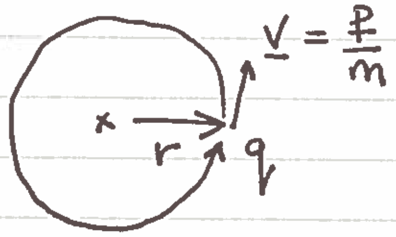
felt a little los .

⑬ Magnetic Moments

[How do you know about ang mom? From the magnetic effects of charges swirling around.]

Circulating charges generate magnetic moments

Classically orbiting charge



$\mu = IA$ out of page (z dirc^n^)

$= \frac{q}{2\pi r/v} . \pi r^2 = \frac{q}{2m} pr$

$= \frac{q}{2m} L_z$ as $\underline{L} = \underline{r} \times \underline{p} = rp\,\hat{z}$

for planar motion.

The energy of μ in a $\underline{B}$ field depends on orientation:

$PE = -\underline{\mu}.\underline{B}$ ie. $\underline{\mu}$ likes to be parallel to field.

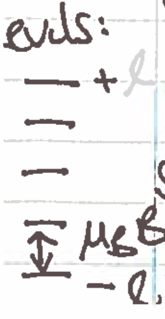
QM: replace with operators $\hat{\underline{\mu}} = \frac{q}{2m} \hat{\underline{L}}$

Example of electron in uniform $\underline{B}$ along z direction:

$q = -e$ $\quad \hat{H} = -\hat{\underline{\mu}}.\underline{B} = + \frac{e}{2m} \hat{\underline{L}}.\underline{B}$ $\quad$ [$\underline{L}$ prefers to be antiparallel to $\underline{B}$ for neg. charge]

$= + \frac{eB}{2m} \hat{L}_z$

Eigenenergies are $\frac{eB}{2m} m_l \hbar$ when $m_l = +l$ to $-l$



Often write using

Bohr magneton $\mu_B = \frac{e\hbar}{2m} = 5.8 \times 10^{-5}$ eV/T in sensible units

So eigenvalues $= m_l \mu_B B$ [so much for orbiting]

Electron has intrinsic spin which should also generate a mom. Assume there is an analogous proportionality to ang. mom.

$$\hat{\mu}_s = \frac{-e}{2m}\hat{S} \times g \leftarrow$$ some proportionality factor called a g-factor.

For an electron: $g = 2$ to within 0.1%

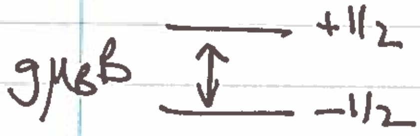
In general $\hat{\underline{\mu}} = \hat{\underline{\mu}}_L + \hat{\underline{\mu}}_S$ but keep thing simple for now with $L=0$.

- Electron just sitting in B field: $\hat{H} = -\underline{\mu}_s . \underline{B} = \frac{eg}{2m}\hat{\underline{S}}.\underline{B}$

$= \frac{egB}{2m}\hat{S}_z$ if $\overset{B}{\hat{}}$ in z direction

~~Main eigenfunctions of~~ $\hat{S}_z$:

Eigenvalues $= \frac{egB}{2m} m_s\hbar = \pm\frac{g}{2}\frac{e\hbar}{2m}B = \pm\frac{g}{2}\mu_B B$



Splittings of order $\mu_B B = 0.5$ to 1×10^{-4} eV even in a strong MRI field of ~ 1T [μwave photons].

- For this electron, how does state develop with time?

TDSE: $i\hbar\frac{d\Psi}{\partial t} = \hat{H}\Psi$

mint if this were ttordau as vac? e.Bz nan By ce.

In this case $\hat{H} \neq \hat{H}(t)$ and can separate

$$\Psi = \chi . T(t)$$

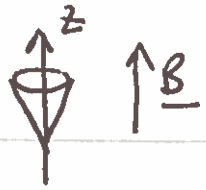
$\alpha_z, \beta_z \leftarrow$ spin time $\rightarrow e^{-i\omega t}$ $\omega = E/\hbar$

Write general soln: superposition of sep soln. $\omega = \frac{g\mu_B B}{2\hbar}$

$$\Psi(t) = a_1 e^{-i\omega t}\alpha_z + a_2 e^{i\omega t}\beta_z$$

$$= \begin{pmatrix} a_1 e^{-i\omega t} \\ a_2 e^{i\omega t} \end{pmatrix} \qquad \Psi(0) = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$\langle S_i \rangle = \chi^+ \hat{S}_i \chi$$



