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APPLICATION OF THE SMALL-PARAMETER METHOD TO THE PROBLEM OF THE SPATIAL FLOW OF A VISCOUS GAS PAST BODIES†

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The solution of the problem of the spatial hypersonic flow of a viscous gas past spherically blunted bodies is considered using the system of equations of a complete viscous shock layer (CVSL). The use of the small-parameter method (SPM) in conjunction with the method of global iterations enables one to reduce the computer time required by a factor of approximately 100 compared with the time needed to calculate similar problems in a strictly spatial formulation by establishment methods [1]. The flow past blunt cones and bicones of long length at low angles of attack is considered as well as the flow past a body, which differs slightly from an axisymmetric one, at zero angle of attack. The applicability of the SPM is confirmed by comparison with experimental and computed data.

THE DIRECT numerical simulation of hypersonic spatial flow of a viscous gas past blunt bodies using the complete unsteady Navier–Stokes equations requires considerable computational resources. In the case of long blunt bodies, this approach becomes unacceptable even when supercomputers are available. Moreover, it has been noted in [2] that the use of numerical methods to solve spatial problems in gas aerodynamics may lead to a loss in accuracy in the case of angles of attack and that the relative error may increase as the angle of attack becomes smaller.

The correct use of the SPM is effective in this connection. It has been used in the case of the flow of an inviscid gas past cones at low angles of attack [2, 3] and in the case of the flow past bodies which are slightly different from solids of revolution [4]. The range of applicability of the SPM to which proper attention has not previously been paid [4], has been indicated in [2].

1. FORMULATION OF THE PROBLEM

The spatial, hypersonic, steady flow of a viscous gas around a blunt body is considered. The gas flow in the shock layer is described using a system of CVSL equations [5] which contains all the terms of a complete system of Navier–Stokes equations up to $O(\mathrm{Re}^{-1/2})$. This system of equation, which is treated in an orthogonal system of coordinates and is normally attached to the surface of the body (x is the length of the generatrix of the contour of the body, y is the distance along the normal to the body surface and φ is the meridional angle measured from the flow plane) has the form [6]

$$\frac{\partial}{\partial x}(H_2\rho u)+\frac{\partial}{\partial\varphi}(H_1\rho w)+\frac{\partial}{\partial y}(H_1H_2\rho v)=0$$

$$\rho\left[Du-\frac{w^2}{H_1H_2}\frac{\partial H_2}{\partial x}+\frac{uw}{H_1H_2}\frac{\partial H_1}{\partial\varphi}+\frac{uv}{H_1}\frac{\partial H_1}{\partial y}\right]= \tag{1.1}$$

$$=-\frac{1}{H_1}\frac{\partial P}{\partial x}+\frac{1}{H_1^2H_2\mathrm{Re}_\infty}\frac{\partial}{\partial y}\left[H_1^3H_2\mu\frac{\partial}{\partial y}\left(\frac{u}{H_1}\right)\right]$$

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$$\rho\,[\,Dw + \frac{wu}{H_1H_2}\frac{\partial H_2}{\partial x} - \frac{u^2}{H_1H_2}\frac{\partial H_1}{\partial \varphi} + \frac{wv}{H_2}\frac{\partial H_2}{\partial y}\,] =$$
$$= -\frac{1}{H_2}\frac{\partial P}{\partial \varphi} + \frac{1}{H_1H_2^2\,\mathrm{Re}_\infty}\frac{\partial}{\partial y}\,[\,H_1H_2^3\mu\frac{\partial}{\partial y}\,(\frac{w}{H_2})\,]$$
$$\rho\,[\,Dv - \frac{1}{H_1}\frac{\partial H_1}{\partial y}u^2 - \frac{1}{H_2}\frac{\partial H_2}{\partial y}w^2\,] = -\frac{\partial P}{\partial y}$$
$$\rho DH = \frac{1}{H_1H_2}\frac{\partial}{\partial y}\left\{\frac{H_1H_2\mu}{\sigma\,\mathrm{Re}_\infty}\,[\,\frac{\partial H}{\partial y} + \frac{V_\infty^2(\sigma-1)}{2H_\infty}\frac{\partial(u^2+w^2)}{\partial y} - \frac{\sigma u^2V_\infty^2}{H_\infty H_1}\frac{\partial H_1}{\partial y} - \frac{\sigma w^2V_\infty^2}{H_\infty H_2}\frac{\partial H_2}{\partial y}\,]\right\}$$
$$D = \frac{u}{H_1}\frac{\partial}{\partial x} + \frac{w}{H_2}\frac{\partial}{\partial \varphi} + v\frac{\partial}{\partial y}, \quad H = h + \frac{V_\infty^2}{2H_\infty}(u^2+w^2+v^2),$$
$$h = \frac{\gamma PV_\infty^2}{(\gamma-1)\rho H_\infty}$$
$$\mathrm{Re}_\infty = \rho_\infty V_\infty R(0)/\mu_\infty, \quad H_1 = 1 + y\kappa(x), \quad H_2 = r_w(x) + y\cos\alpha(x)$$

Here u, w and v are the physical components of the velocity vector along the x, φ and y directions, respectively, $R(x,\varphi)$ is the radius of curvature of the body surface Re_∞ is the Reynolds number, $\kappa(x,\varphi) = 1/R(x,\varphi)$ is the curvature of the body surface, σ is the Prandtl number, α is the angle of inclination of the generatrix of the body to the axis of symmetry of the body, and H_∞ is the total enthalpy of the free stream.

