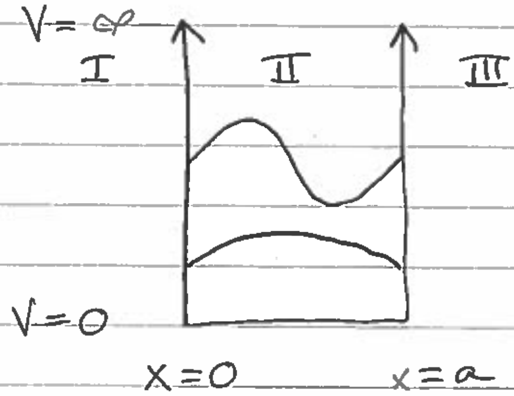
④ Multiple Barriers and Resonant Scattering

ⓐ **Bound states**: so far sol$^{\underline{n}}$s all travelling waves; continuous E
bound states – particles localised, usually with discrete energy levels.

simplest is infinite sq well



II = classically allowed
→ harmonic functions.
I + III = forbidden, $V \to \infty$

$$\left[\frac{-\hbar^2}{2m}\frac{d^2}{dx^2} + V\right]\psi = E\psi$$

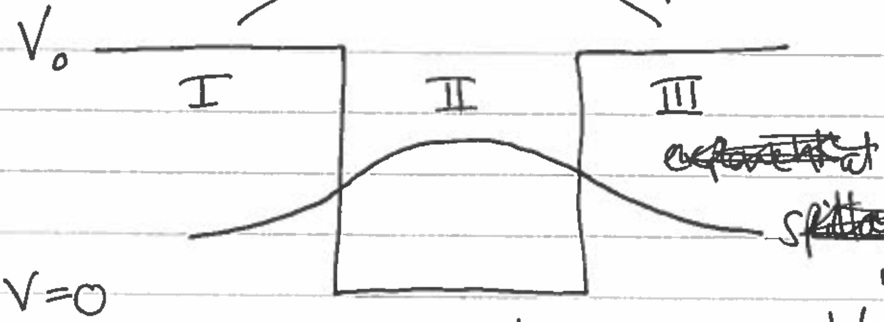
only get out of infinite issue if $\psi = 0$
i.e. completely forbidden.

BC: $\psi = 0$ $x = a$ and $x = 0$
→ standing waves
Sinusoidal sol$^{\underline{n}}$ $\psi = 0$ $x = 0$
$\psi = N\sin kx$. $\psi = 0$ $x = a$ → $k = \frac{n\pi}{a}$ $n = 1, 2, 3 \ldots$

$p = \frac{h}{\lambda}$ $k = \frac{2\pi}{\lambda}$ $E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 n^2 \pi^2}{2ma^2}$ discrete levels

never get $V \to \infty$, so more realistic finite sq well:

now only classically forbidden

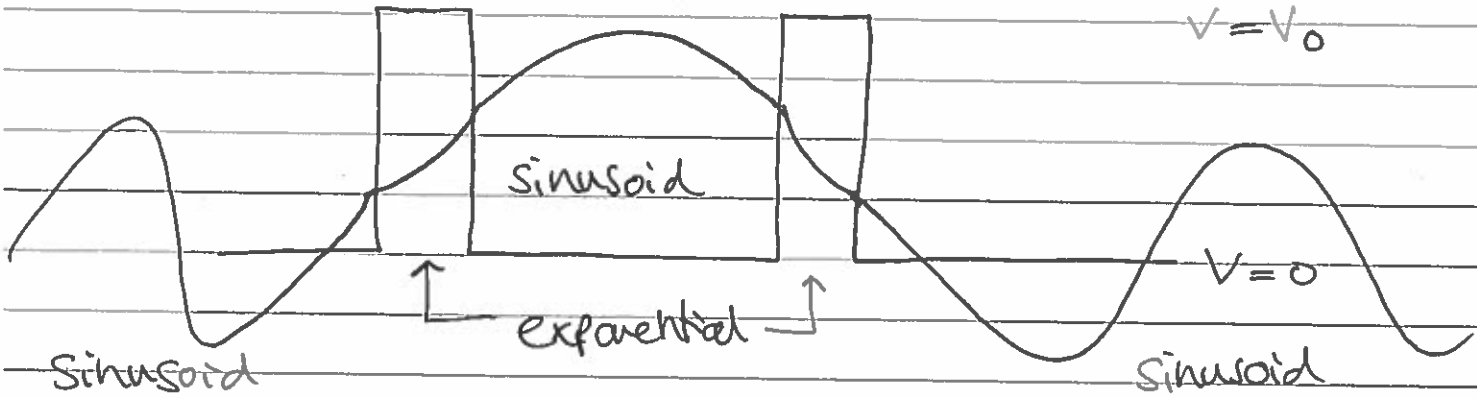


Sketch gs sol$^{\underline{n}}$

gs wave fnc: due to exponential spillage, less curved than infinite sq well
"wavelength" longer
$p = h/\lambda$ → mom lower
energy levels lower $E^n_{finite} = E^n_{infinite}$

READ ABOUT HOW TO SOLVE PROPERLY

(b) Particle inside double barrier
drop the sides of the finite sq well



If you pop a particle in — it isn't bound as non-localised wavefunction — leaks out by tunnelling
Not strictly a stationary state with ~~discrete~~ definite energy.
Wave packet inside will decay away with a certain lifetime τ related to the tunnelling prob.
Remember uncertainty principle

$\tau \Gamma \sim \hbar$

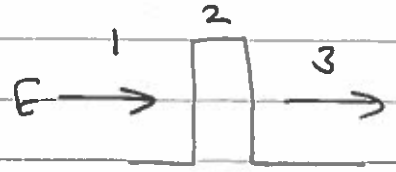
Γ — uncertainty in energy ie. a "width" rather than discrete level
τ — don't know when it pops out but mean lifetime.

