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Non-stationary problem of active sound control in bounded domains

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ABSTRACT

The present paper deals with the non-stationary problem of active shielding of a domain from undesirable external sources of noise. Active shielding is achieved by constructing additional (secondary) sources in such a way that the total contribution of all sources leads to the desirable effect. The problem is formulated as an inverse source problem with the secondary sources positioned outside the domain to be shielded. Along with the undesirable field (noise) to be shielded the presence of a desirable component ("friendly" sound) is accepted in the analysis. The constructed solution of the problem requires only the knowledge of the total field (noise) on the perimeter of the shielded domain. Some important aspects of the problem are addressed in the paper for the first time, such as the non-stationary formulation of the problem, existence of the resonance regimes and sensitivity of the solution to the input errors. The obtained solution is applicable to aeroacoustics problems.

0. Introduction

The active shielding (AS) of some domains from the effect of the external field (noise) generated in other domains is achieved by constructing additional (secondary) sources in such a way that they cancel each other. Along with the undesirable (noise) sources situated outside the domain to be shielded, the existence of internal (desirable) sources is accepted in the analysis. The problem can be formulated as an inverse source problem. The reviews of some theoretical and experimental methods related to the problem subjects can be found in [1,2]. Most theoretical approaches assume that rather detailed information about the undesirable sources and the properties of the medium is known. Unlike these approaches, the JMC method [3,2], based on Huygens' construction, requires only the information on the undesirable field on the perimeter of the shielded domain. However, it has not yet been used in the case when the desirable field ("friendly" sound), generated in the shielded domain, needs to be taken into account. In addition, the JMC method, as many others, has only been used for problems formulated in unbounded domains.

The Difference Potential Method (DPM), proposed in [4,5], allows us to circumvent many limitations of other approaches in that it requires much less information of the total noise field. For example, in [6,4], the solution of the problem is obtained by this method in a finite-difference formulation and requires only the knowledge of the total field (both desirable and undesirable) at the mesh boundary of the shielded domain; any other information on the sources and medium is not required. One can say that the solution procedure uses the minimum information *a priori* available. In [7], the DPM method was applied to the Helmholtz equation. The questions of optimization for this equation were comprehensively studied by Lončarić and Tsynkov; see, e.g., [8].

The DPM-based solution was extended by Ryaben'kii and Utyuzhnikov to arbitrary hyperbolic systems of equations, including the Euler acoustics equations with constant and variable coefficients, in [9]. In [10], the authors considered the AS problem in bounded domains, showed that the control sources are capable of not disturbing even the echo of the "friendly"

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sound component and explained the mechanism of this property of the solution. For the system of first-order equations in continuous spaces, the AS solution was first obtained in [11] for time-harmonic waves under rather general assumptions. The DPM-based discrete solution was shown to approach to this continuous solution as the spatial mesh is refined. The secondary single-layer term is obtained for the Euler acoustics equations.

The solution [11] is also applicable to higher-order equations by means of reducing them to a set of first-order equations. In particular, the Helmholtz equation is considered in this way. The AS solution is represented by a linear combination of single- and double-layer potentials and coincides with the solution first obtained in [7]. The general statement of essentially non-stationary problem (broad band) is first addressed in [12]. The solution of the AS inverse problem is obtained in both linear non-stationary and nonlinear stationary formulations. In [13], the problem of AS in composite domains is formulated and its general solution is provided in the general finite-difference formulation. The counterpart of the problem in continuous spaces is solved in [14]. The principal novelty of the problem considered in [13] is that it allows a selective communication between different sub-domains. The solution of the problem is constructed by means of a predictor–corrector algorithm.

In the present work, the AS solution is considered for the general formulation of the wave equation in the time domain. It is demonstrated that the solution is applicable even to resonance regimes. Since in practical applications errors in the input and output parameters are unavoidable, the sensitivity analysis on the noise cancelation is given for the wave equation. The application to aeroacoustics is also considered.