If initially in, say, α_z ie. $a_1 = 1$ $a_2 = 0$

$$\Psi(t) - \alpha_z e^{-i\omega t}$$

└ inconsequential phase factor

eigenstate of S_z

$$\langle S_z \rangle = \hbar/2$$
$$\langle S_x \rangle = 0$$
$$\langle S_y \rangle = 0$$

} no time dep in expectation values
→ STATIONARY STATE

$= \frac{1}{\sqrt{2}}\{\alpha_z + \beta_z\}$

Compare with an initial state of, say, $\alpha_x = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$

$$\Psi(t) = \frac{1}{\sqrt{2}}\alpha_z e^{-i\omega t} + \frac{1}{\sqrt{2}}\beta_z e^{i\omega t} = \frac{1}{\sqrt{2}}\begin{pmatrix}e^{-i\omega t}\\ e^{i\omega t}\end{pmatrix}$$

$a_1 = 1/\sqrt{2}$
$a_2 = 1/\sqrt{2}$

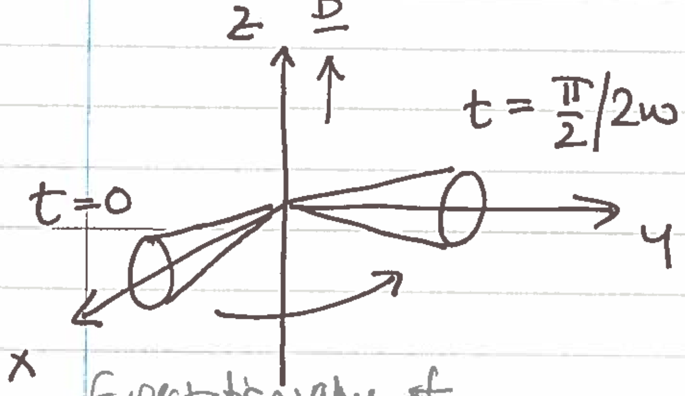
$$\langle \hat{S}_z \rangle = \Psi^+ \hat{S}_z \Psi = \frac{1}{2}\frac{\hbar}{2}\left(e^{i\omega t}\ e^{-i\omega t}\right)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}e^{-i\omega t}\\ e^{i\omega t}\end{pmatrix}$$

$$= \frac{\hbar}{4}\left(1 - 1\right) = 0$$ ie. equal prob of $\pm\hbar/2$

$$\langle \hat{S}_x \rangle = \Psi^+ \hat{S}_x \Psi = \frac{1}{2}\frac{\hbar}{2}\left(e^{i\omega t}\ e^{-i\omega t}\right)\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}e^{-i\omega t}\\ e^{i\omega t}\end{pmatrix}$$

$$= \frac{\hbar}{4}\left(e^{2i\omega t} + e^{-2i\omega t}\right) = \frac{\hbar}{2}\cos(2\omega t)$$

Similarly $\langle \hat{S}_y \rangle = \frac{\hbar}{2}\sin(2\omega t)$ not stationary state!



Expectation value of
Spin rotates around z axis "precesses"

Rate of precession:

$$\omega_p = 2\omega$$
$$= g\mu_B B/\hbar$$
$$= egB/2m$$

LARMOR FREQ.

so a property of the average result!

→ BEWARE: not classical precession that would be, since fixes when y component

After $\omega_p t = 2\pi$ process around back to start.
But since $\omega_p = 2\omega$ the state vector $= -\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$
due to factors $e^{\mp i\omega t}$.

This is just like rotating the coordinate system by 2π and it doesn't get you what you started with.
— general property of spin-1/2 eigenvalues
needs a 4π rotation to get the same thing.

The minus sign is just a phase factor and doesn't normally change the physical properties; unless there is some interference effect.
→ double slit interference experiments
with neutrons have measured
these effects ⇒ RAE.

Spinors not the only thing that needs 4π rot^n.

- arm + cup of water
- binasuan. phillapines.
- dirac belt trick.

Can show that in any initial eigenstates $(\alpha_{x,y} \;\; \beta_{x,y})$ precession occurs. So if you have a bunch of electrons and switch on a field, in general they'll precess around the field.

