

COMPARISON OF GAS-DYNAMIC MODELS FOR HYPERSONIC FLOW PAST BODIES†

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Various gas-dynamic models for describing chemically non-equilibrium flows are compared using the example of the steady flow past the blunt nose of the "Buran" [1] and "Space Shuttle" vehicles during their descent from orbit. Models of locally self-similar approximations of the Navier-Stokes equations [2], of a chemically equilibrium and non-equilibrium complete viscous shock layer (CVSL) [3] and a model of a thin viscous shock layer (TVSL) [4] are considered. In all the models the occurrence of physicochemical processes was taken into account in the same way using fixed values of the constants for the gas-phase chemical reactions (their effect has been considered in [5]). Good agreement between the results of calculations of the heat flux at the critical point is found.

Chemically non-equilibrium flows have been considered earlier using the approximate Navier-Stokes equations [6], within the framework of a TVSL [7] and a CVSL [8, 9] (for more detail, see the review [10]). The TVSL and CVSL models were compared in [11] in the case of flows of a uniform gas.

1. LET US consider the complete system of steady Navier-Stokes equations which describe the flow of a multicomponent, chemically reacting mixture of gases when there are no external mass forces. In the curvilinear system of coordinates x, y, φ , where x is the length of an arc of the contour of the body measured from the front critical point, y is the distance along a normal from the body and φ is the meridional angle, the equations have the form

$$\frac{\partial}{\partial x} (r^\nu \rho u) + \frac{\partial}{\partial y} (H_1 r^\nu \rho v) = 0 \tag{1.1}$$

$$\begin{aligned} \rho \left(\frac{u}{H_1} \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{\bar{u}v}{RH_1} \right) = - \frac{1}{H_1} \frac{\partial P}{\partial x} + \\ + \frac{1}{H_1 r^\nu} \left\{ \frac{\partial}{\partial x} (r^\nu \tau_{xx}) \right\} + \frac{\partial}{\partial y} (H_1 r^\nu \tau_{xy}) + \frac{\bar{\tau}_{xy}}{RH_1} \left\{ \frac{\nu \sin \alpha}{r} \tau_{\varphi\varphi} \right\} \end{aligned} \tag{1.2}$$

$$\begin{aligned} \rho \left(\frac{u}{H_1} \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} - \frac{u^2}{RH_1} \right) = - \frac{\partial P}{\partial y} + \\ + \frac{1}{H_1 r^\nu} \left\{ \frac{\partial}{\partial x} (r^\nu \tau_{xy}) \right\} + \left\{ \frac{\partial}{\partial y} (H_1 r^\nu \tau_{yy}) \right\} - \left\{ \frac{\tau_{xx}}{RH_1} + \frac{\nu \cos \alpha}{r} \tau_{\varphi\varphi} \right\} \end{aligned} \tag{1.3}$$

$$\begin{aligned} \rho c_p \left(\frac{u}{H_1} \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{u}{H_1} \frac{\partial P}{\partial x} + v \frac{\partial P}{\partial y} + \\ + \frac{1}{H_1 r^\nu} \left\{ \frac{\partial}{\partial x} \left(\lambda \frac{r^\nu}{H_1} \frac{\partial T}{\partial x} \right) \right\} + \frac{\partial}{\partial y} (\lambda r^\nu H_1 \frac{\partial T}{\partial y}) + F + \\ + \sum h_i \rho \dot{\omega}_i + \left\{ \sum c_{pi} \frac{J_{ix}}{H_1} \right\} \frac{\partial T}{\partial x} + \left\{ \sum c_{pi} J_{iy} \right\} \frac{\partial T}{\partial y} \end{aligned} \tag{1.4}$$

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$$\rho \left(\frac{u}{H_1} \frac{\partial c_i}{\partial x} + v \frac{\partial c_i}{\partial y} \right) + \frac{1}{H_1 r^\nu} \left[\frac{\partial}{\partial x} (r^\nu \tau_{ix}) + \frac{\partial}{\partial y} (H_1 r^\nu J_{iy}) \right] = w_i, \quad i = 1, \dots, N \tag{1.5}$$

$$P = R_G \rho \mathcal{F} (\sum c_i / m_i)^{-1}, \quad \sum c_i = 1, \quad H_1 = 1 + \overline{y/R}$$

$$F = \{ \tau_{xx} e_{xx} \} + \{ \tau_{yy} e_{yy} \} + \{ \tau_{\varphi\varphi} e_{\varphi\varphi} \} + 2\tau_{xy} e_{xy}$$

