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Benchmark solutions

A practical algorithm for real-time active sound control with preservation of interior sound

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ABSTRACT

In the active sound control problem a bounded domain is protected from noise generated outside via implementation of secondary sound sources on the perimeter. In the current paper we consider a quite general formulation in which sound sources are allowed to exist in the region to be shielded. The sound generated by the interior sources is considered as desired. It is required to remain it unaffected by the control in the protected area. This task proves to be much more complicated than the standard problem of active sound control because of the reverse effect of the controls on the input data. A novel practical algorithm is proposed that can be used for a real-time control. It accepts a preliminary tuning of the control system. In the algorithm the only input information eventually needed is the total acoustic field near the perimeter of the region to be shielded. It includes the contribution from both primary and secondary sources. In the algorithm the noise component to be attenuated is automatically extracted from the total acoustic field. The control system can potentially operate in a real-time regime since it only requires a consequent solution of a quadratic programming problem.

1. Introduction

The active sound control (ASC) proved to be a very efficient approach to attenuate a low frequency noise. This approach has been intensively developed for the last fifty years. It provides an acoustic protection of a region from noise generated outside. The noise attenuation is realized via implementation of additional (secondary) sources on the perimeter of the region to be shielded. The approach is based on the use of Huygens' principle. As a result, secondary sources can be implemented along the perimeter to generate anti-noise. The required input information on the incoming noise can be immediately gained from the measurements. In case no desired acoustic field is presumed in the protected area, there are two principal approaches to the ASC: feed-back and feed-forward controls (see, e.g. [8,9]). The operation of the feed-back system is based on the minimization of the sound intensity at a set of sensors situated in the protected region. A common approach uses the filtered-x least mean square (FxLMS) algorithm [9] to minimize the level of noise. The feed-forward system operates in a predetermined way that depends on the acoustic field measured before the controls [10]. Practically the ASC is applicable to the attenuation of low-frequency noise because of space and time limitations on the control system. Thus, the ASC can be an effective supplement to the passive control that is basically efficient in application to mid- and high-frequency noise (basically, above 500 Hz). Currently most applications are related to noise attenuation in ventilation systems, headphones and propeller driven aircrafts. It is to be noted most of modern ASC systems are adaptive to small changes in the system to be controlled [9].

The problem of ASC becomes much more complicated if interior sound sources are present. In particular, the feed-back approach becomes unapplicable since it is impossible to immediately distinguish the noise component to be minimized from the total field. There is an additional challenge if it is required to retain the interior (desired) sound unaffected by the controls. Apparently, for the first time this problem was formulated by Fedoryuk in [2]. Afterwards it has been considered in a number of publications for time-harmonic waves (see, e.g., [1,11,12,18]). The problem formulation was extended to composite regions in [13,14,20]. In [19] the nonlinear problem of ASC was tackled for the first time with the use of nonlinear potentials. The unsteady ASC problem was considered in [3,15–17]. The potential-based approach to ASC developed in these papers was realized in experiments [6,21–23]. In all these papers a local control is used. This means each control source can operate independently from the others on the basis of the local field from all primary desired and undesired sound sources. In fact

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this is a strong assumption since the field generated by the primary sources cannot be immediately measured. As can be shown, if the interior sound exists, the local control inevitably must affect the input data [16]. Such a problem arises with noise attenuation in the cabin of a car or aircraft as well as with noise reduction in a room having an open window. Technically, this problem can be partially overcome with the use of directional measurements. This approach has been experimentally realized with a different level of success in [24–27]. A principally different approach is suggested in [28,29]. It requires a local solution of the wave equation across the boundary of the protected region with the controls. The boundary conditions for the wave equation are supposed to be taken from the measurements. The ASC problem is entirely formulated and considered in a discrete space.

In [5,7], nonlocal controls were derived in the frequency and time domains, respectively. In the approach the input data are supposed to be measured on the external side of the surface with the controls. The contribution of the desired field and controls is removed from the total field via the calculation of surface potentials which have a projection property. Thus, the control becomes nonlocal since it is based on the total field over the entire closed surface. This field is used as the density for the surface potentials. In the current paper we propose a practical algorithm to realize the control proposed in [5,7]. It is shown that to calculate the surface potentials the knowledge of Green's function is not necessary. Instead, a preliminary tuning of the control system can be used.

The rest of the paper is organized as follows. In the next section a mathematical formulation of the problem is given. The solution of the problem is based on the use of surface potentials, and in Section 3 a brief introduction to the generalized Caderón potentials is provided. The key property of the potentials is their projection property that is used for the ASC as described in Section 4. It is shown that the projection-based control is capable of retaining the interior sound field unaffected. Moreover, the approach can be applicable if only the total field from both primary and secondary sources is available nearby the perimeter of the region to be protected. A practical approach presumes the use of discrete sets of both sensors and controls. Such an algorithm is first described in Section 5 in the frequency domain. Then, in the next section, it is extended to an unsteady formulation.

2. Problem formulation

We suppose that some bounded region is acoustically protected from the noise generated outside. This means ideally there is no noise inside the protected region as the ASC operates. In addition, we allow a desired sound to be generated inside the protected region. It is required that the desired sound retains inside the protected region without changes. The boundary of the protected region is supposed to be acoustically transparent. The only way to tackle this problem is to distribute secondary acoustic sources outside the protected region. Practically the secondary sources should be situated on the boundary of the domain to be shielded in such a way that the total field satisfies the requirements formulated above.

Next, consider a mathematical formulation of the problem. For this purpose, introduce domain D: $\overline{D} \subseteq \mathbb{R}^3$ and a bounded subdomain $D^+ : \overline{D^+} \subset D$. We suppose that the boundaries of domains D and D^+ are the Lipschitz, and they are noted by Γ_0 and Γ, respectively.