Larmor freq $\omega_p = \frac{eB}{2m} g$

If $g = 2$ exactly $\omega_p = \frac{eB}{m}$

i.e. equal to the electron cyclotron freq the rate at which linear mom precesses in a magnetic field

If $g=2$ spin and direction of motion precess at the same rate.

Expt: $g = 2 \times 1.001....$ measured to 12 dp $\}$ expt
= theoretical calcul$^{\underline{n}}$ to 11 dp only! $\}$ wins!

v.high precision test of Standard Model.

Dirac showed QM+relativity
– electron is a point particle with some property that behaves like half-integer ang mom. with $g=2$ automatically
$g \neq 2$ due to QED corrections.

Started with 4 products, used 2 completely, partially used the $\alpha\beta$ combinations with $M=0$.

Can form another, single $M=0$ eigenfunction, making it orthogonal to the others:

$$\frac{1}{\sqrt{2}}\left[\alpha(1)\beta(2) - \beta(1)\alpha(2)\right]$$

$\hat{S}_\pm$ on this gives zero (SHOW IT!)

} 1 state with $M=0$ so S must be 0

NB// if swap $1 \Leftrightarrow 2$ introduces a minus sign.

→ this is an $S=0$ asymmetric singlet.

NB// started with four products: ended up with 4 combinations of them; if you add up squared coefficient of each set both equal 4.

NB// the coefficients are called Clebsch-Gordan coeffs.

NB// Could write this addition or coupling of two spin-$\frac{1}{2}$ as $S = \frac{1}{2} \otimes \frac{1}{2} = 0, 1$

What about something more complicated

$\underline{J} = \underline{L} + \underline{S}$ or $\underline{L} = \underline{L}^{(1)} + \underline{L}^{(2)}$

Can do a similar thing:

Largest M values are $M_L = L$ $M_S = S$

giving a state with $M_J = M_L + M_S = L + S$

Prove top of the ladder hitting with $\hat{J}_+ = \hat{L}_+ + \hat{S}_+$

Step down a rung with $\hat{J}_-$

Get a mixture of products with $M_J = L + S - 1$

Carry on until get to $M_J = -L - S$

Complete ladder with $M_J = L+S \ldots\ldots -|L+S|$

ie. substates of $J = L+S$

eg. $\hat{S}_+\alpha(1)\alpha(2) = \left[\hat{S}_+^{(1)}\alpha(1)\right]\alpha(2) + \alpha(1)\hat{S}_+^{(2)}\alpha(2)$

$= 0$ ie. $\alpha(1)\alpha(2)$ must be the eigenfunction of top of ladder ie. $M = m_1 + m_2 = +1$ as we thought.

Okay let's step down:

$$\hat{S}_-\alpha(1)\alpha(2) = \alpha(2)\hat{S}_-^{(1)}\alpha(1) + \alpha(1)\hat{S}_-^{(2)}\alpha(2)$$

$$= \hbar\left[\beta(1)\alpha(2) + \alpha(1)\beta(2)\right]$$

This is a combination of products with $M = m_1 + m_2 = 0$.
[but not normalised!]

And one more step:

$$\hat{S}_-\left[\beta(1)\alpha(2) + \alpha(1)\beta(2)\right] = \overbrace{\alpha(2)\hat{S}_-^{(1)}\beta(1)}^{=0} + \beta(2)\hat{S}_-^{(1)}\alpha(1)$$
$$+ \beta(1)\hat{S}_-^{(2)}\alpha(2) + \underbrace{\alpha(1)\hat{S}_-^{(2)}\beta(2)}_{=0}$$

$$= 2\hbar\,\beta(1)\beta(2)$$

State with $M = m_1 + m_2 = -1$

Another $\hat{S}_-\beta(1)\beta(2) = 0$ bottom of ladder.

M	Normalised Eigenfunction
$+1$	$\alpha(1)\alpha(2)$
0	$\frac{1}{\sqrt{2}}\left[\alpha(1)\beta(2) + \beta(1)\alpha(2)\right]$
-1	$\beta(1)\beta(2)$

} 3 states $M = +1$ to -1 so total quantum no. S must be 1

NB// if swap particles around ie. $1 \Longleftrightarrow 2$ these states remain the same.

→ form an $S = 1$ symmetric triplet

mentioned $\mu = \mu_s + \mu_L$ last time
how do you do that in QM?
same as how do you add two ang. mom.