Here u and v are the physical components of the velocity in the x and y directions, P , ρ and T are the pressure, density and temperature of a mixture of gases consisting of N chemical components, μ and λ are the coefficient of viscosity and thermal conductivity of the mixture, c_i , m_i , h_i , c_{pi} , J_i and w_i^* are the mass concentration, the molecular mass, the specific enthalpy and the heat capacity, the components of the diffusion flux vector and the rate of formation of mass of the i th component, τ_{ij} and e_{ij} are the components of the viscous stress tensor and the rate of strain tensor, R_G is the universal gas constant, $R(x)$ is the radius of curvature of the contour of the body, H_1 is a Lamé coefficient, $r = r_w - y \cos \alpha$ is the distance from a given point in space to the axis of the body, α is the angle between a tangent to the contour of the body and the axis of symmetry, ν is a symmetry parameter, and summation is carried out from $i = 1$ to $i = N$.

Let us now reduce the equations to dimensionless form. We obtain a system of parabolic Navier–Stokes equations, retaining the terms $O(1)$ and $O(\text{Re}^{-1/2})$ and omitting the terms $O(\text{Re}^{-1})$. The omitted terms are enclosed in the braces. We note that the term $\partial(r^\nu \tau_{xy})/\partial x$, which is of the order of $\text{Re}^{-1/2}$, is also omitted in Eq. (1.3). However, the additional estimates, associated with the smallness of the parameter ρ_∞/ρ_s (the ratio of the densities in the free stream and at the boundary of the shock wave) as well as the values of the derivatives with respect to x and y in the shock wave domain and around the body show that the remaining terms are of the greatest order of magnitude, at least in the shock wave. Hence, the terms which are responsible for molecular transport along the body are neglected; this is justified in the case of smooth blunt bodies when there are no discontinuities in the boundary conditions on the surface.

When the effects of thermal and pressure diffusion are neglected, the remaining components of the viscous stress tensor and the Stefan–Maxwell relationship are expressed in the following way (D_{ij} are the binary diffusion coefficients)

$$\tau_{xy} = \mu \left(\frac{\partial u}{\partial y} - \frac{\bar{u}}{RH_1} + \left\{ \frac{1}{H_1} \frac{\partial v}{\partial x} \right\} \right)$$

$$\tau_{yy} = 2\mu \left[\frac{2}{3} \frac{\partial v}{\partial y} - \left\{ \frac{1}{3r^\nu H_1} \left(\frac{\partial ur^\nu}{\partial x} + v \frac{\partial r^\nu H_1}{\partial y} \right) \right\} \right]$$

$$F = \mu \left[\left(\frac{\partial u}{\partial y} - \frac{\bar{u}}{RH_1} \right)^2 + \left\{ \left\{ \frac{4}{3} \left(\frac{\partial v}{\partial y} \right)^2 \right\} \right\} \right]$$

$$\frac{\partial mc_i}{\partial y} = \sum \frac{m^2}{\rho m_i D_{ij}} (c_p J_{iy} - c_p J_{jy}), \quad i = 1, \dots, N-1, \quad \sum J_{iy} = 0$$

The boundary conditions on the body surface, when the effects of slippage and temperature and concentration discontinuities are neglected, are as follows:

$$y = 0, \quad u = v = 0, \quad q = \epsilon \sigma_B T^4$$

$$q = \lambda \partial T / \partial y - \sum h_i J_{iy}, \quad J_{iy} = -\rho K_{wi} c_i$$

where ϵ is the coefficient of blackness of the surface, σ_B is the Stefan–Boltzmann constant, and K_{wi} is the heterogeneous recombination rate constant.

Conditions are specified at the external boundary which correspond to the free stream parameters.