Assume that sound field U is described by the following boundary-value problem (BVP) with homogeneous boundary conditions:

$$LU = f, \tag{1}$$

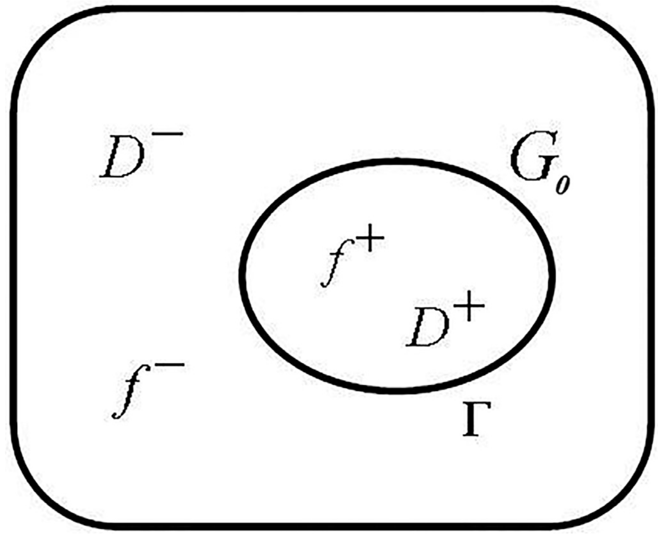

Fig. 1. Domain sketch.

$$U \in \Xi_D. \tag{2}$$

Here, the operator L is a linear differential operator. It can correspond to the linearized Euler equations (LEE) or the Helmholtz equation. Ξ_D is some functional linear space such that the inclusion (2) implicitly implies the boundary and initial (if needed) conditions.

To consider the LEE, introduce a first-order operator by

$$L_f \overset{def}{=} A^0 \frac{\partial}{\partial t} + \sum_1^3 A^i \frac{\partial}{\partial y^i}, \tag{3}$$

where $\{y^i\}$ $(i = 1, 2, 3)$ is the Cartesian coordinate system; A^k, $(k = 0, \ldots, 3)$ are 4×4 matrices: $A^k = A^k(\mathbf{y}) \in C^1(\overline{D})$. In the case of the unsteady formulation, we presume homogeneous initial conditions for the case of simplicity.

We also consider a second-order operator to analyze the Helmholtz equation:

$$L_s \overset{def}{=} -\nabla(p\nabla) - q, \tag{4}$$

where $p \in C^1(\overline{D})$, $q \in C(\overline{D})$ and $p > 0$.

In the further analysis, the solution of BVP (1) and (2) is considered in a weak sense. Thus, a function U is a solution of BVP (1) and (2) if $\langle LU, \Phi \rangle = \langle f, \Phi \rangle$ for any test function $\Phi(\overline{D}^0) \in C_0^\infty(\overline{D}^0)$. Here, $\langle f, \Phi \rangle$ denotes a linear continuous functional associated with a given generalized function f.

It is supposed that the acoustic sources are distributed both in D^+ and outside D^+ (see Fig. 1):

$$f = f^+ + f^-,$$
$$\mathrm{supp} f^+ \subset D^+,$$
$$\mathrm{supp} f^- \subset D^- \overset{def}{=} D \backslash \overline{D^+}. \tag{5}$$

Presume that we are going to protect region D^+ from noise generated outside D^+, in D^-. Thus, f^+ are interpreted as desired sources, while f^- generating noise.

The ASC problem can be formulated as an inverse source problem. It is required to find controls G_0 such as $\mathrm{supp} G_0 \subset \Gamma$ and the solution of BVP:

$$LV = f + G_0,$$
$$V \in \Xi_D \tag{6}$$

on D^+ coincides with the desired sound U^+. Thus, we require

$$U_{D^+} = V_{D^+}.$$

Here and further, U_{D^+} means the restriction of U to D^+.

Since the entire problem is linear, the desired field in D is described by the following BVP

$$LU^+ = f^+,$$
$$U^+ \in \Xi_D. \tag{7}$$

A key problem in the formulation is related to the input data. For a practical purposes, the input data are supposed to be available from immediate measurements (or calculations) in a vicinity of the boundary Γ.

The general solution to the ASC problem, formulated above, is based on the theory of surface potentials. A brief introduction to this theory is given in the next section.

3. Calderón projectors

The solution to ASC problem can be realized via the Calderón projectors in the form first introduced by Ryaben'kii [1] for regular functions. This form makes the Calderón projectors more suitable for practical usage, in particular for computations.

Following [1], see also [5], consider an operator $P_{D^+} : \Xi_{D^+} \to \Xi_{D^+}$, where $\Xi_{D^+} = \{V_{D^+} | \ V \in \Xi_D\}$, as follows:

$$P_{D^+}V_{D^+} \stackrel{def}{=} L_{D^+}^{-1}(\theta(\overline{D^-})LV), \tag{8}$$

where $L_{D^+}^{-1}g \stackrel{def}{=} L^{-1}g_{|D^+}$, $\theta(\overline{D^-})$ is the characteristic function of $\overline{D^-}$.

One can show (see [1,3,5]) that in the case of the Poisson and Helmholtz equations potential (8) can be reduced to the classical single- and double-layer potentials formulated via Green's function.

One can prove that the potential $P_{D^+}V_{D^+}$ is fully determined by the Cauchy data [1,5].