Quantities are reduced to a dimensionless form in the following manner: the components of the velocity vector are referred to the velocity of the free stream V_∞, the pressure P is referred to $\rho_\infty V_\infty^2$, the total enthalpy H is referred to H_∞, and quantities having the dimensions of length are referred to the radius of the bluntness $R(0)$.

In the case of an ideal gas, the coefficient of viscosity μ is assumed to be a known function of the absolute temperature T. Either Sutherland's law or a power law was used in the calculations.

The system of (1.1) is closed by the following boundary conditions. Generalized Rankine–Hugoniot conditions are imposed on the shock wave which, on passing to the limit, is replaced by a surface of pronounced discontinuity. In the system of coordinates (x, φ, y) these conditions have the form [6]

$$v_s = u_s\,\mathrm{tg}\,\beta_s + w_s\,\mathrm{tg}\,\gamma_s + K_sV_\infty(3)$$
$$P_s = \frac{1}{\gamma M_\infty^2} + \frac{V_\infty^2(3)\,(1-K_s)}{1+\mathrm{tg}^2\beta_s+\mathrm{tg}^2\gamma_s}$$
$$u_s = V_\infty(1)\cos^2\beta_s - \frac{w_s}{2}\sin 2\beta_s\,\mathrm{tg}\,\gamma_s - \frac{K_s}{2}V_\infty(3)\sin 2\beta_s + \frac{\mu_s}{\mathrm{Re}_\infty V_\infty(3)}\left(\frac{\partial u}{\partial y} - \frac{u}{H_1}\frac{\partial H_1}{\partial y}\right)_s \qquad (1.2)$$
$$w_s = V_\infty(2)\cos^2\gamma_s - \frac{u_s}{2}\sin 2\gamma_s\,\mathrm{tg}\,\beta_s - \frac{K_s}{2}V_\infty(3)\sin 2\gamma_s + \frac{\mu_s}{\mathrm{Re}_\infty V_\infty(3)}\left(\frac{\partial w}{\partial y} - \frac{w}{H_2}\frac{\partial H_2}{\partial y}\right)_s$$
$$H_s = 1 + \frac{\mu_s}{\sigma\,\mathrm{Re}_\infty V_\infty(3)}\left(\frac{\partial H}{\partial y} + \frac{V_\infty^2(\sigma-1)}{2H_\infty}\frac{\partial(u^2+w^2)}{\partial y} - \frac{\sigma u^2V_\infty^2}{H_\infty H_1}\frac{\partial H_1}{\partial y} - \right.$$

$$- \frac{\sigma w^2 V_\infty^2}{H_\infty H_2} \frac{\partial H_2}{\partial y} \Big)_s$$

$$\operatorname{tg} \beta_s = \frac{1}{H_{1s}} \frac{\partial y_s}{\partial x}, \quad \operatorname{tg} \gamma_s = \frac{1}{H_{2s}} \frac{\partial y_s}{\partial \varphi}, \quad K_s = \frac{1}{\rho_s}$$

The subscript s denotes the value of a quantity behind the surface of the shock wave, $V_\infty(i)$ ($i = 1, 2, 3$) are the components of the velocity vector of the free stream in the system of coordinates (x, φ, y), β_s and γ_s are the angles of inclination between the surface of the shock wave and the surface of the body, and $y_s = y_s(x, \varphi)$ is the shock wave stand-off distance.

The conditions of no-slip, impermeability and of a cooled wall

$$u(x, \varphi, 0) = w(x, \varphi, 0) = v(x, \varphi, 0) = 0, \quad H(x, \varphi, 0) = H_w^0 \tag{1.3}$$

are specified as boundary conditions on the body surface.

The subscript w denotes a quantity on the body surface.

The problem is solved in the domain between the surfaces of the body and the detached shock wave: $0 \leqslant x \leqslant x_f$, $0 \leqslant \varphi < 2\pi$, $0 \leqslant y \leqslant y_s$. "Soft" boundary conditions are imposed at the surface $x = x_f$.