1. General formulation of the AS problem

First, let us introduce some domain D: $\overline{D} \subseteq \mathbb{R}^m$ with smooth boundary Γ_0 and a sub-domain D^+: $\overline{D^+} \subset D$, having smooth boundary Γ.

Let us assume that some field (sound) U is described by the following boundary value problem (BVP):

$$LU = f, \tag{1}$$

$$U \in \Xi_D. \tag{2}$$

Here, the operator L is a linear differential operator, Ξ_D is some functional space specified further. In particular, the operator L can correspond to the acoustics equations. It is supposed that BVP (1), (2) is well-posed for any right-hand side $f: f \in L_2^{\mathrm{loc}}(D)$. Thereby, the boundary conditions are supposed to be implicitly included in the definition of the space Ξ_D. To avoid any possible confusion, we assume that the boundary conditions are homogeneous and formulated locally on the boundary Γ_0.

We assume that the sources on the right-hand side can be placed both on D^+ and outside D^+:

$$f = f^+ + f^-, \tag{3}$$

$$\operatorname{supp} f^+ \subset D^+, \quad \operatorname{supp} f^- \subset D^- \stackrel{\mathrm{def}}{=} D \setminus \overline{D^+}.$$

Thus, f^+ are "friendly" field (sound) sources, while f^- generates an "adverse" field (noise).

Suppose that we know the trace of the function U on the boundary Γ of the domain D^+. We note that only this information is assumed to be available. In particular, the distribution of the sources f on the right-hand side is unknown. The AS problem is then reduced to finding additional sources G in $\overline{D^-}$ (see Fig. 1) such that the solution of the following BVP

$$LV = f + G, \tag{4}$$

$$\operatorname{supp} G \subset \overline{D^-},$$

$$V \in \Xi_D$$

coincides on the domain D^+ with the solution of BVP (1), (2) if $f^- \equiv 0$:

$$LU^+ = f^+, \tag{5}$$

$$U^+ \in \Xi_D.$$

Thus, we seek a source term G such that on the domain D^+ the functions U and V coincide with each other: $V_{D^+} = U^+_{D^+}$.

2. Non-stationary AS problem

Suppose that the field U is the solution of a well-posed initial-boundary value problem (IBVP) in the cylinder $K_T = D \times (0, T) \subseteq \mathbb{R}^{m+1} (T > 0)$:

$$LU \stackrel{\mathrm{def}}{=} L_t^{(p)} U + \sum_1^m A^i \frac{\partial U}{\partial x^i} = f, \tag{6}$$

$$U \in \Xi_D, \tag{7}$$

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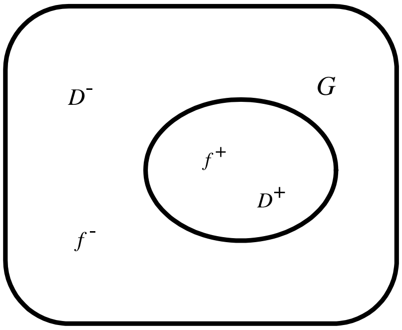

Fig. 1. Domain sketch.

where $\{x^i\}$ $(i = 1, \ldots, m)$ is a Cartesian coordinate system, U and f are vector functions of dimension n, A^i are $n \times n$ matrices: $A^i(\mathbf{x}) \in C^1(D)$ $(i = 1, \ldots, m)$, $L_t^{(p)}$ is a linear differential operator of order p with respect to the time variable t. We assume that space Ξ_D includes functions which are smooth enough with respect to the variable t and satisfy homogeneous initial conditions. That is, if $U \in \Xi_D$, then

$$\frac{\mathrm{d}^k}{\mathrm{d}t^k} U(\mathbf{x}, 0) = 0, \quad (k = 0, \ldots, p-1). \tag{8}$$

We note that without any loss of generality we can suppose that initial conditions are homogeneous. Indeed, we can always represent the solution of IBVP (6), (7) as follows: $U = U^{(f)} + U^{(t)}$, where $U^{(f)}$ is the solution of the IBVP problem with the homogeneous initial data (8), while $U^{(t)}$ is the solution of the IBVP with the homogeneous right-hand side. It is clear that the function $U^{(f)}$ can only represent a "residual" noise.

