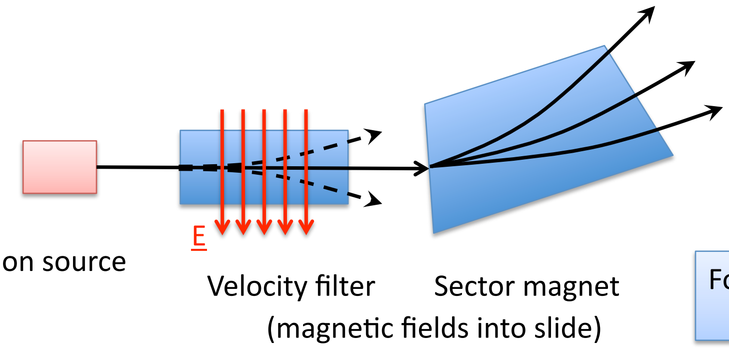
Main methods involve measuring the motion of ions in an electromagnetic field i.e. dynamics driven by the Lorentz force:

$$\underline{F} = q\left(\underline{E} + \underline{v} \times \underline{B}\right)$$

Or the measurement of reaction Q values.

NB: Remember masses normally quoted as *atomic masses*.

# Traditional Mass Spectrometers




Bend ion paths into circular arcs using uniform field:

$$F = qvB = mv^2/\rho$$
$$q/m = v/B\rho$$

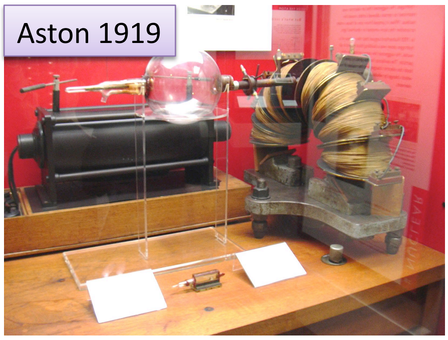
For common field:

$$q/m = E/B^2\rho$$

Select ions of one velocity:

$$\underline{F} = q\left(\underline{E} + \underline{v} \times \underline{B}\right) = 0$$
$$v = E/B$$

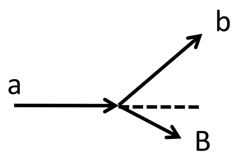
Aston 1919



Many variants.
Often measure relative to a well known mass.
Difficult to measure unstable isotopes.

# Measurement of Reaction Q Values

Take a reaction A(a,b)B: a + A ➔ b + B




If reaction has positive Q, more kinetic energy in exit channel.
If reaction has negative Q, less kinetic energy in exit channel.

Energy conversation: $T_a + m_a c^2 + m_A c^2 = T_b + m_b c^2 + T_B + m_B c^2$
Define Q value: $Q = m_a + m_A - m_b - m_B = T_b + T_B - T_a$

If you know three of the four masses, from a measured Q value get fourth mass.

Usually know the beam energy, so measuring the kinetic energy of the products gives Q…
…except measuring B, the slow-moving heavy target-like particle is often difficult!

Combine energy and momentum conservation. In classical limit:

$$T_a/m_a c^2 << 1,\ T_b/m_b c^2 << 1,\ T_B/m_N c^2 << 1,$$

$$Q = T_b\left(1 + \frac{m_b}{m_B}\right) - T_a\left(1 - \frac{m_a}{m_B}\right) - \frac{2\ \cos\theta}{m_B}\sqrt{T_a T_b m_a m_b}$$

MANCHESTER 1824
The University of Manchester

# Other methods

There are many other types of mass measurements:

- *Time-of-flight measurements*: accelerate ions under the same electric potential and measure time of flight across a known distance
- *Quadrupole mass analyzers:* selectively stabilize ions of different masses passing through a radio frequency (RF) quadrupole field.
- *Ion traps:* Confine ions using electric/magnetic fields, use RF to excite them into harmonic modes and extract mass from resonant frequencies.
- *Storage rings:* Confine ions to a ring using e/m fields. Time how long they take to go around the racetrack.