2. THE SMALL-PARAMETER METHOD

As has already been mentioned above, the use of numerical methods to simulate spatial flows around blunt bodies can lead to a loss of accuracy in the case of low angles of attack [2]. Actually, let a certain blunt axisymmetric body be placed in a stream of a viscous gas at a small angle of attack ϵ. In the system of coordinates (x, φ, y) the coefficient of the lift force, without taking account of the forces due to viscous friction, will be as follows:

$$C_y \sim \int_0^{2\pi} \int_0^{x_f} P_w \cos \alpha \cos \varphi r_w \, dx d\varphi \tag{2.1}$$

where α is the angle of inclination of the generatrix to the axis of the body. On linearizing the pressure with respect to a small parameter and expanding the correction to the pressure in a Fourier series, we obtain

$$C_y \sim \epsilon \int_0^{x_f} P_w^{(1)} \cos \alpha r_w \, dx d\varphi \tag{2.2}$$

It follows from the latter relationship that the relative error in C_y is not proportional to the relative error in the pressure P_w but to the relative error in the coefficient of the Fourier series $P_w^{(1)}$

$$\Delta P^{(1)}/P^{(1)} \sim \epsilon^{-1} \Delta P/P \tag{2.3}$$

Relationship (2.3) explains the well-known fact that, if spatial numerical methods are employed when solving problems of the aerodynamics of solids of revolution at small angles of attack, the relative accuracy in determining the lift is far less than the relative accuracy in determining the drag and, moreover, this situation becomes more aggravated as the angle of attack decreases.

The use of the SPM enables one to overcome the above-mentioned difficulties. Furthermore, this method also possesses a number of advantages. The use of the SPM can yield effective results for the following classes of problems: (1) finding the flow around an axisymmetric body at a small angle of attack [1–3, 6], (2) determining the flow around a body which differs only slightly from a solid revolution [4]. The approach in which perturbations are expanded in formal Fourier series, that is traditional both for the first as well as the second class of problems, enables one to reduce both problems to essentially a single problem. This paper is concerned with solving these classes of problems.

For convenience in obtaining the numerical solution, we will change to the new independent variables $\xi = x$, $n = y/y_s$ and $\nu = \varphi$, where $y_s = y_s(x, \varphi)$ is the shock wave stand-off distance. We note that the domains of definition of the spatial and axisymmetric solutions are different. Such a change of variables leads to a situation where these domains are formally identical.

The essence of the SPM lies in the fact that the spatial solution is represented in the form of a sum of the axisymmetric solution and a certain small perturbation which has a linear form. Here, all the basic non-linear effects in the spatial solution are referred to its axisymmetric component and the asymmetry in the spatial solution is taken into account in terms of small perturbations which have a linear character. The required solution can therefore be represented in the form

$$\begin{aligned} F(\xi, \nu, n) &= F_0(\xi, n) + \epsilon\delta F(\xi, \nu, n) + o(\epsilon) \\ w(\xi, \nu, n) &= \epsilon\delta w(\xi, \nu, n) + o(\epsilon) \\ r_w(\xi, \nu) &= r_{0w}(\xi) + \epsilon\delta r_w(\xi, \nu) + o(\epsilon) \end{aligned} \qquad (2.4)$$

The corresponding quantities in the axisymmetric solution are denoted by a zero subscript, ϵ is the small parameter which characterizes the extent of the non-axisymmetric nature of the flow which may be either the angle of attack or a parameter which indicates the extent to which the body differs from a solid of revolution, w is the meridional component of the velocity vector, F are the remaining unknown functions and $r = r_w$ is the equation of the surface of the body.

Let us now expand the perturbations in Fourier series in the meridional coordinate ν. The coefficients of the Fourier harmonics in the expansions of the perturbations in formal series in the angular variable ν will be the required functions. Hence

$$\begin{aligned} \delta F(\xi, \nu, n) &= F^{(0)}(\xi, n) + \sum_{k=1}^{\infty} F^{(k)}(\xi, n) \cos k\nu \\ \delta w(\xi, \nu, n) &= \sum_{k=1}^{\infty} w^{(k)}(\xi, n) \sin k\nu \\ \delta r_w(\xi, \nu) &= r_w^{(0)}(\xi) + \sum_{k=1}^{\infty} r_w^{(k)}(\xi) \cos k\nu \end{aligned} \qquad (2.5)$$

Here, account has immediately been taken of the fact that the functions $F = (P, \rho, H, u, v, w, y_s, r_w)$ are even with respect to ν but the function w is odd with respect to ν by virtue of the symmetry of the problem about the $\nu = 0$ plane.

Higher terms in ϵ are not considered in expansion (2.4) since the aerodynamic forces and moments solely depend on the coefficients F_0 and $F^{(1)}$ in expansions (2.4) and (2.5), and, by virtue of this, it is unreasonable to consider the quadratic terms of expansion (2.4), especially as taking account of them yields a correction of 1–3% compared with the linear case [1].