(a bit like a classical damped oscillator)

But if $E < V_0$ tunnelling might be low prob
τ might be a long time
Γ might be a small width
then talk about "quasi-bound" states with energies similar to a sq well.

(c) Particle Incident on Double Barrier

One barrier:



$\psi_1 = e^{ikx} + \frac{B}{A} e^{-ikx}$

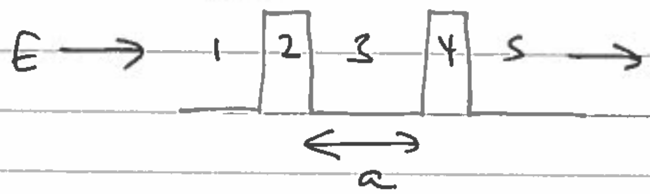
$\psi_3 = \frac{F}{A} e^{ikx}$

reflection amp $= \frac{B}{A} = r e^{i\varphi_r}$

Transmission amp $= \frac{F}{A} = t e^{i\varphi_t}$

calculate as in lecture 2
in general complex so write here as
modulus and phase

Double barrier with identical humps:



$$\psi_5 = t e^{i\varphi_t} \left[1 + r e^{i\varphi_r} r e^{i\varphi_r} e^{2ika} + \left(r e^{i\varphi_r} r e^{i\varphi_r} \right)^2 e^{4ika} + \dots \right] t e^{i\varphi_t} e^{ikx}$$

tunnel straight thru both

reflects twice then escapes travels extra 2a

reflects four times

higher terms

$$= t^2 e^{2i\varphi_t} \sum_{n=0}^{\infty} \left(r^2 e^{2i\varphi_r} e^{2ika} \right)^n . e^{ikx}$$

geometric series

$$= \frac{t^2 e^{2i\varphi_t} . e^{ikx}}{1 - r^2 e^{2i\varphi_r} e^{2ika}}$$

Double Tunnelling prob $= \left| \frac{t^2 e^{2i\varphi_t}}{1 - r^2 e^{2i(\varphi_r + ka)}} \right|^2$

Max if $e^{2i(\varphi_r + ka)} = 1$

Max Double Tunnelling prob $= \left| \frac{T}{1-R} \right|^2$ as single barrier $T = |te^{i\varphi_t}|^2 = t^2$

- $T+R=1 \rightarrow$ Max double Tunnelling prob $= 100\%$!

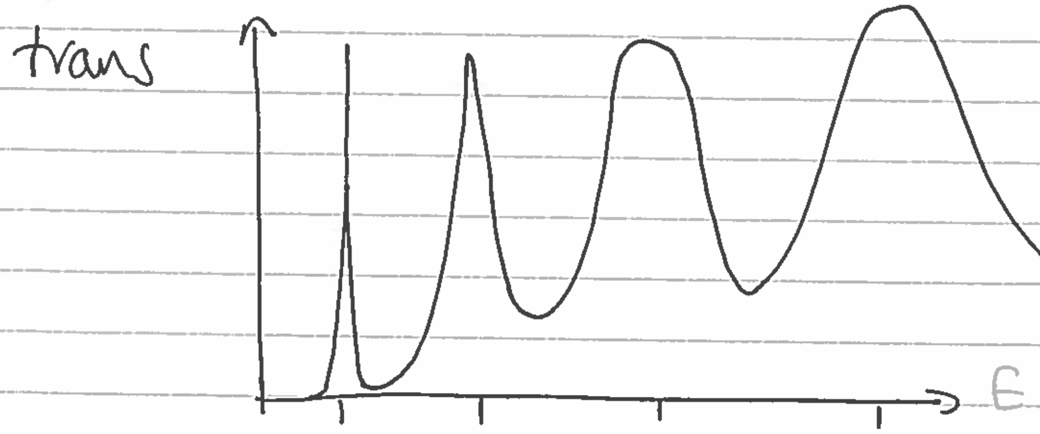
Crazy: if one barrier small tunnelling prob
if two barriers all goes through!!

Tunnelling condition: $e^{2ita} = 1 \rightarrow \cos 2ta = 1$
(if neglect φ_r)
$t = \frac{n\pi}{a}$ ie. half λ fits in between barriers.

- Resonance when reflected waves travel an extra distance which is multiple of λ
  - $\rightarrow$ constructive interference.
  - $\rightarrow$ large prob of finding particle in well AND large tunnelling prob.

(route to fill q. dots)

Also corresponds to matching of incoming energy to that of quasi-bound levels in well
$t = \frac{n\pi}{a}$ quantisation condition for inf sq well



different quasi bound levels

- higher quasi-bound level. larger tunnelling out prob so larger widths

Examples
loading q. dots
resonant tunnelling diodes
graphene-BN-graphene layers
resonances in nuclear fission...