Let us further consider the generalized solution of IBVP (6):

$$\int_0^T \int_D (LU - f, \Phi) \mathrm{d}\mathbf{x} \mathrm{d}t = 0 \tag{9}$$

where for any basic function $\Phi \in C_0^\infty(K_T)$, $(\cdot, \cdot)$ means a scalar product.

We define the functional space Ξ_D in such a way that the weak solution of BVP (1) satisfies the governing equation in the classical sense (6) almost everywhere, and it is bounded. More precisely, we assume that for any $0 < t < T$: $\Xi_D \subset H^s(D \setminus \Gamma) := H^s(D^+) \cap H_0^s(D^-)$, $s > 1/2$, $s \neq$ integer $+ 1/2$, H_0^s and H^s are Sobolev spaces.

The formulated AS problem is an inverse source problem and therefore its solution is not unique [11]. From the point of view of applications, the most interesting solution is probably the one represented by a single-layer source term and provided by the following proposition [12]:

Proposition 1. *A solution of the AS problem (1), (2), (4) and (6) is given by:*

$$G = G_0 \stackrel{\text{def}}{=} A_n U_\Gamma \delta(\Gamma), \tag{10}$$

where $U_\Gamma \stackrel{\text{def}}{=} U(\Gamma)$, $A_n \stackrel{\text{def}}{=} \sum_1^m n_i A^i$, $\mathbf{n} = (n_1, n_2, n_3)^{\mathrm{T}}$ *is the outward unit vector of the boundary* Γ, $\delta(\Gamma)$ *is the Dirac delta-function assigned to the boundary* Γ.

It is important to note that if we reverse the direction of the normal vector $\mathbf{n}$ to the inward one, then the source term given by the proposition provides shielding the domain D^- from the field generated in D^+.

We now consider the application of the AS solution (10) to the wave equation.

3. Wave equation

3.1. Solution of the inverse problem

In the case of the wave equation

$$\gamma \rho_{tt} - \nabla(\sigma \nabla \rho) + q\rho = f, \tag{11}$$

the operator L in (1) is the following: $L := \gamma \frac{\partial^2}{\partial t^2} + L_x$, $L_x \stackrel{\text{def}}{=} -\nabla(\sigma \nabla) + q$. We assume that $\gamma > 0$, $\sigma > 0$, $q \geq 0$, $\gamma(\mathbf{x}) \in C(\overline{D})$, $\sigma(\mathbf{x}) \in C^1(\overline{D}) > 0$, $q(\mathbf{x}) \in C(\overline{D})$.

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One can rewrite (11) as the system of first-order equations with respect to the space variables:

$$\gamma\rho_{tt} - \nabla\mathbf{a} + q\rho = f, \qquad (12)$$
$$\nabla\rho = \mathbf{a}/\sigma.$$

In $\mathbb{R}^4$, we have:

$$U = (a_1, a_2, a_3, \rho)^{\mathrm{T}}, \qquad (13)$$

where a_i $(i = 1, 2, 3)$ are the coordinates of the vector $\mathbf{a}$. The matrix A_n coincides with the matrix corresponding to the Helmholtz equation [11]. Then,

$$G_0(\Gamma) = -(a_n, \rho n_1, \rho n_2, \rho n_3)^{\mathrm{T}}\delta(\Gamma). \qquad (14)$$

Eliminating the auxiliary vector $\mathbf{a}$, we obtain at the following expressive for the source term:

$$g_0 = -\delta(\Gamma)\sigma_\Gamma \frac{\partial\rho}{\partial\mathbf{n}_{|\Gamma}} - \frac{\partial\delta(\Gamma)\sigma_\Gamma\rho_\Gamma}{\partial\mathbf{n}}. \qquad (15)$$

The AS term is represented via the sum of single-layer and double-layer additional source terms. Here, the density of the potentials includes the values ρ_Γ and $\frac{\partial\rho}{\partial\mathbf{n}}_{|\Gamma}$, which are assumed to be known. In the stationary case, if $\sigma \equiv 1$, this solution is identical to the secondary source term obtained in [7] for the Helmholtz equation. As shown in [7], one can obtain the AS solution in the form of either single layer or double layer but this requires solutions of some boundary value problem.