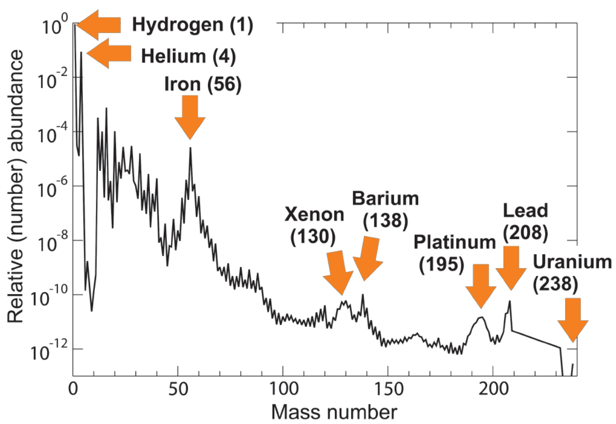
You could READ about some of the methods!

MANCHESTER 1824
The University of Manchester

# Isotopic Abundance

Relative intensity of different isotopes in a mass spectrum gives the *isotopic abundances*.

Abundance is not an intrinsic property. Arises due to differences in origin (production mechanism, time and location).




Example: relative abundance of different masses in the solar system. Labeled peaks are where one isotope dominates.

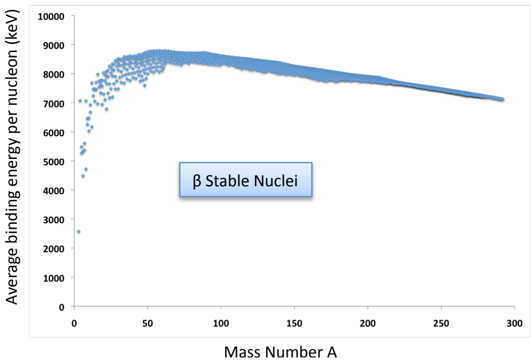
Isotopic abundances are important:
clues to astrophysical nuclear synthetic mechanisms
basis of many different dating techniques e.g. $^{14}C$
small variations in isotopic abundances can be used to track the origin of different samples.

MANCHESTER 1824
The University of Manchester

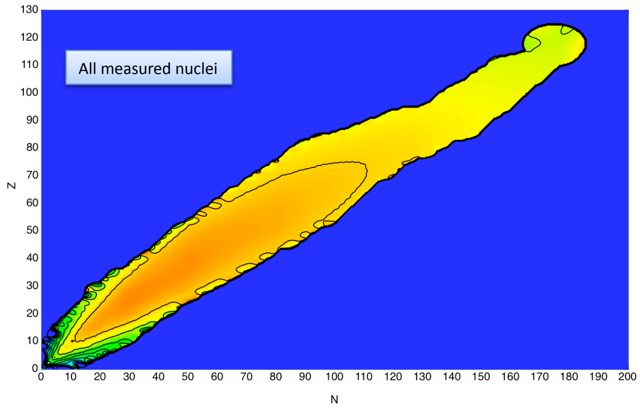
# Binding Energy, Average Binding Energy per nucleon and Nucleon Separation Energy

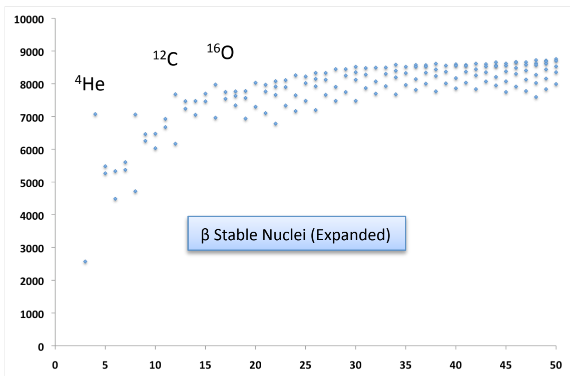
MANCHESTER 1824
The University of Manchester