Next, introduce the trace operator $\mathrm{Tr}_\Gamma : H^s(D^+) \to H^{s-1/2}(\Gamma)$:

$$\mathrm{Tr}_\Gamma U_{D^+} \stackrel{def}{=} \lim_{\epsilon \to 0} \mathrm{Tr}_{\Gamma_\epsilon^+} U_{D^+}, \tag{9}$$

where

$$\mathrm{Tr}_{\Gamma_\epsilon^+} U_{D^+} \stackrel{def}{=} U_{D^+}(\mathbf{x}), \quad \mathbf{x} \in \Gamma_\epsilon^+.$$

Here, Γ_ϵ^+ are smooth manifolds parallel to Γ [4]: $\Gamma_\epsilon^+ \subset D^+$, $\Gamma_\epsilon^+ \to \Gamma$ if $\epsilon \to 0$.

Then, the Cauchy data $\mathrm{Tr}_\Gamma V_{D^+}$ are given by

$$\mathrm{Tr}_\Gamma V_{D^+} = \begin{cases} V_\Gamma, & \text{if } \ L := L_f, \\ (V, \frac{\partial V}{\partial \mathbf{n}})_\Gamma^T, & \text{if } \ L := L_s. \end{cases} \tag{10}$$

Here $\mathbf{n}$ is the outward normal to the boundary Γ.

As shown in [3,5],

$$P_{D^+\Gamma}\xi_\Gamma = -L_{D^+}^{-1}(A_\Gamma \zeta(\xi_\Gamma)), \tag{11}$$

represents a surface potential with density $\xi_\Gamma = \mathrm{Tr}_\Gamma V_{D^+}$ if

$$A_\Gamma = \begin{cases} A_n \stackrel{def}{=} \sum_1^3 A_i n^i, & \text{if } \ L := L_f, \\ -p_\Gamma(1,1), & \text{if } \ L := L_s, \end{cases}$$

and

$$\zeta(\xi_\Gamma) = \begin{cases} \xi_\Gamma \delta(\Gamma), & \text{if } \ L := L_f, \\ (\xi_\Gamma^{(1)} \frac{\partial}{\partial \mathbf{n}} \delta(\Gamma), \xi_\Gamma^{(2)} \delta(\Gamma))^T & \text{if } \ L := L_s, \end{cases}$$

where $\xi_\Gamma = (\xi_\Gamma^{(1)}, \xi_\Gamma^{(2)})^T$, $\delta(\Gamma)$ is the surface delta–function associated with Γ.

One can show (see, e.g. [5]), that in application to the Helmholtz equation the potential (11) can be reduced to the classical single- and double-layer potentials by setting either $\xi_\Gamma = (0,1)^T$ or $\xi_\Gamma = (1,0)^T$, respectively.

In the general case of L_s, the potential $P_{D^+}\xi_\Gamma$ is represented by a linear combination of single- and double-layer potentials:

$$P_{D^+\Gamma}\xi_\Gamma = L_{D^+}^{-1}\left(\xi_\Gamma^{(2)}\delta(\Gamma) + \frac{\partial}{\partial \mathbf{n}}(\xi_\Gamma^{(1)}\delta(\Gamma))\right). \tag{12}$$

For our further analysis, it is very important that the potentials (8) are projectors:

$$P_{D^+\Gamma}\mathrm{Tr}_\Gamma(L_{D^+}^{-1}f^+) = 0_{D^+}, \tag{13}$$

and

$$P_{D^+\Gamma}\mathrm{Tr}_\Gamma(L_{D^+}^{-1}f^-) = L_{D^+}^{-1}f^-. \tag{14}$$

The solution to the ASC problem is based on the property of projection and given in the next section.

4. Active sound control

4.1. Active sound control without a reverse effect

Next, we decompose the total acoustic field into desired sound U^+ and noise U^-:

$$U = U^+ + U^-.$$

It is clear that

$$U_{D^+}^- = L_{D^+}^{-1}f^-.$$

Then, from the property of projection it follows that

$$P_{D^+\Gamma}\mathrm{Tr}_\Gamma U_{D^+} = U_{D^+}^-. \tag{15}$$

Thus, the potential operates as a filter to subtract the noise component from the total field. Then, to cancel the noise, it is required to generate the field that is equal to U^- with an opposite noise or "anti-noise" [31]. From the Huygens principle it is sufficient to generate anti-noise in a vicinity of the boundary Γ.

The required result can be achieved (see, e.g., [3]) if the secondary source G_0 is distributed on boundary Γ:

$$G_0 = A_\Gamma \zeta(\mathrm{Tr}_\Gamma U). \tag{16}$$

In more detail, in the case of the first order operator L_f the control (16) is given by

$$G_0 = A_n U_\Gamma \delta(\Gamma)), \tag{17}$$

whilst with the second order operator L_s it is represented by

$$G_0 = \frac{\partial p_\Gamma}{\partial \mathbf{n}}\delta(\Gamma) + \frac{\partial}{\partial \mathbf{n}}(p_\Gamma \delta(\Gamma)), \tag{18}$$

where $\mathbf{n}$ is the outward normal to Γ.

4.1.1. Linearized Euler equations

Next, consider an application of the control (17) to the LEE:

$$\frac{1}{\rho_0 c_0^2}(p_t + (\mathbf{u}_0, \nabla)p) + \frac{1}{\rho_0 c_0^2}(\mathbf{u}, \nabla)p_0 + \nabla \cdot \mathbf{u}$$
$$= \frac{1}{\rho_0 c_0^2} f^{(p)} + q_{vol},$$
$$\rho_0(\mathbf{u}_t + (\mathbf{u}_0, \nabla)\mathbf{u} + (\mathbf{u}, \nabla)\mathbf{u}_0) + \nabla p = \mathbf{f}^{(u)} + \mathbf{f}_{vol}. \tag{19}$$

Here u_j $(j = 1, 2, 3)$ are the components of the particle velocity $\mathbf{u}$ in some Cartesian coordinate system; p, the sound pressure; c_0, the speed of sound; ρ_0, the density; the functions marked by 0 correspond to some mean flow; q_{vol}, the volume velocity per a unit volume; $\mathbf{f}_{vol}$, the force per a unit volume [8] and $f^{(p)}$ and $\mathbf{f}^{(u)}$ are possible additional sound sources.