Lecture a little short, did sums on identical electrons.

(14) Adding Angular Momentum

Classically if a particle had spin and orbital ang mom, just add vectorially: $\underline{J} = \underline{L} + \underline{S}$
In QM not quite so easy, so start with a simpler case adding spin of two spin-1/2 particles.
Combining the eigenfunctions of each particle can make 4 combinations: ($\alpha \equiv \alpha_{\frac{1}{2}}$ $\beta \equiv \beta_{\frac{1}{2}}$ in this lecture)

	$\alpha(1)\alpha(2)$	$\alpha(1)\beta(2)$	$\beta(1)\alpha(2)$	$\beta(1)\beta(2)$
$m_1 + m_2 =$	$+1$	0	0	-1

Since the z components are scalars and should just add, but what about the total spin of the two?

For each particle, can write down ladder operators:

$$\hat{S}_+ = \hat{S}_x + i\hat{S}_y = \hbar\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \hat{S}_- = \hat{S}_+ - i\hat{S}_y = \hbar\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\hat{S}_+\alpha = \hbar\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = 0$$ top of ladder

$$\hat{S}_-\beta = 0$$ bottom of ladder

$\alpha \equiv \alpha_{\frac{1}{2}}$ $\beta \equiv \beta_{\frac{1}{2}}$ in this lecture.

$$\hat{S}_-\alpha = \hbar\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \hbar\beta$$ move down

$$\hat{S}_+\beta = \hbar\alpha$$ move up.

The total spin $\underline{\hat{S}} = \underline{\hat{S}}^{(1)} + \underline{\hat{S}}^{(2)}$ — label says which particle to operate on.
is ang mom so should have ladder operators:

$$\hat{S}_\pm = \hat{S}^{(1)}_\pm + \hat{S}^{(2)}_\pm$$

see what this does to our eigenfunction products.

Find some "probability" left over since only partially used some combinations of eigenfunctions of $\hat{L}$ and $\hat{S}$.
Form a second state with $m_J = L + S - 1$
and run down its ladder to $m_J = -|L + S - 1|$
ie. a set with $J = L + S - 1$.

And carry on forming additional ladders until you "run out" of probability .....

What you end up with is:

Eigenvalues of $\underline{\hat{J}}^2 = (\underline{\hat{L}} + \underline{\hat{S}})^2$ are $J(J+1)\hbar^2$

where $J = L+S,\ L+S-1,\ \ldots\ldots,\ |L-S|$

Eigenvalues of $\hat{J}_z = \hat{L}_z + \hat{S}_z$ are $m_J \hbar$

where $m_J = +J,\ J-1,\ \ldots\ldots\ -J$

ie. each J value has $(2J+1)$ "magnetic" substates.
If interested in eigenfunctions can look up Clebsch Gordan's

Example:
An electron with $L=3$ and $S=1/2$ forms states with $J_1 = 3 + 1/2$ or $3 - 1/2$
$= 7/2$ or $5/2$.

If you added a second ~~electron~~ ~~particle~~ electron. with $J_2 = 1/2$ the combined system would have.
$$J = 7/2 + 1/2,\ 7/2 - 1/2,\ 5/2 + 1/2,\ 5/2 - 1/2.$$
$$= 4,\ 3,\ 3 \text{ or } 2$$

~~NB, I was careful not to say a second electron → identical particles need careful handling!~~

NB, if combining two identical particles with same spin needs more care! Pauli principle comes into play

For example :

~~eg~~ two electrons with $j = 3/2$. think $J = 3, 2, 1, 0$.
but

highest $M = 3/2 + 3/2$ } $J = 3$ → except can't do this!
lowest $m = 3/2 - 3/2$

violates Pauli
can make $M = \pm 3$
so can't have $J = 3$

~~next highest / lowest~~
~~$M = 3/2 + 1/2 = 2$~~
~~[illegible]~~
~~$M = 3/2 - 1/2 = 1$~~
~~$M = 1/2 - 1/2 = 0$~~
~~$m = -1/2 - 3/2$~~

What other combinations of m_1 and m_2?

m_1	m_2	M
$3/2$	$1/2$	2
	$-1/2$	1
	$-3/2$	0
$1/2$	$3/2$ × can't use electrons indisting: already had $3/2, 1/2$	
	$1/2$ × violates Pauli .	
	$-1/2$	0
	$-3/2$	−1
$-1/2$	~~3~~ $3/2$ ×	
	$1/2$ ×	
	$-1/2$ ×	
	$-3/2$	−2
$-3/2$	$3/2$ ×	
	$1/2$ ×	
	$-1/2$ ×	
	$-3/2$ × .	