Let us now analyze how the AS source term affects the field in the domain D. Assume that the boundary conditions on the boundary Γ_0 are as follows:

$$\alpha\rho + \beta\frac{\partial\rho}{\partial\mathbf{n}_0} = 0, \qquad (16)$$

where $\alpha = \alpha(\mathbf{x}) \in C(\Gamma_0)$, $\beta = \beta(\mathbf{x}) \in C(\Gamma_0)$, $\alpha \geq 0$, $\beta \geq 0$, $\alpha + \beta > 0$, $\mathbf{n}_0$ is the outward normal vector to the boundary Γ_0.

Then, the space and time variables are separated, and we can obtain the analytical solution of the problem via the Fourier method, see, e.g. [15].

3.2. *The solution of the direct problem*

The operator L_x is the Hermitian. Therefore, its eigenvalues are positive, and there exists the full system of eigenfunctions which are orthogonal with respect to the following scalar product: $(a, b)_\gamma \stackrel{\text{def}}{=} \int_D \gamma ab\mathrm{d}\mathbf{x}$ [15].

Assume that $f \in C(0, T)$. Then, the solution of IBVP (11), (8) and (16) can be represented by the uniformly convergent Fourier series [15]:

$$\rho(\mathbf{x}, t) = \sum_1^\infty T_k(t)X_k(\mathbf{x}),$$

where X_k, $(k = 1, 2, \ldots)$ are the eigenfunctions of the operator L_x and the coefficients $T_k(t) = (\rho, X_k)_\gamma$ $(k = 1, 2, \ldots)$ satisfy the following initial problem:

$$T_k''(t) + \mu_k T_k(t) = c_k(t), \qquad (17)$$
$$T_k(0) = 0, \qquad T_k'(0) = 0,$$

where

$$c_k(t) = (f, X_k).$$

In (17), $\mu_k > 0$ are the eigenvalues of the operator L_x corresponding to the eigenfunctions X_k, respectively. If the function f is regular, then the coefficients c_k are given by the integrals:

$$c_k(t) = (f, X_k) = \int_D f(\mathbf{x}, t)X_k(\mathbf{x})\mathrm{d}\mathbf{x}. \qquad (18)$$

The solution of the Cauchy problem (17) is given by the following expression:

$$T_k(t) = -\mathrm{i}\frac{1}{2\sqrt{\mu_k}}\mathrm{e}^{\mathrm{i}\sqrt{\mu_k}t}\int_0^t c_k(\tau)\mathrm{e}^{-\mathrm{i}\sqrt{\mu_k}\tau}\mathrm{d}\tau + \mathrm{i}\frac{1}{2\sqrt{\mu_k}}\mathrm{e}^{-\mathrm{i}\sqrt{\mu_k}t}\int_0^t c_k(\tau)\mathrm{e}^{\mathrm{i}\sqrt{\mu_k}\tau}\mathrm{d}\tau.$$

It is clear that the "friendly" field ρ^+ can similarly be obtained with the replacement of the right-hand side f by f^+. Thus,

$$\rho^+(\mathbf{x},t)=\sum_1^\infty T_k^+(t)X_k(\mathbf{x}),$$

where

$$T_k^+(t)=-\mathrm{i}\frac{1}{2\sqrt{\mu_k}}\mathrm{e}^{\mathrm{i}\sqrt{\mu_k}t}\int_0^t c_k^+(\tau)\mathrm{e}^{-\mathrm{i}\sqrt{\mu_k}\tau}\mathrm{d}\tau+\mathrm{i}\frac{1}{2\sqrt{\mu_k}}\mathrm{e}^{-\mathrm{i}\sqrt{\mu_k}t}\int_0^t c_k^+(\tau)\mathrm{e}^{\mathrm{i}\sqrt{\mu_k}\tau}\mathrm{d}\tau$$

and $c_k^+=(f^+,X_k)=(f^+,X_k)_{D^+}$. Here, $(a,b)_{D^+}\stackrel{\mathrm{def}}{=}\int_{D^+}a(\mathbf{x})b(\mathbf{x})\mathrm{d}\mathbf{x}$.