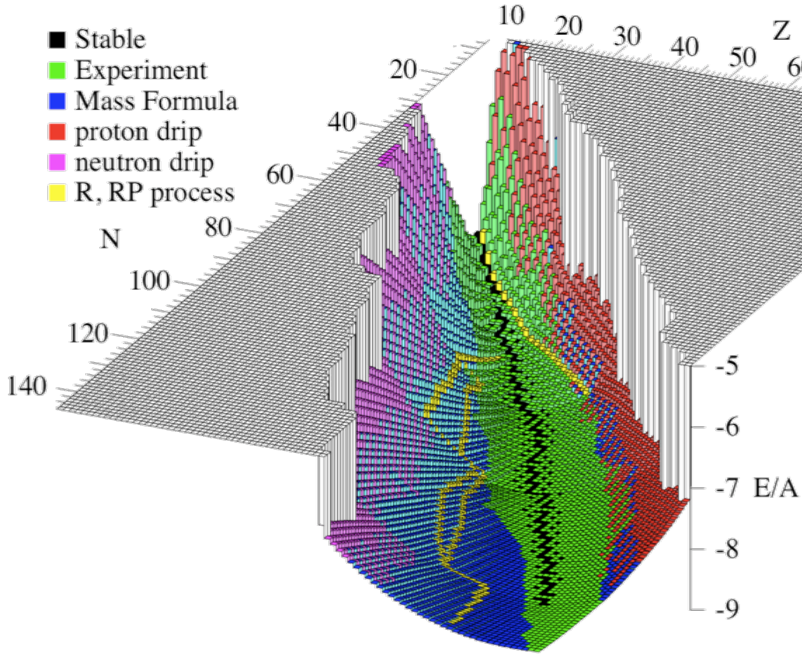
# Average Binding Energy per Nucleon




- Most nuclei BE/A is pretty constant around 7 to 8.5 MeV/A.
- Very steep rise in light nuclei until around A=10-12.
- Some nuclei have unusually high BE/A compared to their neighbours e.g. $^{4}$He has highest BE/A for A<12.
- Highest BE/A is 8794.549 keV/A for $^{62}$Ni.
- Lowest BE/A for a stable, bound nucleus is $^{2}$H with 1112.283 keV/A



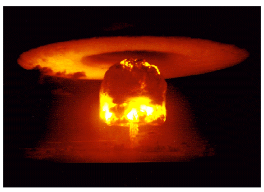


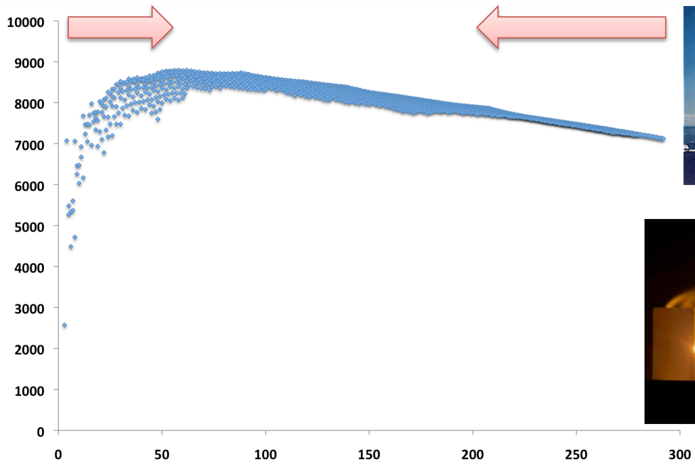

MANCHESTER 1824
The University of Manchester

# Energy Release from Nuclei

Up to $^{62}$Ni, *fusion* of two nuclei moves from low to high BE/A, liberating energy.

Down to $^{62}$Ni, *fission* of a heavy nuclei moves from low to high BE/A, liberating energy.







BUT inducing these reactions in a controlled manner is difficult!

# Describing BE and Masses

Try characterise the BE, and therefore masses of nuclei:

$$\mathrm{BE} = Zm_{1H} + Nm_n - m\left(A, Z\right)$$

Write down a formula for BE driven by what we know about nuclei.