So possible

$$M = \underbrace{\pm 2, \pm 1, 0}_{J=2} \text{ and } \underbrace{0}_{J=0}$$

ie. Pauli
+ indistinguish
limits to even J .

Spin $m_j = \pm \frac{\hbar}{2}$

⑮ Other Two State Systems

Important : (1) in many areas of physics
(2) in a new field called quantum computing.
~~as~~ to generate units of quantum information
known as qbits.

Classical bit : either 0 or 1 (usually realised as a voltage level)
if you have a random bit } no information.
50% of either
chance

Qbit : two level system for example spin up $\alpha_z \equiv 1$
down $\beta_z \equiv 0$
Unlike classical bits:
Can form linear superpositions
eg. $\frac{1}{\sqrt{2}}(\alpha_z + \beta_z)$

This has 50% chance of a 1 or 0 by measuring S_z but contains additional information because it is identical to α_x.

The quantum superposition available in qbits parallels the classical bit, but has additional information content that has no classical analogue. (more later!)

There are other systems that are can generate that have two states eg. linear polarisation of photons / circular pol
these can be described by the same mathematical language as spin-$\frac{1}{2}$. using Pauli spin matrices .....

Example.... a ~~double~~ quantum dot.
~~Either slices of vertically stacked semiconductor~~
~~[double decker of example used before]~~
~~or 2 regions etched on a Si wafer~~

replace with next sheet.

Photon polarisation:

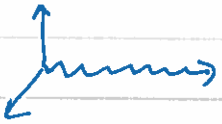
phase

Jone's vector $\begin{pmatrix} E_{0x} e^{i\phi_x} \\ E_{0y} e^{i\phi_y} \end{pmatrix}$

amplitude of $\underline{E}$ vector.

Linear polarisation: $|H\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ or $|V\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ like $\alpha_z \beta_z$.

along axes



For axis at 45° $|D\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ or $|A\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ like $\alpha_x \beta_x$

diag/antidiag.

Circular polarisation $|R\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$ $|L\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$

like $\alpha_y \beta_y$.

[important for QI as easily transported
difficult to ship electron spin around without
disturbing it].

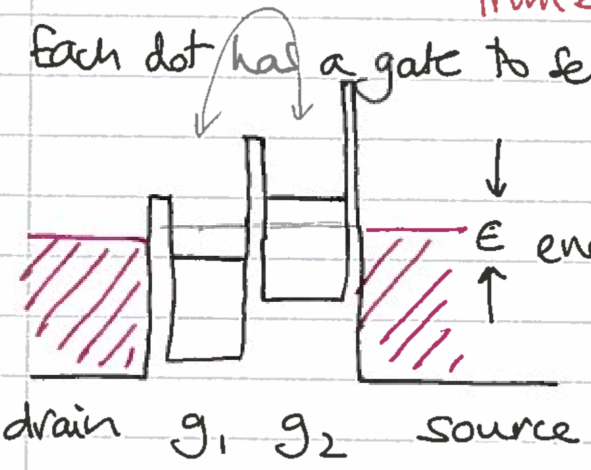
[Some systems can be approximately two level
if other levels are high excitation
and inaccessible]

Another example:

Double quantum dot
— ~~two~~ like semiconductor dots from before, but two etched on a wafer.

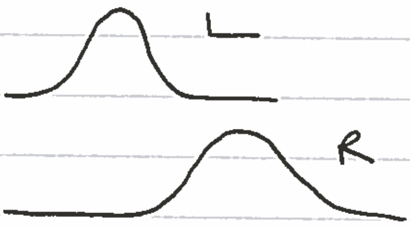
draw other way around!

Each dot has a gate to set its potential



ϵ energy difference $\propto e\,(V_{g_1} - V_{g_2})$
between states

Lowest states:



If other states are much higher in excitation energy — ignore them.

Effectively a two state system

$$L = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad R = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\hat{H} = \frac{1}{2}\begin{pmatrix} \epsilon & 0 \\ 0 & -\epsilon \end{pmatrix}$$

where energy is ~~the~~ relative to the midpoint between states

$\hat{H} L = \frac{\epsilon}{2} L \quad \hat{H} R = -\frac{\epsilon}{2} R$ in a minute!