Let us also introduce $\overline{\rho}$: $\overline{\rho}\stackrel{\mathrm{def}}{=}L^{-1}g_0$, where g_0 is determined by (15), then it can also be represented by a Fourier series:

$$\overline{\rho}(\mathbf{x},t)=\sum_1^\infty \overline{T}_k(t)X_k(\mathbf{x})$$

where

$$\overline{T}_k(t)=-\mathrm{i}\frac{1}{2\sqrt{\mu_k}}\mathrm{e}^{\mathrm{i}\sqrt{\mu_k}t}\int_0^t \overline{c}_k(\tau)\mathrm{e}^{-\mathrm{i}\sqrt{\mu_k}\tau}\mathrm{d}\tau+\mathrm{i}\frac{1}{2\sqrt{\mu_k}}\mathrm{e}^{-\mathrm{i}\sqrt{\mu_k}t}\int_0^t \overline{c}_k(\tau)\mathrm{e}^{\mathrm{i}\sqrt{\mu_k}\tau}\mathrm{d}\tau$$

and $\overline{c}_k=(g_0,X_k)=-\int_\Gamma\sigma\left(\frac{\partial\rho}{\partial\mathbf{n}}X_k-\rho_\Gamma\frac{\partial X_k}{\partial\mathbf{n}}\right)\mathrm{d}\sigma_\Gamma$.

From Green's identity [15], we obtain

$$\begin{aligned}\overline{c}_k=\int_{D^+}(X_kL_x\rho-\rho L_xX_k)\mathrm{d}V&=-\mu_k(\rho,X_k)_{\gamma|D^+}+(X_k,f^+)_{D^+}-(X_k,\rho_{tt})_{\gamma|D^+}\\&=-\mu_k(\rho,X_k)_{\gamma|D^+}+c_k^+-\frac{\mathrm{d}^2}{\mathrm{d}t^2}(X_k,\rho)_{\gamma|D^+}.\end{aligned}$$

We can now rewrite $(u,X_k)_{\gamma|D^+}$ as follows:

$$(\rho,X_k)_{\gamma|D^+}=(\rho,X_k)_\gamma-(\rho,X_k)_{\gamma|D^-}=(\rho,X_k)_\gamma+(\widetilde{\rho},X_k)_\gamma,$$

where

$$\widetilde{\rho}=\begin{cases}0, & \text{on } D^+\\ -\rho, & \text{on } D^-.\end{cases}$$

Hence, since $(u,X_k)_\gamma=T_k$, we have:

$$\overline{c}_k=-\mu_kT_k-T_k''+c_k^+-\mu_k(\widetilde{\rho},X_k)_\gamma-\frac{\mathrm{d}^2}{\mathrm{d}t^2}(\widetilde{\rho},X_k)_\gamma=c_k^+-c_k-\widetilde{c}_k, \tag{19}$$

where $\widetilde{c}_k=\widetilde{T}_k+\mu_k\widetilde{T}_k$, $\widetilde{T}_k=(\widetilde{\rho}_k,X_k)_\gamma=-(\rho,X_k)_{\gamma|D^-}$. Thus, from (19) it follows that

$$\overline{\rho}=\rho^+-\rho-\widetilde{\rho}. \tag{20}$$

Let v be the solution of the direct AS problem: $v\stackrel{\mathrm{def}}{=}L^{-1}(f+g_0)$. From the linearity of the problem we obtain $v=\rho+\overline{\rho}$. Hence, $v_{D^+}=\rho^+_{D^+}$.