It should be noted that the use of series (2.5) is only effective in those cases when the first terms of series (2.5) for fixed ξ and n decay rapidly as the number of the harmonic k increases, which also ensures their applicability for practical purposes. In the case of the flow past an axisymmetric body at a small angle attack, where the small parameter ϵ is the angle of attack, series (2.5) only contains a single term corresponding to $k = 1$ [1, 3]. In the case of the flow past a body which is slightly different from an axisymmetric body at zero angle of attack, all the terms of series (2.5) are non-zero. For the effective use of series (2.5) in the latter case, it is necessary that the surface of the body $r_w(\xi, \theta)$ should belong to the class of periodicity $\tilde{C}^{(m)}$, and, moreover, the greater the value of m, the more rapidly the terms of the Fourier series (2.5) decay as the number of the harmonic k increases and, in fact, their rate of decay is equal to $o(k^{-m})$. Belonging to the periodicity class $\tilde{C}^{(m)}$ means that [18]: (1) the function $f(\nu)$ is continuous in the interval $[-\pi, \pi]$ together with its derivatives up to the mth derivatives, inclusive, and (2) $f(-\pi+0) = f(\pi-0)$.

On substituting an expansion of the form of (2.4) and (2.5) into the system of CVSL equations (1.1) and into the boundary conditions (1.2) and (1.3), on picking out the terms $O(I)$ and $O(\epsilon)$ we obtain that the spatial system is equations is subdivided into a set of two-dimensional systems of equations: an axisymmetric system and a number of independent linear systems with two independent variables for determining the coefficients accompanying the first harmonics of the gas-dynamic quantities.

Actually, let us represent the system of CVSL equations (1.1) in the form

$$A\frac{\partial \mathbf{U}}{\partial \xi}+B\frac{\partial \mathbf{U}}{\partial n}+C\frac{\partial \mathbf{U}}{\partial \nu}+\mathbf{F}=\mathbf{D}\frac{\partial \mathbf{U}}{\partial n^2},\quad \mathbf{U}=[\rho,u,w,v,H]^{\mathrm{T}} \tag{2.6}$$

Let us apply the SPM to (2.6) and, in fact, represent the matrices A, B, C and D and the vectors $\mathbf{U}$ and $\mathbf{F}$ in the form (2.4) and (2.5). We substitute these expressions into (2.6). After picking out the terms $O(I)$ and $O(\epsilon)$, we obtain that the spatial system of equations reduces to the set of two-dimensional systems of equations

$$A_0\frac{\partial \mathbf{U}_0}{\partial \xi}+B_0\frac{\partial \mathbf{U}_0}{\partial n}+\mathbf{F}_0=D_0\frac{\partial^2 \mathbf{U}_0}{\partial n^2} \tag{2.7}$$

$$A_0\frac{\partial \mathbf{U}^{(k)}}{\partial \xi}+B_0\frac{\partial \mathbf{U}^{(k)}}{\partial n}+\mathbf{F}_0^{(k)}=D_0\frac{\partial^2 \mathbf{U}^{(k)}}{\partial n^2},\quad k=0,1,2\ldots \tag{2.8}$$

$$\mathbf{U}^{(k)}=(\rho^{(k)},u^{(k)},w^{(k)},v^{(k)},H^{(k)})^{\mathrm{T}}$$

where $\mathbf{U}^{(k)}$ are the coefficients of the Fourier harmonics, A_0, B_0 and D_0 are matrices which correspond to an axisymmetric system of equations, and the axisymmetric solution is denoted by the zero subscript. We also carry out a similar procedure in the case of the boundary conditions.

We shall consider the coefficients of the Fourier harmonics for linear perturbations as the required functions.

The proper application of SPM in the case of a problem involving spatial flow past blunt bodies assumes [2]: (1) continuous differentiability of the gas-dynamic quantities with respect to the small parameter ϵ in the neighbourhood of $\epsilon=0$, (2) the equivalence of the axisymmetric and spatial solutions in the sense that they must possess one and the same system of singularities (shock waves, contact discontinuities, etc.), (3) the applicability of expansion (2.4) in the case of axisymmetric bodies at angles of attack imposes the following constraint on the small parameter ϵ

$$\epsilon\,|\,z/r_w+\partial r_w/\partial z\,|\ll 1$$

where z is the axial coordinate (as was pointed out by Livinskii) and, in the case of a body which differs slightly from an axisymmetric one: $\epsilon\ll 1$.

Spatial calculations of the flow past a body with a specified geometry and specified free stream parameters using the approach described above immediately enables one to find the entire single parameter family of solutions with the parameter ϵ which is either the angle of attack or a parameter characterizing the extent to which the body differs from a solid of revolution.

3. THE METHOD OF GLOBAL ITERATIONS

After using the SPM, the spatial system of CVSL equations (1.1) was reduced to a set of two-dimensional systems: to the axisymmetric system (2.7) and to a series of independent systems of equations for determining the coefficients accompanying the first harmonics of the gas-dynamic quantities (2.8).