In this paper, the system of parabolic Navier–Stokes equations is solved using a local self-similar approximation in the critical line. We expand the required solution in a series [2] confining ourselves to the first terms but also retaining the second term in the expansion for the pressure

$$u = u_1(y) \cos \alpha + \dots, \quad v = v_0(y) \sin \alpha + \dots$$

$$p = p_0(y) + p_2(y) \cos^2 \alpha + \dots, \quad G = G_0(y) + \dots \tag{1.6}$$

where G is any of the function ρ , T , μ , λ and c_i .

Substituting the expansions (1.6) into Eq. (1.3) and equating the terms accompanying $\cos^2 \alpha$, we obtain the equation for finding the longitudinal pressure gradient

$$\frac{1}{r_w} \frac{\partial P}{\partial x} = 2 \frac{\cos \alpha}{r_w} \frac{\sin \alpha}{R} P_2, \quad \frac{\partial P_2}{\partial y} = -\frac{\partial P}{\partial y} + \frac{\rho}{RH_1} \frac{u}{\cos \alpha} \left(\frac{u}{\cos \alpha} + \frac{v}{\sin \alpha} \right)$$

The problem is therefore closed on the critical line.

The model of a complete viscous shock wave is derived from the parabolic Navier–Stokes equations by neglecting the viscous terms in Eq. (1.3) (indicated by double braces). As a result, the order of the momentum equation in the projection onto the normal is reduced and the remaining boundary condition serves for determining the shock wave stand-off distance. The transitional domain across the shock waves is replaced by a surface where there is a pronounced discontinuity. The Rankine–Hugoniot relationships serve as boundary conditions at this surface. When the terms $O(k/Re)_s$ are neglected (the values of quantities at the internal boundary of the shock are denoted by the subscript s , $k_s = \rho_\infty/\rho_s$), these relationships take the form

$$\begin{aligned} u_s &= \cos \beta \cos \beta_s + K_s \sin \beta \sin \beta - \xi \tau_{xy}|_s \\ v_s &= u_s \operatorname{tg} \beta_s - K_s / \xi, \quad P_s = \bar{P}_\infty + (1 - K_s) \sin^2 \beta \\ H_s &= \bar{H}_\infty + \xi J_{Hy}|_s; \quad c_{is} = c_{i\infty} + \xi J_{iy}|_s; \quad \xi = \cos \beta_s / \sin \beta \end{aligned} \tag{1.7}$$

Here H and J_{hy} are the total enthalpy and the energy flux, $\beta(x)$ is the angle between the tangent to the shock wave and the axis of symmetry, and $\beta_s = \beta - \alpha$.

The complete viscous shock layer (CVSL) equations contain all the terms of the Euler equations as well as terms which make a contribution to the second approximation of the boundary layer with respect to a small parameter $Re^{-1/2}$ both in the case of the inner and outer expansions.

The thin viscous shock layer (TVSL) equations are an asymptotic form of the CVSL equations at high mach and Reynolds numbers and $K_s \rightarrow 0$. The terms which are omitted in the equations are marked with a bar over them. It is necessary to put $K_s = 0$ and $\beta_s = 0$ in the boundary conditions (1.7) at the shock wave. The TVSL equations contain terms of the boundary layer and inviscid shock layer equations in the hypersonic approximation.

2. The simplified Navier–Stokes equations in the locally self-similar approximation were integrated on the critical line using the method proposed in [12]. The CVSL equations retain elliptic properties because of the longitudinal component of the pressure gradient and the form of the shock wave and an iteration method based on the execution of global iterations [13] was used to solve them. A system of TVSL equations of parabolic type and a method for solving it have been described in [14].

A scheme [15] of fourth-order accuracy with respect to the transverse coordinate was used in order to integrate the equations within the framework of the above-mentioned models. In treating the chemical reactions it was assumed that there were five components N_2 , O_2 , N , O and NO in the perturbed flow domain and that dissociation, recombination and exchange reactions occur between them. The system of reactions, the rate constants of the reactions and the transport coefficients were assumed to be identical for all of the models and to those previously used in [14].

Let us now consider the flow past bodies during motion along gliding entry trajectories into the Earth’s atmosphere. The free-stream conditions corresponded to a standard atmosphere.

Three models of catalytic surface activity were considered: ideally catalytic (model 1); non-catalytic (model 2); it was assumed that the surface possesses finite catalytic properties with rate constants $K_{wO} = K_{wN} = 3$ m/s, $K_{wNO} = 0$ (model 3). Model 3 corresponds to the catalytic activity of the carbon–carbon surface coating of the “Buran” space vehicle [16].