In the case of

$$U = (u_1, u_2, u_3, p)^T, \tag{20}$$

matrix A_n reads

$$A_n = \begin{pmatrix} n_1 & n_2 & n_3 & \frac{u_{0n}}{\rho_0 c_0^2} \\ \rho_0 u_{0n} & 0 & 0 & n_1 \\ 0 & \rho_0 u_{0n} & 0 & n_2 \\ 0 & 0 & \rho_0 u_{0n} & n_3 \end{pmatrix}, \tag{21}$$

where $u_{0n} = \mathbf{u}_0 \cdot \mathbf{n}$.

Then, from (17) the control is given by [3]

$$q_{vol} = \left(\mathbf{u} \cdot \mathbf{n}_{|\Gamma} + \frac{u_{0n}}{\rho_0 c_0^2} p_{|\Gamma} \right) \delta(\Gamma),$$
$$\mathbf{f}_{vol} = (p_{|\Gamma}\mathbf{n} + \rho_0 u_{0n} \mathbf{u}_{|\Gamma}) \delta(\Gamma). \tag{22}$$

Obviously, if $u_0 = 0$, then we arrive at the acoustics Euler equations which are reduced to the Helmholtz equation in the case of a harmonic sound.

4.1.2. Helmholtz equation

Next, consider the Helmholtz equation with the controls:

$$\Delta p + k^2 p = f + G_0,$$

where k is the wave number.

Then, from Eq. (18) the control function is given by

$$G_0 = \frac{\partial p}{\partial \mathbf{n}} \delta(\Gamma) + \frac{\partial p \delta(\Gamma)}{\partial \mathbf{n}}. \tag{23}$$

The same control can be obtained from the wave equation (see, e.g., [3]). It is clear that in the case of $u_0 = 0$, control (22) is reduced to control (23).

As can immediately be proven, the secondary source (16) generates the anti-noise field equal to $-U^-$. It provides the entire noise cancellation in the protected region, whilst retaining the desired sound unchanged. This exact solution to the inverse source problem on the ASC is based on two essential assumptions which are hardly realizable in practice.

First, the control distribution is supposed to be continuous over the whole boundary Γ. This assumption can be overcome via a compromise with the accuracy. In other words, a continuous distribution of the control can be approximated by a discrete distribution. Another assumption is more problematic to be resolved. It presumes the total field from the primary sources is available nearby the boundary Γ and can be immediately measured. This assumption is quite natural for many systems of ASC in which the existence of desired sound is not taken into consideration. As can be shown, see e.g. [17], with the secondary source (16) the desired sound is doubled immediately outside the control, in D^-. Thus, the field from the primary sources becomes unavailable for immediate measurements. In other words, the ASC affects the input information. The problem is significantly simplified if the total primary field can be recorded in advance as it was realized in [21,22]. However, it is clear that this approach is practically very limited.

4.2. Active sound control with the reverse effect. Frequency domain

To take into account the reverse effect of the controls, in [5,7] a two-stage procedure is proposed that can be successively realized. First, the noise component is subtracted from the total field that can contain a contribution from the secondary sources. Then, the noise can be cancelled by the anti-noise that can be generated by the source (16) with the input data obtained at the previous stage. In this approach the total field from both the primary and secondary sources has a jump across the boundary Γ due to the control. It is supposed that the total field is measured on the external side of the boundary Γ.

Next, to formulate the control, introduce a trace operator

$$\mathrm{Tr}_{\Gamma^-} U_{D^-} \overset{def}{=} \lim_{\epsilon \to 0} \mathrm{Tr}_{\Gamma_\epsilon} U_{D^-}, \tag{24}$$

where

$$\mathrm{Tr}_{\Gamma_\epsilon^-} U_{D^-} \overset{def}{=} U_{D^-}(\mathbf{x}), \quad \mathbf{x} \in \Gamma_\epsilon^-.$$

Here, Γ_ϵ^- are smooth manifolds parallel to Γ: $\Gamma_\epsilon^- \subset D^-$, $\Gamma_\epsilon^- \to \Gamma$ if $\epsilon \to 0$.

Then

$$G_0 = A_\Gamma \zeta (P_{\Gamma\Gamma^-} \mathrm{Tr}_{\Gamma^-} W), \tag{25}$$

where W is the total field from both the primary and secondary sources measured on the external side Γ^- of the boundary Γ.

It is important to note that in contrast to the control (16), the secondary source (25) is nonlocal because of the operator $P_{D^+\Gamma}$. This means, the contribution of both the desired sources and control is integrated and automatically eliminated from the total field. The realization of the control (25) is not obvious.

First, for a practical realization the sensors cannot be situated nearby the controls. Therefore, it is better to separate surfaces Γ^- and Γ from each other. Assume that the surface Γ^- is smooth enough and $\Gamma^- \subset D^-$. In addition, we assume that there is no noise sources situated between Γ and Γ^-.

Second, the following requirement should be acoustically satisfied:

$$d_H(\Gamma, \Gamma^-) \ll \lambda_{min},$$

where $d_H(\Gamma, \Gamma^-)$ is the Hausdorff distance between the surfaces and λ_{min} is the minimal wave length to be attenuated.

Then, the control (25) can be used to protect the domain D^+. In this case, the input data can be taken from the boundary Γ^- whilst the controls are situated on Γ.

For further consideration presume Γ^- is the boundary of domain $D_-^+ : D^+ \subset D_-^+$. If the field at the boundary Γ^- is known from either the measurements or calculations, then the noise component is given by

$$p^-(\mathbf{x}) = \int_{\Gamma^-} \left(p \frac{\partial G}{\partial \mathbf{n}} - \frac{\partial p}{\partial \mathbf{n}} G \right) d\sigma, \quad \mathbf{x} \in D_-^+. \tag{26}$$

Here, G is Green's function.

