BETA DECAY

In β-decay processes, the mass number A is constant, but the proton and neutron numbers change as the parent decays to a daughter that is better bound with lower mass. In β^- decay, a neutron transforms into a proton, and an electron is emitted to conserve charge. Overall, $Z \rightarrow Z+1$. In β^+ decay, a proton transforms into a neutron, an electron is emitted and $Z \rightarrow Z-1$.

In a third process, called electron capture or EC decay, the nucleus swallows an atomic electron, most likely from the atomic 1s state, and a proton converts to a neutron. The overall the effect on the radioactive nucleus is the same in β^+ decay.

The electrons (or positrons) emitted in the first two types of β decay have continuous kinetic energies, up to an endpoint which depends on the particular species decaying, but is usually of the order of 1 MeV.

The electrons (or positrons) cannot pre-exist inside the nucleus. A simple uncertainty estimate suggests that if they did, the endpoint would be much higher in energy:

If it did pre-exist, then the electron would be somewhere in the nucleus beforehand, with an uncertainty in position equal to the size of the nucleus, $\Delta x \sim r_0 A^{1/3} \sim 5$ fm, in a medium mass nucleus.

The uncertainty in momentum is then $\Delta p = \frac{\hbar}{\Delta x} = \frac{197 \text{ MeV.fm/c}}{5 \text{ fm}} = 40$ MeV/c. So the smallest momentum that the endpoint could be would correspond to 40 MeV/c. The corresponding smallest endpoint energy would be: $E^2 = c^2p^2 + m^2c^4 \approx c^2p^2 = 40$ MeV. This is far too big compared to experimentally known endpoints, so the electrons cannot have pre-existed in the nucleus.

The continuous energy spectrum of the emitted electron/positron implies that there are more than two bodies in the final state, otherwise energy and momentum conservation would give a single discrete kinetic energy. The third body is a neutrino or antineutrino. These are elementary particles that interact weakly, with half integer spin and a very small, but non-zero mass, known to be less than 2 eV.

In detail, the processes and their Q values can be written as:

$$\beta^- \text{ Decay}: \; X \rightarrow Y + \beta^- + \bar{\nu}_e$$

$$Q_{\beta^-} = m_x - m_y$$

$$\beta^+ \text{ Decay}: \; X \rightarrow Y + \beta^+ + \nu_e$$

$$Q_{\beta^-} = m_x - m_y - 2m_e$$

$$\text{EC Decay}: \; X + e^- \rightarrow Y + \nu_e$$

$$Q_{EC} = m_x - m_y - BE_{\text{captured atomic electron}}$$

Note these masses are *atomic* masses. Make sure that you can derive these expressions from the basic definitions of Q value and nuclear masses m^N.

Example:

$$\beta^- \text{ Decay}: \; X \rightarrow Y + \beta^- + \bar{\nu}_e$$

$$\text{Relationship between nuclear and atomic mass}: \; Xm^N = m - Zm_e + \sum B_i$$

$$\text{From definition of Q} - \text{value}: Q_{\beta^-} = m_X^N - \left(m_Y^N + m_e + m_{\bar{\nu}_e}\right)$$

$$= m_X - Zm_e + \sum_X B_i - \left[m_Y - (Z+1)m_e + \sum_Y B_i + m_e + m_{\bar{\nu}_e}\right]$$

$$= m_X - m_Y + \sum_X B_i - \sum_Y B_i - m_{\bar{\nu}_e}$$

$$= m_X - m_Y$$

The differences in the atomic binding energies and the neutrino mass are less than a few eV and are neglected. Note that in the same calculation for β^+ decay, where the atomic number decreases in the decay, the various electron/positron masses do not cancel out in the same way. For EC, you need to take careful note of the binding energy of the electron swallowed in the processes; since this is in a deeply bound atomic state, the energy is order keV and cannot be so easily neglected.

Lifetimes for β decay vary from a few milliseconds to 10^{16} years or so. The details of the mechanism are more complicated than for α decay, but the half lives are still sensitive to the Q value: $\tau_{1/2} \propto Q^{-5}$

As with α decay, β decay can populate excited states. If these states have energies that are above the nucleon separation energy S_N (the lowest energy needed to remove a nucleon from a nucleus), then a nucleon will be emitted. This leads to the processes of β-delayed neutron emission, if $E^* > S_n$, and β-delayed proton emission if $E^* > S_p$. The existence of β-delayed neutron decay is important, since it allows us the time needed to control a nuclear reactor.

If E^* is less than both neutron and proton separation energies, the states populated usually γ decay to the ground state.

By considering the semi-empirical mass formula (SEMF) for a constant A, the masses of the nuclei form a parabola as a function of Z (see pictures on the lecture handouts). Nuclei therefore β decay in order to reach the bottom of the parabola where they have the lowest mass and highest binding energy.