3.3. Resonance regime

The AS source term (15) gives the noise cancelation even in the case of resonance regimes. In order to demonstrate this, let us consider a time-harmonic noise source: $f^-=\gamma(\mathbf{x})\widehat{f}^-(\mathbf{x})\mathrm{e}^{\mathrm{i}\omega_0t}$, with supp $f^-\subset D^-$.

Then, $c_k=\mathrm{e}^{\mathrm{i}\omega_0t}(\widehat{f}^-,X_k)_\gamma$ and

$$T_k(t)=\frac{(\widehat{f}^-,X_k)_\gamma}{\omega_0^2-\mu_k}\left(\cos\sqrt{\mu_k}t-\cos\omega_0t+\mathrm{i}\left(\frac{\omega_0}{\sqrt{\mu_k}}\sin\sqrt{\mu_k}t-\sin\omega_0t\right)\right). \tag{21}$$

If $\omega_0\to\sqrt{\mu_k}$, then we have the resonance and one can show that:

$$T_k(t)=\frac{(\widehat{f}^-,X_k)_\gamma}{2\sqrt{\mu_k}}\left(t\sin\sqrt{\mu_k}t+\mathrm{i}\left(\frac{\sin\sqrt{\mu_k}t}{\sqrt{\mu_k}}-t\cos\sqrt{\mu_k}t\right)\right). \tag{22}$$

In this case the amplitude of the field linearly grows in time, and the growth velocity is higher for lower frequencies.

We can repeat the previous analysis and find out that if the AS source term (15) exactly corresponds to the field f^-, then it provides a complete noise cancelation in the domain D^+.

In the case of implementation of the AS source terms, some errors in the input data are unavoidable. Next, we consider the influence of both the frequency error and the phase error on the field in the domain to be shielded.

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4. Sensitivity analysis

Assume first that there is a frequency error and the frequency of the AS source term is given by $\widetilde{\omega}_0 = \omega_0 + \delta\omega$. Let us consider the following equivalent noise source: $(f^- - \widetilde{f}^-) + \widetilde{f}^-$ where $\widetilde{f}^- = \gamma(\mathbf{x})\widehat{f}^-(\mathbf{x})e^{i\widetilde{\omega}_0 t}$. it follows from the linearity of the problem that in D^+ the AS source term completely cancels the field generated by the second term $\widetilde{f}^-$ while the field generated by the first term $f^- - \widetilde{f}^-$ remains. As the result, on D^+, we have the following field: $\delta\rho = \rho^-(\omega_0) - \rho^-(\omega_0 + \delta\omega_0)$. If the original BVP (1), (2) is well-posed according to Hadamard, then there is a continuous dependence of the solution on the input parameters. Hence, there exists a constant $C > 0$:

$$\|\delta\rho\|_{L^2} < C\|\gamma(\mathbf{x})\widehat{f}^-(\mathbf{x})\|_{L^2} t|\delta\omega|. \tag{23}$$

The sensitivity analysis in the vicinity of a resonance frequency: $\omega_0 = \sqrt{\mu_k}$ gives the following perturbations of the Fourier coefficients:

$$\frac{\delta T_k(t)}{T_k(t)} = (F(\xi_k) - 1)\frac{\delta\omega}{2\sqrt{\mu_k}},$$

where $\xi_k = \sqrt{\mu_k}t$ and

$$F(\xi_k) = \frac{\xi_k^2(\cos\xi_k + i\sin\xi_k)}{\xi_k\sin\xi_k + i(\sin\xi_k - \xi_k\cos\xi_k)}. \tag{24}$$