Then, the control function can be obtained immediately

$$G_0 = \frac{\partial p^-}{\partial \mathbf{n}} \delta(\Gamma) + \frac{\partial p^- \delta(\Gamma)}{\partial \mathbf{n}} \tag{27}$$

since $D^+ \subset D_-^+$ and the field generated by G_0 in D^+ is

$$p_c(\mathbf{x}) = \int_\Gamma \left(\frac{\partial p^-}{\partial \mathbf{n}} G(\mathbf{x}|\Gamma) - p^- \frac{\partial G}{\partial \mathbf{n}}(\mathbf{x}|\Gamma) \right) d\sigma, \quad \mathbf{x} \in D^+.$$

On the other hand, the distribution of p^- in D^+ is given by

$$p^-(\mathbf{x}) = \int_\Gamma \left(p^- \frac{\partial G}{\partial \mathbf{n}}(\mathbf{x}|\Gamma) - \frac{\partial p^-}{\partial \mathbf{n}} G(\mathbf{x}|\Gamma) \right) d\sigma, \quad \mathbf{x} \in D^+.$$

Thus, in D^+ we have the noise cancellation $p_c(\mathbf{x}) = -p^-(\mathbf{x})$ while the desired field retains.

4.3. Active sound control with the reverse effect. Time domain

One can apply the same approach to first-order equations such as (19) in time domain. Then, from the projection property

$$U^-(\mathbf{x}, t) = \int_{\Gamma^-, \tau} \widehat{G}(\mathbf{x}, t|\Gamma^-, \tau) A_n U(\Gamma^-, \tau) d\sigma d\tau \quad \mathbf{x} \in \Gamma. \tag{28}$$

Here, $\widehat{G}$ is Green's matrix function, the convolution takes into account the retarded time.

Then, the control on Γ is given by

$$G_0(\Gamma, t) = A_n U^-(\mathbf{x}, t) \delta(\Gamma). \tag{29}$$

As can be seen from (28), such a control is nonlocal not only in space but also in time. In the protected region it generates the field

$$U_c(\mathbf{x}, t) = \int_{\Gamma,\tau} \widehat{G}(\mathbf{x}, t|\Gamma, \tau) A_n U d\sigma d\tau \quad \mathbf{x} \in D^+.$$

In the continuous case, the entire noise attenuation is realized: $U_c(\mathbf{x}, t) = -U^-(\mathbf{x}, t)$, $\mathbf{x} \in D^+$ [7].

It is clear that the continuous distribution of controls (27) and (29) is practically impossible. To realize these controls, a discrete approximation of the surface potentials is needed.

Consider first a discrete approximation of the surface controls in the frequency domain. For this purpose in the next section the Helmholtz equation is addressed.

5. Discrete distribution of sensors on Γ^- and controls on Γ. Frequency domain

5.1. Discrete distribution of sensors

First consider a discrete distribution of the sensors on Γ^-.

Next, let us approximate the first integral in (26):

$$\int_{\Gamma^-} \frac{\partial p}{\partial \mathbf{n}}_{|\Gamma^-} G(\mathbf{x}|\Gamma^-) d\sigma \approx \sum_{i=1}^{N^-} \frac{\partial p}{\partial \mathbf{n}}(\mathbf{x}_i) G(\mathbf{x}|\mathbf{x}_i) \Delta\sigma_i^- + \sum_{i=1}^{N^-} \int_{\sigma_i^-} [\frac{\partial p}{\partial \mathbf{n}}_{|\Gamma^-} G(\mathbf{x}|\Gamma^-) d\sigma - \frac{\partial p}{\partial \mathbf{n}}(\mathbf{x}_i) G(\mathbf{x}|\mathbf{x}_i)] \Delta\sigma_i^-, \quad (30)$$

where $\Delta\sigma_i^- \quad (i = 1, \dots, N^-)$ are nonintersecting elementary areas σ_i^- which cover the entire surface Γ^-, $\mathbf{n}$ is the outward normal to Γ^-.

Next, assume that $l_i = \sqrt{\Delta\sigma_i^-} \ll r_i =| \mathbf{x} - \mathbf{x}_i | \quad (i = 1, \dots, N^-)$. Then, on an ith segment $\Delta\sigma_i^-$ of boundary Γ^- we have

$$G(\mathbf{x}|\Gamma_i^-) \approx G(\mathbf{x}|\mathbf{x}_i)$$

since for the free-space Green's function

$$G(\mathbf{x}|\Gamma_i^-) - G(\mathbf{x}|\mathbf{x}_i) \approx \frac{1}{|\mathbf{x} - \mathbf{y}_i|} - \frac{1}{r_i} \approx \frac{\mathbf{x} \cdot \mathbf{y}_i}{r_i^3},$$

where $\mathbf{y}_i \in \Delta\sigma_i^-$.

Thus, the discrete approximation error is given by

$$e = | \int_{\Gamma} \frac{\partial p}{\partial \mathbf{n}}_{|\Gamma^-} G(\mathbf{x}|\Gamma^-) d\sigma - \sum_{i=1}^{N^-} \frac{\partial p}{\partial \mathbf{n}}(\mathbf{x}_i) G(\mathbf{x}|\mathbf{x}_i) \Delta\sigma_i^- | \approx | \sum_{i=1}^{N^-} \int_{\sigma_i^-} \left[\frac{\partial p}{\partial \mathbf{n}}_{|\Gamma^-}(\mathbf{x}) - \frac{\partial p}{\partial \mathbf{n}}(\mathbf{x}_i) \right] G(\mathbf{x}|\Gamma^-) d\sigma |. \quad (31)$$

Thus, the approximation error corresponds to the difference between the radiation from entire elementary surfaces with distributed and fixed strengths.