It was shown in [9] that the axisymmetric system of CVSL equations (2.7) shows elliptic properties in subsonic flow domains, in particular, close to the surface of the body and, as a consequence of this, it is impossible to use any marching method with respect to the variable ξ to solve this problem directly. A previously described approach [10] was employed when solving the axisymmetrical system of equations.

The set of systems of equations for determining the coefficients of the Fourier harmonics (2.8) is identical as regards its basic properties with the system of CVSL equations in the axisymmetric case

(2.7) and, by virtue of this, the matrices A_0, B_0 and D_0 in the systems of equations (2.7) and (2.8), which determine the type of the systems of equations, are completely identical.

On account of this, an approach was used to solve the systems of equations for the coefficients of the Fourier harmonics which is based on the execution of global iteration. In order to do this, we represent the derivative $\partial P^{(k)}/\partial \xi$ in the momentum equation in the projection on the tangent to the body in the form

$$\partial P^{(k)}/\partial \xi = \alpha_T(\partial P^{(k)}/\partial \xi)^{(m)} + (1 - \alpha_T)(\partial P_g^{(k)}/\partial \xi)^{(m)} \tag{3.1}$$

The current mth global iteration is calculated for a specified field $P_g^{(k)}$ and a specified coefficient of the Fourier harmonic of the angle of inclination of the shock wave $\beta_s^{(k)}$ which are obtained in the calculation of the preceding global iteration. The Fourier coefficients of the pressure field and of the inclination of the shock wave to the surface of the body for the next $(m+1)$th global iteration are determined from the relaxation relationships

$$(P_g^{(k)})^{(m+1)} = \tau_p(P^{(k)})^{(m)} + (1 - \tau_p)(P_g^{(k)})^{(m)} \tag{3.2}$$
$$(\beta_s^{(k)})^{(m+1)} = \tau_s(\beta_s^{(k)})_T + (1 - \tau_s)(\beta_s^{(k)})^{(m)}$$

Here $(\beta_s^{(k)})_T$ and $(P^{(k)})^{(m)}$ are the Fourier coefficients of the angle of inclination of the shock wave and the pressure respectively, obtained as a result of the calculation of the mth global iteration, and τ_p and τ_s are relaxation parameters.

A new method is proposed for calculating the inclination of the shock wave on a bluntness which is unlike that previously adopted in [10]. In fact, the Fourier image of the angle of inclination of the shock wave on the current global iteration was found not from the mass balance [10] but from the Rankine–Hugoniot boundary condition for the Fourier image of the normal component of the velocity vector $v_s^{(k)}$. In the case of corrections to the axisymmetric solution which are linear with respect to the angle of attack, the integral law of conservation of mass is identically satisfied and does not yield any additional information whatsoever. We note that such an approach to the calculation of the inclination of the shock wave is fundamental in the spatial case since when solving the spatial problems it is not clear how the mass balance relationship can be used to find the position of the shock wave, while the approach which has been described above enables one to overcome this difficulty.

The derivatives in (3.1) have to be approximated by difference expressions in such a manner that the propagation of perturbations upstream in subsonic flow domains is taken account of in carrying out the iterations. For this purpose, the approximation was employed which uses backward differences in the first term and forward differences in the second term in expression (3.1).

A numerical method was employed when solving the mixed problem for the coefficients of the Fourier harmonics. It is based on the use of a difference which is of the first order of approximation with respect to the coordinate which is longitudinal to the body and of fourth order with respect to the transverse coordinate.

A grid which is adaptive with respect to n was employed for calculating flows with large flux gradients (at high Reynolds numbers). The step size was chosen at each point depending on the first and second derivatives of the longitudinal component of the velocity vector u with respect to the coordinate n [11]. This enabled calculations to be carried out over a wide range of Reynolds numbers ($10^2 \leq \mathrm{Re}_\infty \leq 10^7$), which encompasses flow conditions ranging from a completely viscous shock layer to an inviscid shock layer with a thin boundary layer at the body.

4. RESULTS OF CALCULATIONS

The following calculations were carried out using the algorithm described above: (1) the flow past spherically blunted cones at small angles of attack of 5–10°, (2) numerical simulation of the flow past a long spherically blunted bicone at a small angle of attack, (3) the flow past an elliptic cone at a zero angle of attack.