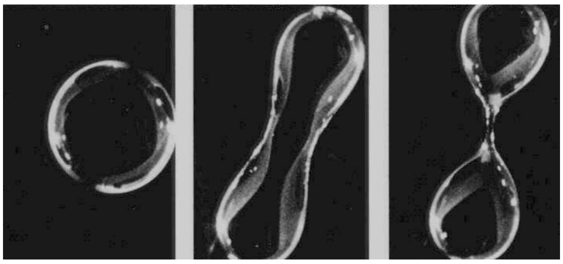
"Most nuclei BE/A is pretty constant around 7 to 8.5 MeV/A."

SO adding more nucleons will just increase the overall BE: $\mathrm{BE} \propto A$

Total BE must be related to the number of nucleon-nucleon interactions…the larger the number, the higher the BE.
In a nucleus, there are *A* nucleons…so there are *A(A-1)/2* pairs…so *A(A-1)/2* interactions.
Why doesn't BE vary as *A(A-1)*?

Well…so-called *saturation* property could arise if nucleons only interact with their nearest neighbours…this would happen if the force between them was short ranged i.e. range is of the order of the size of a nucleon around 1-2 fm. (More next week!)

This is similar to the Van de Waals forces holding molecules together in a liquid drop….hence *liquid-drop model* of nuclear masses.



So if each nucleon interacts with *x* neighbours, then BE goes as *xA*.

Hang on….those at the surface don't have as many nearest neighbours!

Add a term which reduces the BE in proportion to the surface area, assuming spherical nuclei. Surface area is: $4\pi R^2 = 4\pi\left[r_0 A^{1/3}\right]^2 \propto A^{2/3}$

First two terms in an empirical formula describing BE and masses:

$$\mathrm{BE} = a_v A - a_s A^{2/3}$$
$$m(A,Z) = Zm_{1H} + Nm_n - \mathrm{BE} = Zm_{1H} + Nm_n - a_v A + a_s A^{2/3}$$

*So-called volume and surface terms.*

Treating the nucleus as a fluid "ignores" its particulate nature….*macroscopic* model.

The University of Manchester

# Coulomb Term

Most stable nuclei have highest BE…so what we have so far is clearly WRONG!
Could keep increasing A forever and BE increases….but nuclei have A<293..
[Although do neutron stars count as whop-off nuclei??]

The more protons, the higher the nuclear charge and the stronger the Coulomb repulsion between them. Electrostatic forces must fight against the nuclear binding.

As part of first problem class you showed that the electrostatic energy of a uniformly charged sphere is:

$$\frac{3}{5}\frac{(Ze)^2}{4\pi\epsilon_0}\frac{1}{R} = \frac{3}{5}\frac{(Ze)^2}{4\pi\epsilon_0}\frac{1}{r_0 A^{\frac{1}{3}}} \propto \frac{Z^2}{A^{1/3}}$$

But protons come already constructed, and so the electrostatic repulsion within the proton should contribute to the proton mass and not the nuclear mass. Each proton electrostatically repels Z-1 other protons: so correct by replacing $Z^2$ with Z(Z-1).

So add a *Coulomb term:*

$$-a_c\frac{Z(Z-1)}{A^{1/3}}$$

Sometimes self-interaction energy is neglected and form above used!

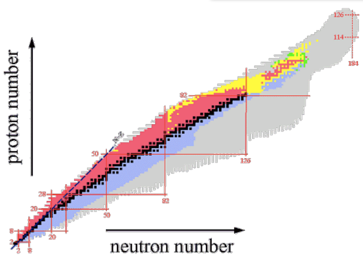
The University of Manchester

# Symmetry Term

Okay….so what is the most stable isotope for a particular mass?

$$\mathrm{BE} = a_v A - a_s A^{2/3} - a_c\frac{Z^2}{A^{1/3}} \qquad \left(\frac{\partial\ \mathrm{BE}}{\partial Z}\right)_A = -2a_c\frac{Z}{A^{1/3}}$$

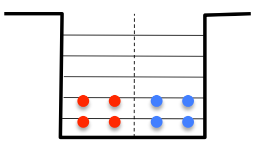
i.e. max when Z=0! WRONG! [Use Z(Z-1) form and get Z=1…still WRONG!].