On the other hand,

$$\frac{\partial p}{\partial \mathbf{n}}_{|\Gamma^-} - \frac{\partial p}{\partial \mathbf{n}}(\mathbf{x}_i) \sim \frac{\partial}{\partial s} \frac{\partial p}{\partial \mathbf{n}}(\mathbf{x}_i) l_i, \quad i = 1, \dots, N^-.$$

Here, $\frac{\partial}{\partial s}$ is the derivative along the surface, $l_i \in \sigma_i^-$. It is proportional to $\frac{1}{\lambda}$, where λ is the wave length.

A similar estimate occurs for the second integral in (26) as well.

From the representation of the difference between the full continuous and discrete control (31), it follows that the error is $O(kl)$ where l is a typical $l_i \quad (i = 1, \dots, N^-)$. However, if the sensor is situated at the center of mass, then, since the local error $e_i \approx \int_{\Delta\sigma_i^-} l_i d\sigma$, the total error is at least $O(k^2 l^2)$. Thus, the entire surface can be split into cells and the location of the sensor in each cell should be at the center of mass of an appropriate cell.

In addition, the error caused by a discrete distribution of the controls and sensors can be maximally reduced via the placement of the sensors and controls at the zero points of the Chebyshev polynomials [30].

It is to be noted that the knowledge of Green's function is not necessarily needed. To avoid it, it is sufficient to make preliminary measurements of the field from unit monopoles and dipoles between the sensors and controls. In other words, the set of Green's functions can be measured in advance. Alternatively, elementary surface potentials can be calculated in advance with either real or approximate boundary conditions.

5.2. Explicit formula for the controls

The control function (27) on Γ can be approximated via a set of monopoles and dipoles:

$$G_0 = \frac{\partial p^-}{\partial \mathbf{n}} \delta(\Gamma) + \frac{\partial p^- \delta(\Gamma)}{\partial \mathbf{n}} \approx \sum_{i=1}^{N} \left[\frac{\partial p^-}{\partial \mathbf{n}}(\mathbf{x}_i) \delta(\mathbf{x} - \mathbf{x}_i) + p^-(\mathbf{x}_i) \frac{\partial \delta}{\partial \mathbf{n}}(\mathbf{x} - \mathbf{x}_i) \right] \Delta\sigma_i, \quad (32)$$

where $\mathbf{x}_i \in \Delta\sigma_i \quad (i = 1, \dots, N)$, $\mathbf{n}$ is the outward normal to Γ.

On the other hand

$$p^-(\mathbf{x}_i) = \sum_{j=1}^{N^-} \Delta\sigma_j^- \left(\frac{\partial p}{\partial \mathbf{n}}(\mathbf{x}_j) G(\mathbf{x}_i|\mathbf{x}_j) - p(\mathbf{x}_j) \frac{\partial G}{\partial \mathbf{n}}(\mathbf{x}_i|\mathbf{x}_j) \right),$$

where $\mathbf{x}_i \in \Delta\sigma_i \quad (i = 1, \dots, N)$, $\mathbf{x}_j \in \Delta\sigma_j^- \quad (j = 1, \dots, N^-)$.

Thus, we arrive at an explicit formula for the controls:

$$G_0 = \sum_{i=1}^{N} \Delta\sigma_i \sum_{j=1}^{N^-} \left[\left(A_{ij} \frac{\partial p}{\partial \mathbf{n}} + B_{ij} p \right)_{|\mathbf{x}_j} \delta(\mathbf{x} - \mathbf{x}_i) + \left(C_{ij} \frac{\partial p}{\partial \mathbf{n}} + D_{ij} p \right)_{|\mathbf{x}_j} \frac{\partial \delta}{\partial \mathbf{n}}(\mathbf{x} - \mathbf{x}_i) \right].$$

Here, matrices A, B, C and D, which have dimensions $[N \times N^-]$, depend only on the Green's function and coordinates of the sensors and speakers. Thus, they represent the response matrices that can be measured (or calculated if possible) in advance. It is clear that the operation of the controls is nonlocal since each control depends on the data from all N^- sensors.

5.3. Surface potentials

A more efficient approach can be based on surface potentials. The main idea is as follows. First, a set of basis functions is introduced on the surface Γ^-. Then, a set of potentials (responses) in which the basis functions are used as their densities are calculated. These basis potentials can be obtained in advance and used for a quick approximation of the potentials with arbitrary densities. The approximation is based on the linearity property of the potentials. A similar algorithm is used in the method of Difference Potentials [1] for finite difference potentials.

Next, consider a set of basis functions $\phi_{k\Gamma^-} \quad (k = 1, \dots, N_b)$. It is worth noting that in the general case $P_{\Gamma^-\Gamma^-}\phi_{k\Gamma^-} \neq \phi_{k\Gamma^-}$.

Then, we can calculate (or measure) in advance the response to signal ϕ_{Γ^-} on Γ: $\nu_{k\Gamma} = P_{\Gamma\Gamma^-}\phi_{k\Gamma^-}$ $(k = 1, \dots, N_b)$. This can be presented via a response from a set of monopoles and dipoles with strengths $\phi_{k,\,i}$ and $\frac{\partial \phi_k}{\partial \mathbf{n}}_{|i}$ $(i = 1, \dots, N^-)$, respectively, which are situated on Γ^-:

$$v_k(\phi_k) = \sum_{i=1}^{N^-} \left(\phi_{k,i} \frac{\partial G}{\partial \mathbf{n}}_{|i} - \frac{\partial \phi_k}{\partial \mathbf{n}}_{|i} G_{|i} \right) \Delta\sigma_i^- \quad (k = 1, \ldots, N_b). \quad (33)$$

Next, for any density $\xi_{\Gamma^-} \in \mathbb{R}^{N^-}$ measured on Γ^- via both the sound pressure and particle velocity we calculate an approximation of ξ_{Γ^-} by means of the basis functions $\phi_{k\Gamma^-} \in \mathbb{R}^{N^-}$ $(k = 1, \ldots, N_b)$:

$$\min_{\alpha_k} \|\xi_{\Gamma^-} - \sum_{k=1}^{N_b} \alpha_k \phi_{k\Gamma^-}\|_{2,\Gamma^-}, \quad (34)$$

subject to $\alpha_k \geq 0$ $(k = 1, \ldots, N_b)$, $\sum_{k=1}^{N_b} \alpha_k = 1$.