Only the term corresponding to $k = 1$ [1, 3] is non-zero in expansion (2.5) in the case of a blunt cone at

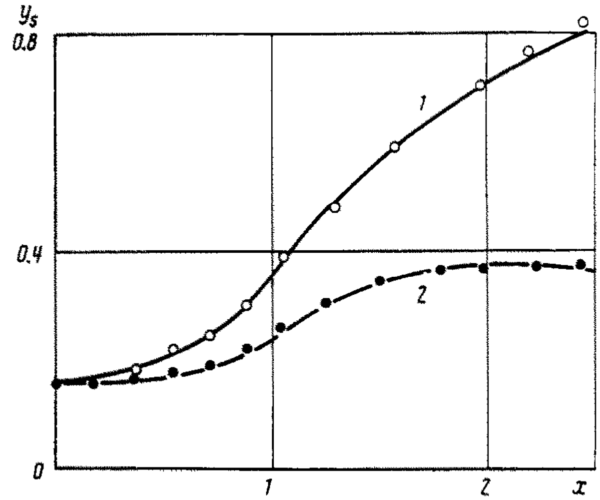

Fig. 1.

moderate angles of attack. By virtue of this, two two-dimensional problems were solved to find the spatial flow. A comparison of the calculations with experimental data [12] for the windward side ($\varphi = 0$, curves 1) and leeward side ($\varphi = \pi$, curves 2) of the cone is presented in Figs 1–3. The computational grid consisted of 35×31 grid points where the first number corresponds to the n coordinate. Data on the shock wave stand-off distance for the following input parameters: $\mathrm{Re}_\infty = 3.95 \times 10^4$, $H_w = 0.701$ is the temperature factor, $M_\infty = 5.9$ is the Mach number, $T_\infty = 53.7$ K and $\theta_c = 25°$ is the cone half-angle and $\epsilon = 10°$ is the angle of attack, are presented in Fig. 1. Data on the pressure distribution when $\mathrm{Re}_\infty = 3.95 \times 10^4$, $M_\infty = 5.8$, $T_w = 300$ K, $T_0 = 428$ K is the stagnation temperature, $\epsilon = 8°$ and $\theta_c = 40°$ are shown in Fig. 2. Data concerning the normalized heat transfer coefficient: $C_H = 2\mu\sigma^{-1}\mathrm{Re}_\infty^{-1}\partial h/\partial y|_w$ when $\mathrm{Re}_\infty = 1.3 \times 10^5$, $H_w = 0.316$, $M_\infty = 10.33$, $T_\infty = 46.82$ K, $\theta_c = 45°$ and $\epsilon = 5°$ are presented in Fig. 3.

Curves of the drag C_τ (the dashed line), of the lift C_y (the dotted and dashed line) and the pitching moment M_z (a dash and two dots) on the angle of attack ϵ, obtained using the present method for spherically blunted cone, when $\mathrm{Re}_\infty = 4.1 \times 10^3$, $M_\infty = 13$, $H_w = 0.27$, $\sigma = 0.72$, $p = 0.75$ is the power index in the viscosity law, $\theta_c = 5°$, $L = 6.98$ is the length of the cone referred to the radius of bluntedness, are presented in Fig. 4. The solid lines correspond to the calculation in [13] while the points are experimental data [14, 15]. As was mentioned above, the proper application of the small-parameter method in this case imposes the following constraint on the magnitude of the angle of attack: $\epsilon \ll 5°$. However, as can be seen from the comparison, there is good agreement with the experimental results up to angles of attack of about 10°. This is indicative of the fact that the SPM is valid in the case of integral characteristics over a wider range than would be expected from theoretical estimates.

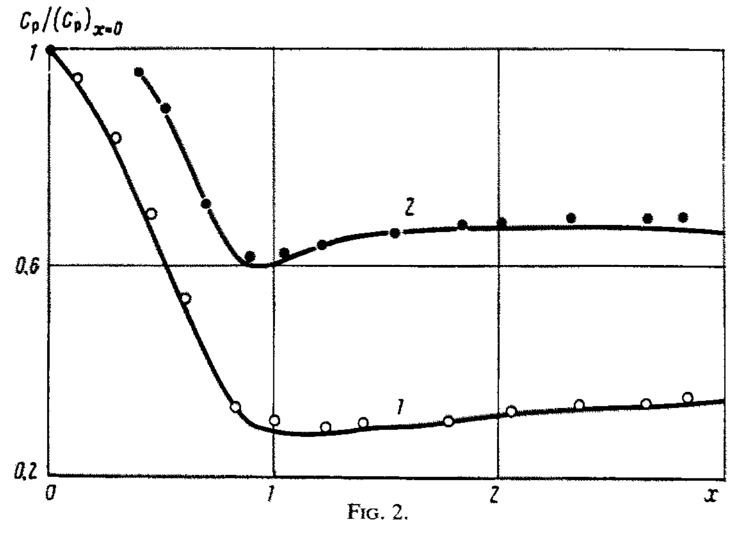

Fig. 2.