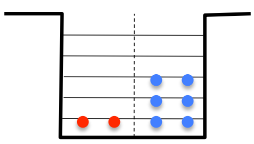

Actually no preference for large excesses of either nucleon; light nuclei stable systems have N similar to Z!

Think of nucleons sitting in a nuclear box…
….must sit in quantum energy levels according to Pauli's principle.

Light nucleus with different N/Z ratios:



Lower total energy
Higher BE



Higher total energy
Lower BE
β decays!

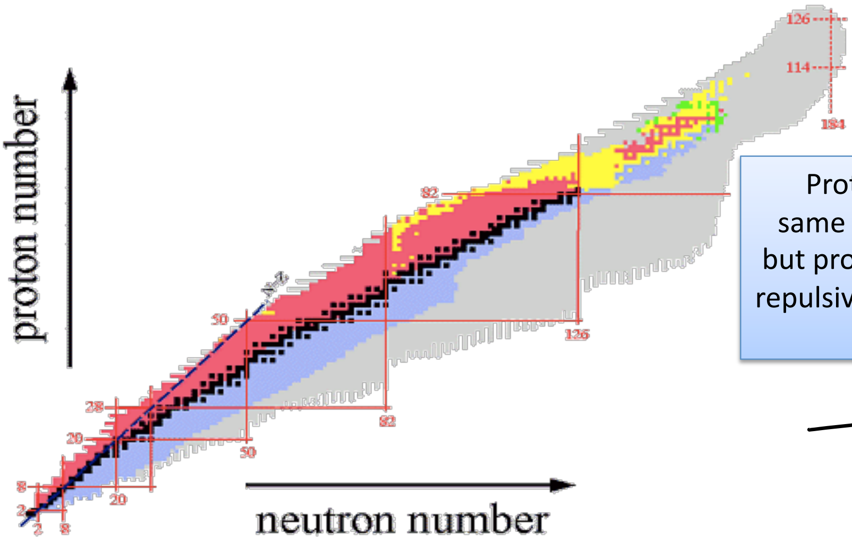
Need to add term to introduce a preference for symmetry…a *symmetry term!*
Analysis of a simple quantum Fermi gas suggests something like:

$$-a_a\frac{(N-Z)^2}{A}$$

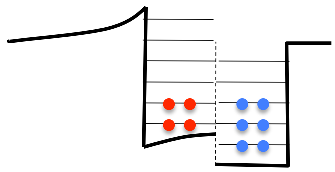
Competition between Coulomb and symmetry terms define line of stability.

$$\mathrm{BE} = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(N-Z)^2}{A}$$

Light nuclei Z is small so strong preference for N=Z.
Heavier nuclei: Coulomb instability grows rapidly with Z, can be minimised by having N>Z i.e. compared to N=Z same volume but smaller charge.