When A is odd, there is one parabola and the bottom of it usually corresponds to a stable isotope. So there is usually just one stable odd-A isotope (see pictures on the lecture handouts).

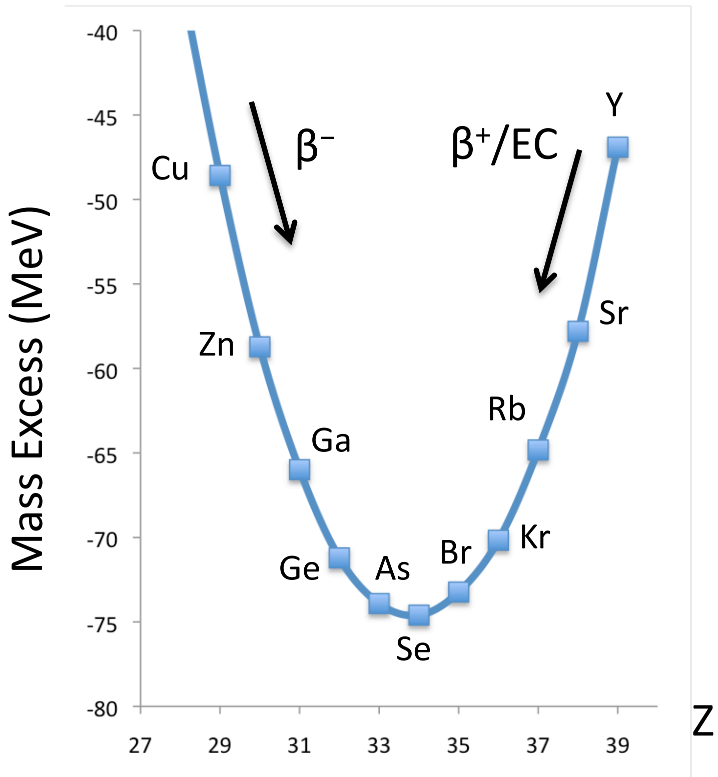
For even A chains, there are two parabolas. One corresponding to odd-odd nuclei, displaced above another, corresponding to even-even nuclei, by the pairing term in the SEMF. Nuclei again β decay, but this time between the two parabolas, trying to reach the minimum mass again. But now, due to the displacement of the odd-odd nuclei upwards by pairing, often two even-even nuclei are left as being stable because the β decay to the odd-odd nucleus between them $requires$ energy and so won't spontaneously happen (see pictures on the lecture handouts).

For example, on the slides the $A = 76$ chain is shown. On the neutron-rich side, ^{76}Ni will decay to Cu, Cu to Zn, Zn to Ga, and Ga to ^{76}Ge. On the proton-rich side, ^{76}Sr decays to Rb, Rb to Kr, Kr to Br, Br to Se. Notice that ^{76}As lies higher in mass than both ^{76}Ge and Se, so it can decay either way. Both ^{76}Ge and ^{76}Se are stable with respect to normal β decays.

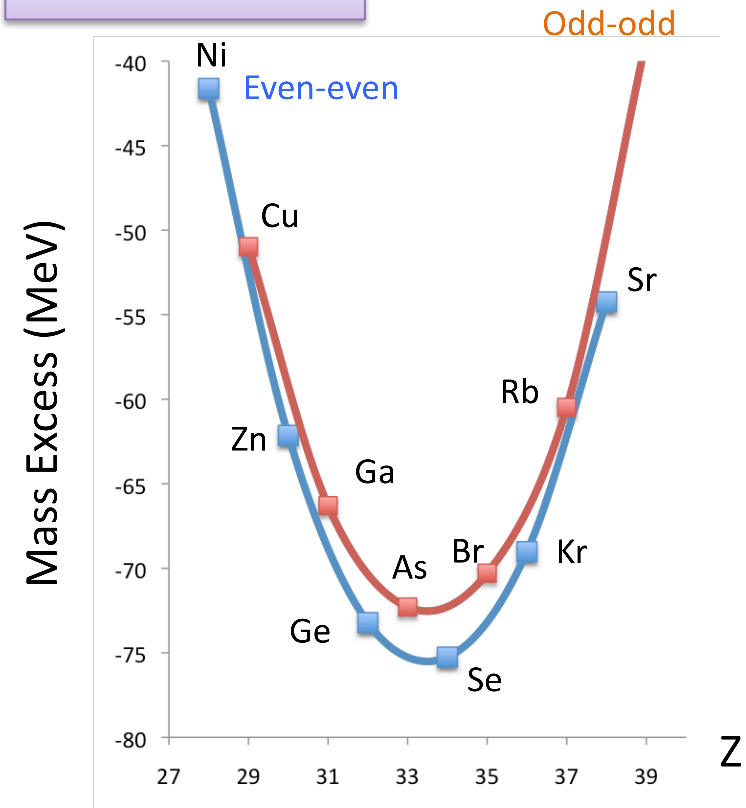
These two stable even-even nuclei could decay IF there were two simultaneous β decays and the overall Q value for the process were positive. It turns out that this can happen in double β decay, but is highly unlikely so has very long half lives $\sim 10^{21}$ years. This process has been observed in some cases.