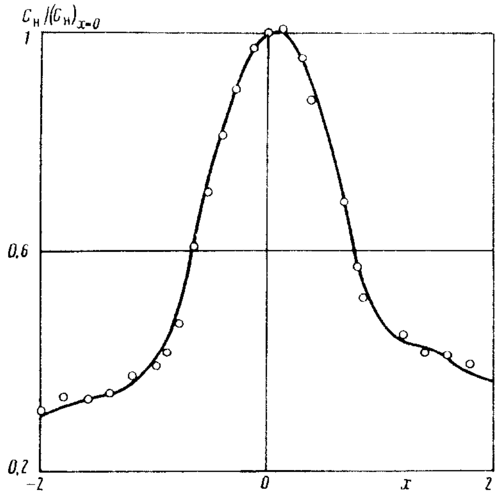

Fig. 3.

The following block marching technique for implementing the method was employed in order to simulate the flow around a long (more than 50 times the bluntness radius), spherical blunted bicone at an angle of attack of 2°. The whole of the computational domain was subdivided into mutually intersecting blocks and the calculations of each block were carried out using the method of global iterations. Mutual intersection of the blocks was introduced in order to take account of the transmission of perturbations upstream in subsonic flow domains. The dimensions of the subsonic domains may vary considerably on different segments of the body. The use of a block marching method enables one to shorten the overall computation time significantly since, for each block, there is a specific number of iterations which is necessary for convergence. The discontinuity in the angle of the generatrix of the bicone was smoothed in the neighbourhood of the point where the cones joined. This smoothing was insignificant and was propagated onto 4–5 grid points of the computational grid along the direction of ξ. In this calculation, the grid was not only adapted in the normal direction but also in directions longitudinal to the body, which enabled us to carry out more accurate calculations in the neighbourhood of the points where the cones joined. The computation grid consisted of 35×119 grid points.

A comparison with the results of an experiment [16] on the pressure distribution on the body both for the windward side (curve 2) and leeward side (curve 1) of the bicone is shown in Fig. 5. The calculations were carried out for $Re_\infty = 3.5 \times 10^4$, $M_\infty = 10.1$, $T_w = 49.3$ K, $H_w = 0.705$, $z_c = 14.81$ is the point where the cones join, $\theta_{c1} = 9.33°$, $\theta_{c2} = 5°$ and $\epsilon = 2°$. The systematic difference of about 2–3% in the magnitude of the pressure on the end of the bicone is explained by the fact that quadratic terms in ϵ in expansion (2.4), which are not taken into account in this approach, start to have an effect on the solution.

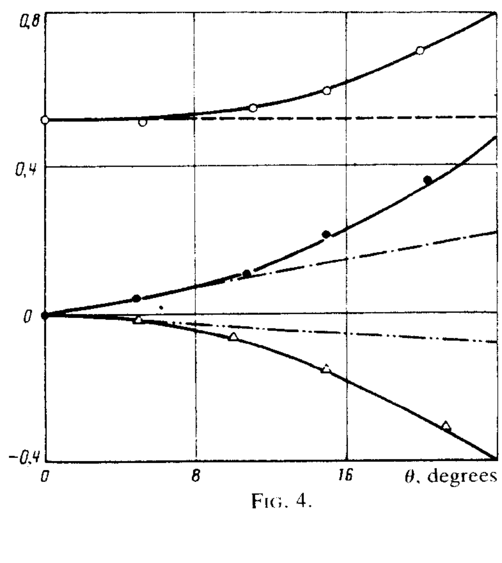

Fig. 4.

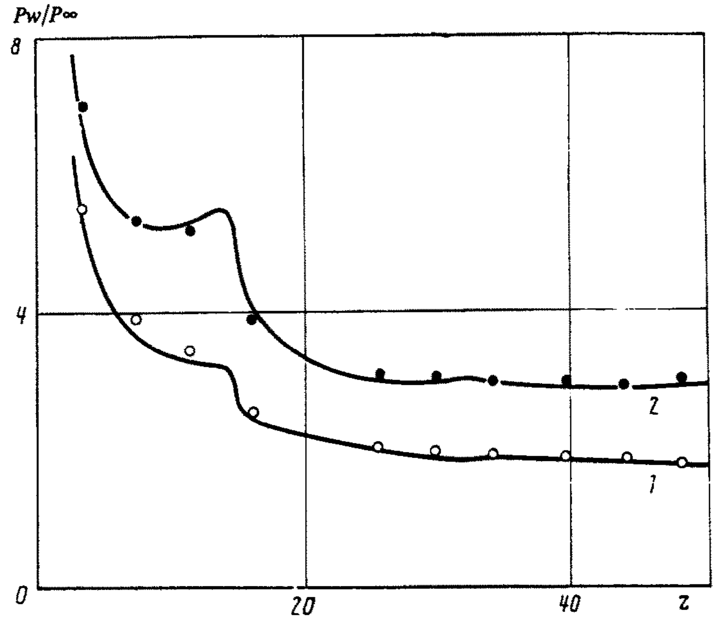

FIG. 5.