Then, from the linearity of the potentials we obtain the value of the potential on Γ:

$$P_{\Gamma\Gamma^-}\xi_{\Gamma^-} \approx \sum_{k=1}^{N_b} \alpha_k v_{k\Gamma}.$$

In this way a transfer function from Γ^- to Γ can be determined.

For practical implementation it is important to be able to quickly calculate the potentials in (18). The number of basis functions can be significantly reduced: $N_b \ll N^-$ if they are selected well enough. The calculation of the potentials can be realized via the method of Difference Potentials [1]. In a practical realization, the use of the Boundary Element Methods (see, e.g., [32]) may be efficient.

To calculate the surface potentials, we should know the boundary conditions on the external boundary of D^-. However, in real applications they can be unavailable. Then, there are two principal ways:

1) experimental identification of the response functions $v_{k\Gamma}$ $(k = 1, \ldots, N_b)$. In this case the measurements of these functions can be obtained in advance.
2) the surface potentials and, hence, response functions $v_{k\Gamma}$ $(k = 1, \ldots, N_b)$ can be calculated with some prescribed boundary conditions. Physically, this means we attenuate noise that would be with different boundary conditions and, hence, a different reflection from the boundary.

In more detail, let us present the total sound pressure at the boundary Γ^- as

$$p_{|\Gamma^-} = \overline{p}_{|\Gamma^-} + \delta p_{|\Gamma^-}.$$

Here, $\overline{p}$ is the sound pressure that would be with the same sources of sound and prescribed boundary conditions. Then,

$$P_{D^+_-\Gamma^-} p_{\Gamma^-} = -\overline{p}^-,$$

where $\overline{p}^-$ means the noise that would be with the prescribed boundary conditions.

Hence, the fraction δp is out of control but practically it might be not essential.

5.4. Discrete distribution of the controls

A discrete distribution of the controls on Γ is an essential issue even if $f^+ = 0$. Indeed, in this case the sound generated by the continuous control makes no effect outside D^+:

$$p_c(\mathbf{x}) = \int_\Gamma \left(p^- \frac{\partial G}{\partial \mathbf{n}} - \frac{\partial p^-}{\partial \mathbf{n}} G \right) d\sigma = 0, \quad \mathbf{x} \in D^-.$$

Thus, the control does not affect the measurements on Γ^-. However, this is the case only with a continuous control on Γ.

In the general case of a discrete distribution of the controls

$$p_c(\mathbf{x}) = \sum_{i=1}^{N} \left(p^-_{|i} \frac{\partial G}{\partial \mathbf{n}}_{|i} - \frac{\partial p^-}{\partial \mathbf{n}}_{|i} G_{|i} \right) \Delta\sigma_i \neq 0, \quad \mathbf{x} \in \Gamma^-.$$

In more detail

$$p_c(\mathbf{x}) = \int_\Gamma \left(p^- \frac{\partial G}{\partial \mathbf{n}} - \frac{\partial p^-}{\partial \mathbf{n}} G \right) d\sigma + \left(\sum_{i=1}^{N} p^-_{|i} \frac{\partial G}{\partial \mathbf{n}}_{|i} \Delta\sigma_i - \int_\Gamma p^- \frac{\partial G}{\partial \mathbf{n}} d\sigma \right) - \left(\sum_{i=1}^{N} \frac{\partial p^-}{\partial \mathbf{n}}_{|i} G_{|i} \Delta\sigma_i - \int_\Gamma \frac{\partial p^-}{\partial \mathbf{n}} G d\sigma \right) \neq 0, \quad \mathbf{x} \in \Gamma^-.$$

As can be seen, the second and third terms represent the error of approximation of surface integrals that is not necessarily equal to zero.

Thus, it is important to have a good enough approximation of the surface integrals on Γ. Otherwise, the input data can be affected by the controls even without a desired sound.

In region D^+ the field generated by the controls is equal to

$$p_c(\mathbf{x}) = \sum_{i=1}^{N} \left(p^-_{|i} \frac{\partial G}{\partial \mathbf{n}}_{|i} - \frac{\partial p^-}{\partial \mathbf{n}}_{|i} G_{|i} \right) \Delta\sigma_i = \int_\Gamma \left(p^- \frac{\partial G}{\partial \mathbf{n}} - \frac{\partial p^-}{\partial \mathbf{n}} G \right) d\sigma + \left(\sum_{i=1}^{N} p^-_{|i} \frac{\partial G}{\partial \mathbf{n}}_{|i} \Delta\sigma_i - \int_\Gamma p^- \frac{\partial G}{\partial \mathbf{n}} d\sigma \right) - \left(\sum_{i=1}^{N} \frac{\partial p^-}{\partial \mathbf{n}}_{|i} G_{|i} \Delta\sigma_i - \int_\Gamma \frac{\partial p^-}{\partial \mathbf{n}} G d\sigma \right) = -p^- + \delta p', \quad \mathbf{x} \in D^+.$$

Thus, a discrete set of controls does not provide the entire noise cancellation. A fraction of noise equal to $\delta p'$ retains and reaches the sensors on Γ^-. To cancel this field, the projection $P_{\Gamma\Gamma^-}\delta p_\Gamma$ should be calculated before transferring the data to the controls.