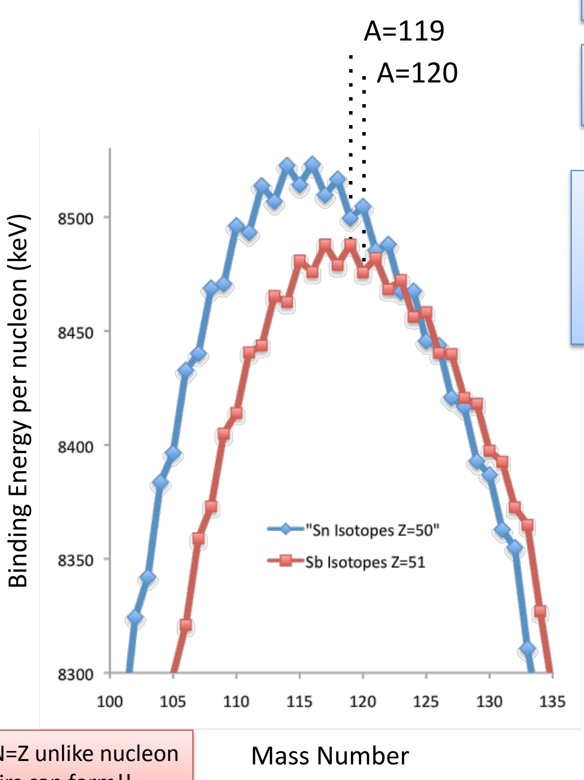

Protons and neutrons fill to same extent (to the Fermi level) but proton levels shifted up by the repulsive Coulomb potential, giving a neutron excess.



# Odd-Even Effects

Sn Z=50: even-even isotopes higher BE than odd-even isotopes

Sb Z=51: even-odd isotopes higher BE than odd-odd isotopes




Of the 284 terrestrially-occurring isotopes, only 9 odd-odd: $^{2}$H, $^{6}$Li, $^{10}$B and $^{14}$N are stable
$^{40}$K, $^{50}$V, $^{138}$La, $^{176}$Lu and $^{180m}$Ta are long lived
165 even-even and 110 odd-even/even-odd
Pure & Appl. Chem. Vol. 70, 217-235 No. 1 (1998)

Indicative of increased BE when like-nucleons form pairs. Add minor correction to the BE formula:

$$\pm a_p A^{-1/2}$$

add for even-even
zero for odd-even
minus for odd-odd

No particular physics argument for this form. It just seems to work! i.e. EMPIRICAL.
Other forms can be used…for example, you might see:

$$\pm a_p' A^{-3/4}$$

plus for even-even, minus for odd-odd, zero for odd-even.

Along N=Z unlike nucleon pairs can form!!

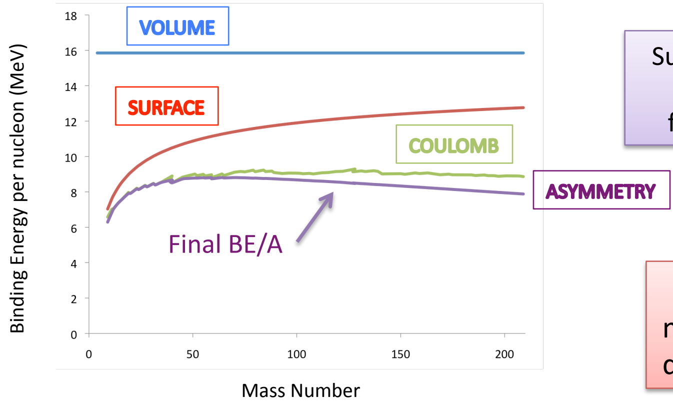
# Semi-Empirical Mass Formula

First version by Carl Friedrich von Weizsäcker in 1935.
*Modern versions available with various improvements, including removing the restriction to spherical nuclei.*
Example: P. Möller et al., At. Data Nucl. Data Tables 59 (1995) 185

$$\mathrm{BE} = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} \pm a_p A^{-1/2}$$

Can perform a least-squares fit to experimental data and find the values of the coefficients....various sets available due to small differences in how this is done and over what nuclei.

$$a_v = 15.85 \text{ MeV}$$
$$a_s = 18.34 \text{ MeV}$$
$$a_c = 0.71 \text{ MeV}$$
$$a_a = 23.21 \text{ MeV}$$
$$a_p = 12 \text{ MeV}$$




Successive effect of various terms for stable nuclei.

Wapstra et al.
Nucl. Data Tables 9 (1971) 267

Useful for general trends in BE and masses, estimating Q values, what things decay and how....and EXAM QUESTIONS!

# Key Ideas … Lecture TWO

- Techniques used for measuring nuclear masses.
- Binding energy and Q values.
- Systematics of average BE per nucleon.
- Generating energy in nuclear reactions.
- Physical ideas in the liquid drop model.
- Understanding of semi-empirical mass formula.