In the case of the flow past a spherically blunted elliptic cone the quantity $\epsilon = 1 - \sin\theta_1/\sin\theta_0$ $(\theta_1 < \theta_0)$ was chosen as the small parameter. The surface of the elliptic cone was specified in the following manner

$$r_w(x, \varphi) = \left(\frac{\sin^2\varphi}{r_{0w}^2(x)} + \frac{\cos^2\varphi}{r_{1w}^2(x)}\right)^{-1/2} \tag{4.1}$$

$$r_{0w}(x) = \begin{cases} \sin x, & 0 \leqslant x \leqslant x_0, x_0 = \pi - \theta_0 \\ \cos\theta_n + \sin\theta_0 (x - x_0), & x > x_0 \end{cases} \tag{4.2}$$

$$r_{1w}(x) = \begin{cases} \sin x, & 0 \leqslant x \leqslant x_0 \\ \cos\theta_0 + \sin\theta_1 (x - x_0), & x > x_0 \end{cases} \tag{4.3}$$

In order to ensure that there was a smooth transition from the spherical bluntness onto the surface of the elliptic cone, the discontinuity in the angle of the generatrix in (4.3) was smoothed. On linearizing (4.1) with respect to the small parameter ϵ and then expanding the resulting perturbations in Fourier series, we obtain that just two terms in the Fourier series (2.5), corresponding to k = 0, 2 will have non-zero values. The surface of the elliptic cone is thereby replaced by a surface which has the following form

$$r_w(x, \varphi) = r_{0w}(x) + \epsilon(x - x_0)\sin\theta_0(-1/2 - 1/2\cos 2\varphi) + O(\epsilon^2)$$

In this case, the spatial problem of finding the flow around an elliptic cone is reduced to solving an

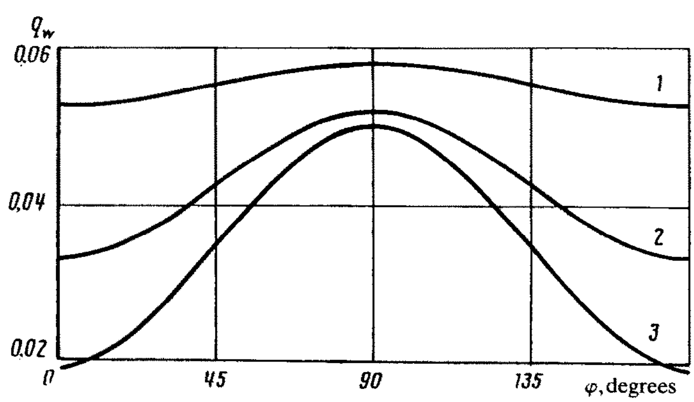

FIG. 6.

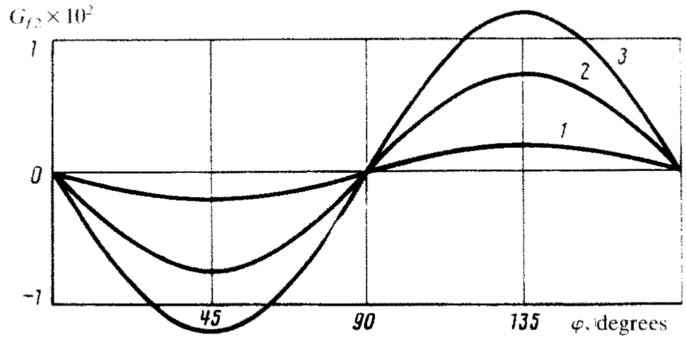

FIG. 7.

axisymmetric problem (the problem of finding the flow around a circular cone) and two two-dimensional problems of determining the Fourier harmonics of gas-dynamic quantities. The calculations of the gas flow were carried out for $\mathrm{Re}_\infty = 10^3$, $H_w = 0.1$, $T_\infty = 200$ K, $M_\infty = 15$ and $\theta_0 = 30°$. The thermal flux $-q_w = \mu\sigma^{-1}\mathrm{Re}_\infty^{-1}\partial H/\partial y|_w$ and the coefficient of friction $C_{f2} = 2\mu\mathrm{Re}_\infty^{-1}\partial w/\partial y|_w$ are presented in Figs 6 and 7 in three cross-sections with respect to x: $x_1 = 1.388$, $x_2 = 1.976$ and $x_3 = 2.6571$. All results for the elliptic cone are presented for the case when $\epsilon = 0.3$.

It can be seen from the above comparisons that the proper use of the approach which has been described above enables one to obtain results of high accuracy and over a wide range of free-stream parameters and also makes it possible to find a one-parameter family of solutions where the parameter is either the angle of attack or a parameter corresponding to the degree of difference between the body under consideration and a solid of revolution. All of this shows the effectiveness of the method proposed.

In concluding, it should be pointed out that the computer time required for the most widely different values of Re_∞ from 10^2 to 10^7 varied only slightly and was 20–30 minutes using an IBM PC AT 386.

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