Complication: states can tunnel through middle barrier an mix together. Don't get hung up on the details result is

$$\hat{H} = \frac{1}{2}\begin{pmatrix} \epsilon & -\Delta \\ -\Delta & -\epsilon \end{pmatrix}$$

where $-\frac{\Delta}{2}$ is called mixing energy.

$$\hat{H} = \frac{1}{2}\left(\epsilon\sigma_3 - \Delta\sigma_1\right)$$

Pauli spin matrices.

Maths looks like a spin $\frac{1}{2}$ in a magnetic field with z and x components.

$$\text{cf.} \quad \hat{H} = -\underline{\mu}\cdot\underline{B} = -\frac{q}{2m}\frac{\hbar}{2}\left(B_z\sigma_3 + B_x\sigma_1\right)$$

In situations where $\Delta = 0$:

$$\hat{H} = \frac{1}{2}\begin{pmatrix} \epsilon & 0 \\ 0 & -\epsilon \end{pmatrix} \propto \sigma_3 \qquad \hat{H}L = \frac{1}{2}\begin{pmatrix} \epsilon & 0 \\ 0 & -\epsilon \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{\epsilon}{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

L and R are eigenfunctions with eigenvalues $\pm\frac{\epsilon}{2}$.

[Just like B along z axis; spin "up"/down z axis].

In situations when $\varepsilon = 0$:

$$\hat{H} = \frac{1}{2}\begin{pmatrix} 0 & -\Delta \\ -\Delta & 0 \end{pmatrix} \quad \propto \sigma_1$$

χ like α_x and β_x

Eigenvectors: $\chi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]$ and $\chi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]$

[do problem sheet 5]

Eigenvalues: $-\frac{\Delta}{2}$ $\qquad$ $+\frac{\Delta}{2}$

mixtures of L and R

How is this realised in practice?

Start with $V_{g_1} \neq V_{g_2} \Rightarrow \epsilon \gg \Delta$

Adjust drain to let one electron tunnel into L

[can't resonantly tunnel into R as energies don't match!]

Hence $\psi(0) = L = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Apply a voltage pulse to give $V_{g_1} = V_{g_2}$ for some time $\Rightarrow \epsilon = 0$

State evolves via TDSE: $i\hbar \frac{d\psi}{dt} = -\frac{1}{2}\begin{pmatrix} 0 & \Delta \\ \Delta & 0 \end{pmatrix}\psi$

[like a spin up along z; B along x $\Rightarrow$ precession]

General sol$^{\underline{n}}$: $\psi(t) = c_1 e^{\frac{i\Delta t}{2\hbar}}\chi_1 + c_2 e^{\frac{-i\Delta t}{2\hbar}}\chi_2$

(a time evolving mix of L and R)

At the end of the pulse back to $\epsilon >> \Delta$
and wavefunction will be in a superposition of L and R.
[Schrödinger's Cat].

To measure which side electron in
  change & drain voltage back ~~[illegible]~~
  if electron in L → tunnels out → current.
  if in R → no current.
Do it for different pulse lengths ie. different t.
If you run through the problem on sheet 5:

$$P_L(t) = \cos^2\left\{\frac{\Delta}{2\hbar}t\right\}$$

ie. prob of L oscillates with time.

Seen in real quantum dots: PRL 95, 090502 (2005)

Fig 1 → picture of double dot.
Fig 4(a) current out of dot as fnc of pulse length "Δt"
  if ~~[illegible]~~ pulse length $= 0$ stays in L max
  $= \frac{\pi\hbar}{\Delta}$ minima* electron in R.
  inbetween → wouldn't know where it was until you measured it.
  [ethical Schrödinger's cat].

write up $\sigma_1, \sigma_2, \sigma_3$

Time-Dependent Hamiltonians:

⑯ Manipulating Spins with Oscillating Fields.

Time dependence in QM:

(a) if $\hat{H} \neq \hat{H}(t)$: TDSE $i\hbar \frac{d\Psi}{dt} = \hat{H}\Psi$

use separable sol$^{\underline{n}}$s : $\Psi = T(t).\chi$

end up with $e^{-i\omega t}$ time evolution $\omega = E/\hbar$

(b) if $\hat{H} = \hat{H}(t)$ can't separate TDSE (try it!)

and things get more difficult → in case of spin-1/2 can do it! analytically.