If $\xi_k \to 0$, then

$$\frac{|\delta T_k(t)|}{T_k(t)} \to \frac{t|\delta\omega|}{3}. \tag{25}$$

For high frequencies $\xi_k \to \infty$ the dependence is similar:

$$\frac{|\delta T_k(t)|}{T_k(t)} \to \frac{t|\delta\omega|}{2}. \tag{26}$$

From Parseval's equality, in the resonance case we obtain that there is a constant $C > 0$:

$$\|\delta\rho\|_{L_2} < C\|\rho\|_{L_2} t|\delta\omega|. \tag{27}$$

In the analysis of the influence of a phase error, we consider a shift at the initial time. Then, in the resonance regime, we have

$$\delta T_k(t) = \frac{(\widehat{f}^-, X_k)_\gamma}{2\sqrt{\mu_k}}\left(\sin\sqrt{\mu_k}t + \sqrt{\mu_k}t\cos\sqrt{\mu_k}t + i\sqrt{\mu_k}t\sin\sqrt{\mu_k}t\right)\delta t.$$

If $\xi_k = \sqrt{\mu_k}t \to 0$, then

$$\frac{\delta T_k(t)}{T_k(t)} = O\left(\frac{\delta\phi}{\phi}\right). \tag{28}$$

In the case of high frequencies $\xi_k \to \infty$, we obtain

$$\frac{|\delta T_k(t)|}{T_k(t)} \to \sqrt{\mu_k}|\delta t|. \tag{29}$$

We can see that high frequencies are the most sensitive to the phase error.

We note that other possible sources of error may exist. In real applications, it is quite difficult or impossible to create a continuous shielding system corresponding to the secondary sources (15). Meanwhile, in [11] it was shown that the discrete analogue of (15) gives the exact noise cancelation on the appropriate grid and there is a consistency between the discrete and continuous source terms. It is well known that the minimal required grid resolution corresponds to the cell size smaller than half the minimal length wave: $h < \lambda_{\min}/2$. This simple evaluation is consistent with the results described in [1]. Finally, we assume that the secondary sources are acoustically transparent that in reality might not be the case.

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5. Application to aeroacoustics

Let us set in (11): $\gamma = 1/c^2$, $\sigma \equiv 1$, $q \equiv 0$, $\rho' = \rho - \rho_0$, $p' = c_0^2\rho'$ and assume that ρ is the density of fluid, ρ_0 and c_0 are the density and sound speed in quiescent medium, respectively. In addition, we assume that the source term is given by the following expression:

$$f = \Delta\widehat{T}, \tag{30}$$

$$\operatorname{supp} f \subset D^-, \tag{31}$$

where $\widehat{T}$ is a second rank tensor: $\widehat{T} \stackrel{\text{def}}{=} \rho\mathbf{u}\bigotimes\mathbf{u} + \widehat{P} - c_0^2\rho'\widehat{g}$, $\mathbf{u}$ is the fluid velocity, $\widehat{P}$ is the stress tensor and $\widehat{g}$ is the metric tensor.

Then, Eq. (11) is reduced to the Lighthill equation well known in aeroacoustics [16]:

$$\rho'_{tt} - c_0^2\Delta\rho' = \Delta\widehat{T}. \tag{32}$$

The Lighthill tensor $\widehat{T}$ called is responsible for noise generation by turbulent flow, in particular. It can be represented by quadruple volume sources.

If the domain D^+ has to be shielded, then according to (15) we have the following counterpart of Eq. (4):

$$\rho'_{tt} - c_0^2\Delta\rho' = \Delta\widehat{T} - \delta(\Gamma)\frac{\partial p'}{\partial\mathbf{n}_{|\Gamma}} - \frac{\partial\delta(\Gamma)p'_{\Gamma}}{\partial\mathbf{n}}. \tag{33}$$

We note that this equation corresponds to Curle's equation.

6. Conclusion

The solution of the AS problem has been obtained for the wave equation in the general formulation. The solution only requires the knowledge of the total field (desirable and undesirable) on the perimeter of the shielded domain and does not use any additional information on either the characteristics of the undesirable sources or the surrounding medium. It has been shown that the solution is applicable to resonance regimes. The sensitivity of the solution to the input errors in the frequency and phase data have been analyzed. The obtained solution is also applicable to aeroacoustics problems.

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