The results of calculations of the heat flux at the critical point of the “Buran” space vehicle as a function of the descent time over a range of heights h from 100 to 52 km are presented for the trajectory of the first flight [1] (Fig. 1) and for a typical trajectory (Fig. 2). The radius of bluntness $R = 0.83$ m and $\epsilon = 0.8$. Results of calculations of the CVSL equations are shown by the solid lines, the results of calculations of the chemically equilibrium CVSL equations by the dotted and dashed lines, and the result of calculations of the TVSL equations by the dashed lines. Lines 1–3 correspond to models 1–3 for the catalytic activity of the surface. The dependence of the speed of flight on the descent time is represented by lines 4.

For practically all of the flow conditions considered, the thermal flux was higher for calculations within the framework of the TVSL model than when the calculations were carried out using the CVSL model. This is due to the fact that the assumption regarding the thinness of the layer leads to a greater degree of compression compared with the model of a complete viscous shock layer where the position of the shock wave is found while solving the problem.

A comparison with the results of calculations of a chemically equilibrium viscous shock layer shows that the

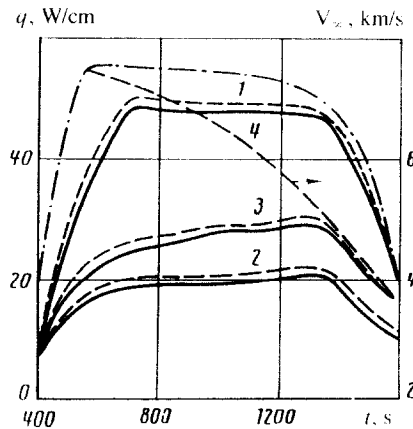


FIG. 1.

principal thermally stressed part of gliding trajectories is characterized by the occurrence of chemical reactions under substantially non-equilibrium conditions. The greater values of the heat flow in the equilibrium case is explained by the fact that, as calculations have shown, the occurrence of reactions under equilibrium conditions causes more intensive dissociation of the molecules compared with the non-equilibrium case. As a result, the more intensive diffusion of atoms to the relatively cold wall and recombination in the boundary region leads to the additional generation of energy close to the surface.

For comparison, the temperature profiles for an ideally catalytic surface are shown in Fig. 3. The free-stream conditions correspond to the trajectory in Fig. 1 when $t = 710$ s.

Thermal flux distributions from the height of the flight in motion along a trajectory which simulates the descent of the "Space Shuttle" space vehicle are shown in Fig. 4. At the critical point, the principal radii of curvature were equal to 0.5 and 1.25 m, and $e = 0.85$.

The solid lines correspond to calculations within the framework of a locally self-similar approximation of the Navier–Stokes equations and the dashed lines to calculations within the framework of the TVSL model. Lines 1 and 2 correspond to the ideally catalytic and non-catalytic surfaces.

At heights of $65 < h < 90$ km, which are characterized by the occurrence of intense chemical reactions, the heat flux when solving the TVSL system was greater than that obtained within the framework of the locally self-similar approximation of the Navier–Stokes equations by not more than 5%. The coefficients of friction are identical to within 1%. As the height of the flight is reduced and there is a reduction in the Mach number at $50 < h < 60$ km ($8.7 < M_\infty < 15$), the difference increases and, at $h = 50$ km, the heat flux is greater by 7% and the coefficients of friction by 4%.

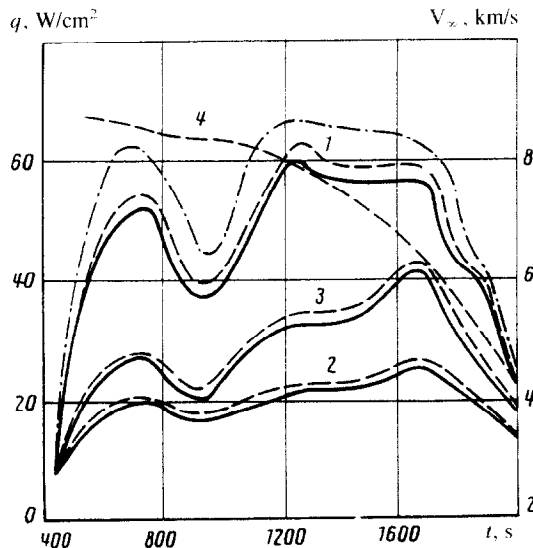


FIG. 2.