There is a lot of interest in looking for a possible neutrino-less double β decay; if this were observed it would indicate that neutrinos were their own antiparticle and is one of a very small number of possible ways to measure the absolute mass of neutrinos. This process has not yet been definitively observed, although lots of experiments are looking for it, but the lifetime is likely to be greater than 10^{25} years.

A=77 Odd Mass



A=76 Even Mass



$$M(A,Z) = ZM_{1H} + (A-Z)M_n - a_v A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + a_a \frac{(A-2Z)^2}{A} \mp \delta . A^{-1/2}$$

$$M(A,Z) = \alpha A - \beta Z + \gamma Z^2 + \frac{\delta}{A^{1/2}}$$

$$\alpha = M_n - a_v + a_s A^{-1/3} + \frac{a_a}{4}$$

$$\beta = a_a + M_n - M_H$$

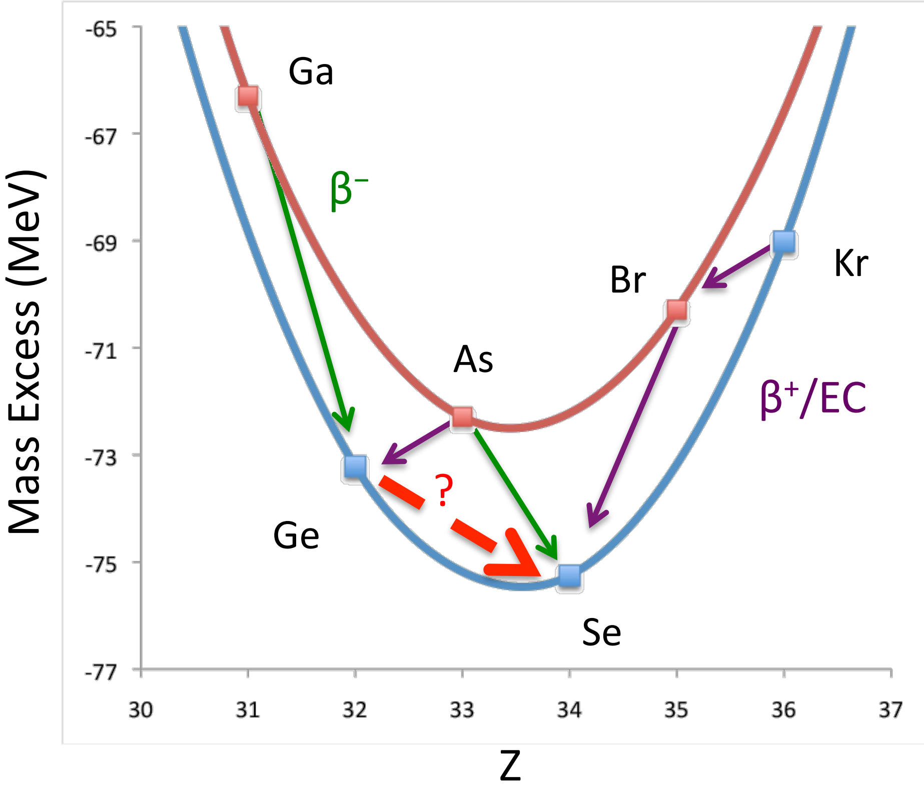
$$\gamma = \frac{a_a}{A} + \frac{a_c}{A^{1/3}}$$

$$\delta = a_p$$

-11.2 MeV for even-even
0
+11.2 MeV for odd-odd

Double beta decay:

Example in A=76 Isobars:
Can ^{76}Ge decay to ^{76}Se?



Two-neutrino double beta decay:

$$^{76}\mathrm{Ge} \rightarrow ^{76}\mathrm{Se} + 2e^{-} + 2\bar{\nu}$$

essentially two simultaneous normal β decays.
Measured with lifetimes of around 10^{21} years.

Neutrino-less double beta decay:

$$^{76}\mathrm{Ge} \rightarrow ^{76}\mathrm{Se} + 2e^{-}$$

occurs if neutrinos are their own antiparticles!
(Probably) not been observed (yet)!
IF it occurs could be the only way to measure the absolute mass of the neutrino.
Lifetimes expected $> 10^{25}$ years!