Electron in constant B_0 in z direction is example of (a):

$$\Psi = a_1 e^{-i\omega_0 t}\alpha_z + a_2 e^{i\omega_0 t}\beta_z \qquad \omega_0 = \frac{eg}{4m}B_0$$

a_1 and a_2 are constants

motion is a precession of spin around $\underline{B}_0$ at rate $2\omega_0$

An example of (b): add to this a time dep field.

eg. $\underline{B} = (B_1\cos\omega t, B_1\sin\omega t, B_0)$ z dir const. B_0

$\underline{S} = \frac{\hbar}{2}(\sigma_1, \sigma_2, \sigma_3)$

$\underline{B}_1$ rotates in xy plane

$\omega > 0$ RH circular pol

$\omega < 0$ LH.

Now: $\hat{H} = -\underline{\mu}.\underline{B} = \frac{eg}{2m}\hat{\underline{S}}.\hat{\underline{B}}$

$$= \frac{eg\hbar}{4m}\left[\sigma_1 B_1\cos\omega t + \sigma_2 B_1\sin\omega t + \sigma_3 B_0\right]$$

$$= \frac{eg\hbar}{4m}\begin{pmatrix} B_0 & B_1(\cos\omega t - i\sin\omega t) \\ B_1(\cos\omega t + i\sin\omega t) & -B_0 \end{pmatrix}$$

$$= \frac{eg\hbar}{4m}\begin{pmatrix} B_0 & B_1 e^{-i\omega t} \\ B_1 e^{i\omega t} & -B_0 \end{pmatrix}$$

[off diagonals non-zero, if multiply out get two eq^n s that mix c_1 and c_2] .

Form TDSE: $$i\hbar \frac{d}{dt}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \frac{eg\hbar}{4m}\begin{pmatrix} B_0 & B_1 e^{-i\omega t} \\ B_1 e^{i\omega t} & -B_0 \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

non-zero off diag → two coupled eq^n s.

when, to overcome the problems of not being able to use separable sol^n s, try sol^n

$$\Psi = a_1(t)e^{-i\omega_0 t}\alpha_z + a_2(t)e^{i\omega_0 t}\beta_z .$$

$$= \begin{pmatrix} a_1(t)e^{-i\omega_0 t} \\ a_2(t)e^{i\omega_0 t} \end{pmatrix}$$ ie. a's now time varying!

Substitute this into TDSE and tidy up:

$$i\hbar \frac{d}{dt}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \frac{eg\hbar}{4m}\begin{pmatrix} 0 & B_1 e^{i(2\omega_0-\omega)t} \\ B_1 e^{-i(2\omega_0-\omega)t} & 0 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

[Two eq^n s for a_1 and a_2 → sub into each other and solve for a_1 and a_2]

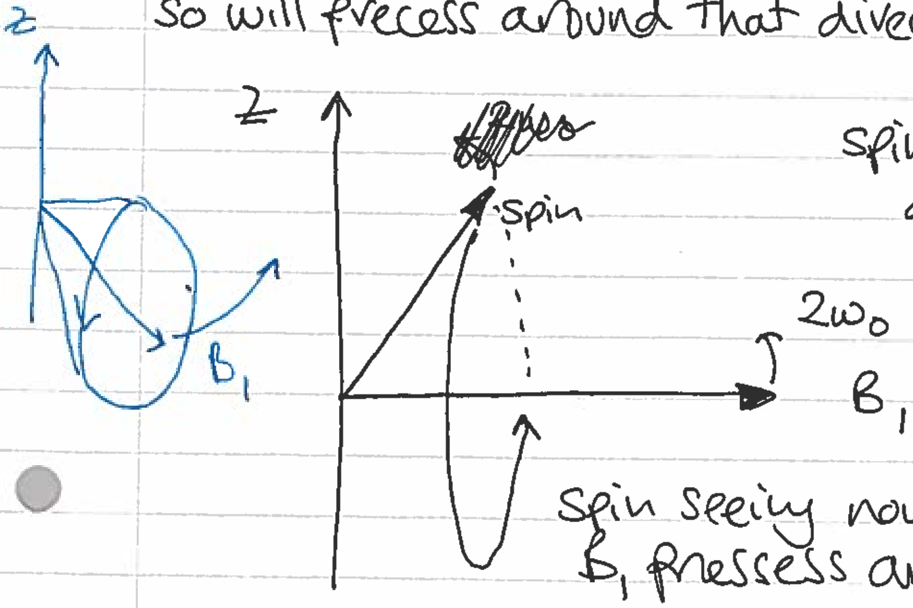
[differentiating $ae^{\pm i\omega_0 t}$ gives two bits from product rule, bring parts over to the other side plus some cancellations!]