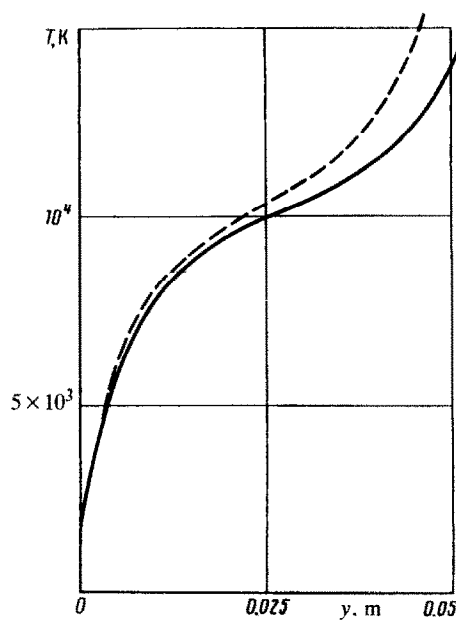


FIG. 3.

The temperature profiles for an ideally catalytic surface at $h = 70$ km ($Re_\infty = 1.33 \times 10^4$) are shown in Fig. 5. The result of a calculation using the TVSL model taking into account the normal coordinate y in the Lamé coefficient is shown by the dotted line. The difference in the heat from fluxes—those for the case when $H_1 = 1$ does not exceed 3% while the difference in the coefficients of friction does not exceed 5%.

These results show that the difference between the values of the heat fluxes at the critical point given by the models considered does not exceed 8% and at the equilibrium surface temperature it is 30 K in the case of the flow conditions considered. Hence, in the case of parametric calculations in the neighbourhood of the stagnation line for conditions of gliding descent, it is advisable to employ the model of a thin viscous shock layer which is simpler and more convenient. However, in a number of cases of the lateral surface of a streamlined body, a TVSL can lead to a drop in pressure on the body, and for these calculations it is necessary to use the model of the complete viscous shock layer or parabolized Navier–Stokes equations.

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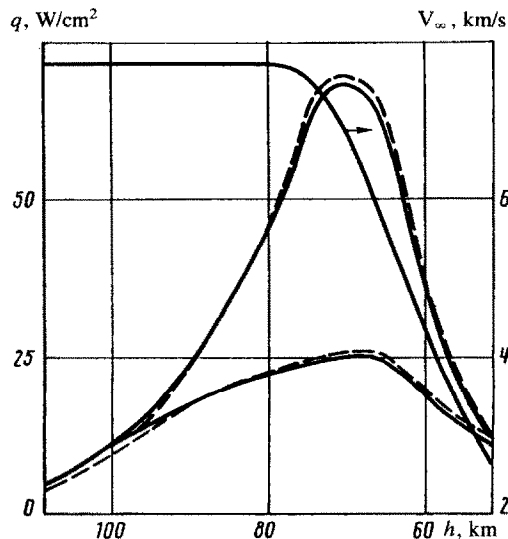


FIG. 4.

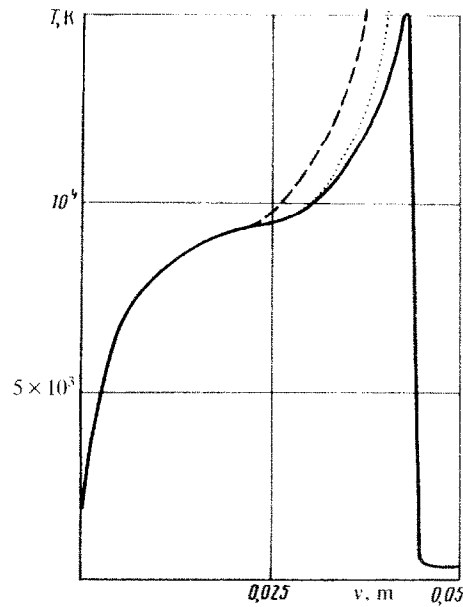


FIG. 5.

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