6. Discrete distribution of sensors on Γ^- and controls on Γ. Time domain

Consider now the effect of a finite number of sensors and controls in the time domain. The key difference is related to the retarded time. As shown further, its effect can be taken into account automatically. The main requirement to the control system is that all computations should be fast enough. More precisely, the computational time t_c required for the control should be much less than the typical acoustic time t_a:

$$t_c \ll t_a = \lambda_{min}/a,$$

where a is the speed of sound, t_c is the period of updating the control.

6.1. Explicit formula for the controls

The input data for the controls are related to the data measured by the sensors on Γ^-:

$$U^-(\mathbf{x}_i, t) = \sum_{j=1}^{N^-} \Delta\sigma_j^- \int_\tau \widehat{G}(\mathbf{x}_i, t|\mathbf{x}_j, \tau) A_n U(\mathbf{x}_j, \tau) d\tau, \quad (35)$$

where $\mathbf{x}_i \in \Delta\sigma_i$ $(i = 1, \ldots, N)$, $\mathbf{x}_j \in \Delta\sigma_j^-$ $(j = 1, \ldots, N^-)$.

Then, the control (29) is approximated by

$$G_0(\mathbf{x}, t) = \sum_{i=1}^{N} A_n U^-(\mathbf{x}_i, t) \Delta\sigma_i \delta(\mathbf{x} - \mathbf{x}_i, t), \quad \mathbf{x}_i \in \Delta\sigma_i. \quad (36)$$

In the unsteady case, the problem can be solved in either the frequency domain or the time domain with the usage of the impulse response function. The impulse response function is the response to a unit impulse applied to an infinitesimally short period of time. It can be represented by Green's function. With the use of the impulse response function the retarded time can be taken into account automatically.

Consider the use of the surface potentials. In the unsteady case the procedure is similar to that described in the previous section but the coefficients α_j depend on t. Thus, they should be steadily updated via the solution of a variation problem.

Next, introduce the surface basis functions $\phi_{k\Gamma^-}$ $(k = 1, \ldots, N_b)$. Then, the impulse response function is given by

$$v_k(\mathbf{x}_i, t) = \begin{cases} \sum_{j=1}^{N^-} \widehat{G}(\mathbf{x}_i, t|\mathbf{x}_j, 0) A_n \phi_k(\mathbf{x}_j) \Delta\sigma_j^-, & \min_j \tau_{ij} \le t \le \max_j \tau_{ij} \\ 0, & \text{else}, \end{cases}$$

$$\mathbf{x}_i \in \Delta\sigma_i \quad (i = 1, \ldots, N), \quad \mathbf{x}_j \in \Delta\sigma_j^- \quad (j = 1, \ldots, N^-),$$

where $\tau_{ij} = \frac{|\mathbf{x}_i - \mathbf{x}_j|}{a}$ is a retarded time.

Then, for any density $\xi_{\Gamma^-}(t)$ measured on Γ^- at time moment $t = t_p$ $(p = 0, 1, \ldots)$ we calculate the approximation of $\xi_{\Gamma^-}(t_p)$ via $\phi_{k\Gamma^-}$ $(k = 1, \ldots, N_b)$:

$$\min_{\alpha_k(t_p)} \|\xi_{\Gamma^-}(t_p) - \sum_{k=1}^{N_b} \alpha_k(t_p) \phi_{k\Gamma^-}\|_{2,\Gamma^-}, \tag{37}$$

subject to $\alpha_k \ge 0$ $(k = 1, \ldots, N_b)$, $\sum_{k=1}^{N_b} \alpha_k = 1$.

Thus, for time interval $[t_p \; t_{p+1}]$ until updating $\xi_{\Gamma^-}(t)$ we have an approximation of the potential

$$P_{\Gamma\Gamma^-}\xi_{\Gamma^-}(t) \approx \sum_{k=1}^{N_b} \sum_{l=0}^{p} \alpha_k(t_l) v_{k\Gamma}(t - t_l)(t_{l+1} - t_l). \tag{38}$$

The knowledge of the response functions is a key factor if the control system must operate in the real-time regime. In this case, in addition, the quadratic optimization problem (37) should be solved as fast as possible. It is to be noted that algorithm (38) is the basic. It can be improved with respect to the time resolution.

Thus, there are the following basic steps in the algorithm.

1. Preliminary tuning. The surface potentials with the densities equal to the basis functions $\phi_{k\Gamma^-}$, $(k = 1, \ldots, N_b)$ are measured (or calculated) in advance. As the output, the set of response functions $v_{k\Gamma}$ $(k = 1, \ldots, N_b)$ is obtained.

2. The input data (density $\xi_{\Gamma^-}(t_p)$) is measured at time step t_p.

3. Variation problem (37) is solved. As the output, the set of coefficients α_k $(k = 1, \ldots, N_b)$ is obtained.

4. The potential (38) is calculated. It provides the noise field $U^- \in \mathbb{R}^N$ to be attenuated with the use of control (36).

7. Conclusion

A practical algorithm has been proposed for the active noise cancellation with preservation of desired sound in the real-time regime. It is based on surface potentials and their projection property. The approach does not presume an explicit knowledge of Green's function. Instead, a preliminary tuning can be used. It is based on preliminary measurements and/ or calculation of either the transfer function in the frequency domain or response function in the time domain. Without preliminary tuning the operation of the ASC is also possible although its efficiency may be reduced. The control system can potentially operate in a real-time regime since it only requires a consequent solution to a quadratic optimization problem. In the future work the proposed algorithm will be realized both numerically and experimentally.

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