Resonant effect when $\omega = 2\omega_0$

ie. when time-varying field rotates around with Larmor precession of the spin

ie. from point of view of spin, looks like B_1 is a constant field

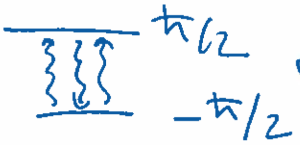
so will precess around that direction also



spin and B_1 precess at $2\omega_0$ around z .

spin seeing now an apparent const B_1 precesses around it at $2\omega_1$ $\omega_1 = \frac{egB_1}{}$

paramagnetic resonance inducing transitions



In the LAB frame: complex motion but looks like ~~spin~~ $\langle \hat{S}_z \rangle$ is oscillating ~~[illegible]~~ between $\pm\frac{\hbar}{2}$ at frequency of $2\omega_1$

but

$+\frac{\hbar}{2}$ has higher energy than $-\frac{\hbar}{2}$ due to presence of B_0.

→ absorbs energy from oscillating field as $\langle \hat{S}_z \rangle$ increases and radiates it as $\langle \hat{S}_z \rangle$ decreases.

Rabi Oscillation, Nobel for NMR 1944

~~ie~~. If in a $\langle S_z \rangle = \frac{\hbar}{2}$ state could move it into a $-\frac{\hbar}{2}$ state by exposure to the right sort of e/m radiation for a set duration, assuming you know g

~~[illegible]~~ → manipulating spins as qbits.

For complicated solids, $g \neq 2$ due to similar effects as m^*, so the absorption/radiation of microwaves is expt signature → measure ω_1 and determine g knowing B_1 → electron spin resonance (EPR or ESR)

If not using electrons, can use to measure spin ⇒ NMR.

Can "in clever ways" use similar spin-flip resonance of protons in water to image → nuclear magnetic imaging.

[For $\omega \neq 2\omega_0$ need to solve coupled ODE's see Gaz.
spin gets out of step with B_1,
stops precessing.
reverses precession
→ ie. just wobbles around].

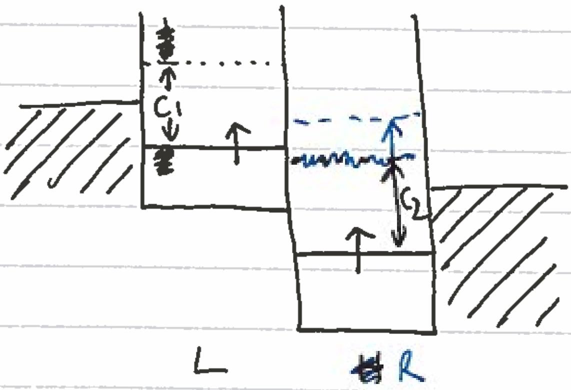
effects v. much lower than resonance.

[Usually use linear polarised μwaves = LH + RH circ. pol
replacing $\omega \Rightarrow -\omega$ in above
terms like $B_1 e^{i(2\omega_0+\omega)t}$ ie. rapid oscillation
→ small wobble].

Ran out of time
and missed this -

Controlling spins in a double dot system:
constant field B_0 close to wire accompanying AC $\Rightarrow B_1 \cos \omega t$

Adjust gate voltages so exactly one electron can tunnel into each dot:



Coulomb energies C_1 and C_2 stop second electrons joining — Coulomb blockade

On energetics alone, electron in L could tunnel into R but would then be in same state as electron already in R if has same spin — Pauli forbids
— Spin blockade.
Can only tunnel L→R if has opposite spin.

Apply AC current creating B_1. At resonant frequency, spin of L electron precesses (not purely spin up) then can escape into R → current flows.
~~Only happens at resonance~~

$$\omega = 2\omega_0 = \frac{eg}{2m} B.$$

For GaAs find that for $B_0 = 0.1\,T$ $f = 500\,MHz$

$\Rightarrow g = 0.36$

ie. very different to free space!