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# 1 Potential-based methodology for active sound control in three 2 dimensional settings

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12 This paper extends a potential-based approach to active noise shielding with preservation of wanted  
13 sound in three-dimensional settings. The approach, which was described in a previous publication  
14 [Lim *et al.*, *J. Acoust. Soc. Am.* **129**(2), 717–725 (2011)], provides several significant advantages  
15 over conventional noise control methods. Most significantly, the methodology does not require any  
16 information including the characterization of sources, impedance boundary conditions and sur-  
17 rounding medium, and that the methodology automatically differentiates between the wanted and  
18 unwanted sound components. The previous publication proved the concept in one-dimensional con-  
19 ditions. In this paper, the approach for more realistic conditions is studied by numerical simulation  
20 and experimental validation in three-dimensional cases. The results provide a guideline to the  
21 implementation of the active shielding method with practical three-dimensional conditions.  
22 Through numerical simulation it is demonstrated that while leaving the wanted sound unchanged,  
23 the developed approach offers selective volumetric noise cancellation within a targeted domain. In  
24 addition, the method is implemented in a three-dimensional experiment with a white noise source  
25 in a semi-anechoic chamber. The experimental study identifies practical difficulties and limitations  
26 in the use of the approach for real applications. © 2014 Acoustical Society of America.  
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Pages: 1–10

## 27 I. INTRODUCTION

28 Active sound control (ASC) is a technique for altering  
29 acoustic field to a wanted one in a given region of space by  
30 means of an active control boundary established by control-  
31 lable secondary sound sources. A typical problem formula-  
32 tion for ASC involves a domain to be protected from an  
33 external unwanted field (noise) by introducing special control  
34 sources positioned on a boundary surface. The problem  
35 becomes more complicated if an internal wanted field is  
36 present and completely mixed up together with the noise in  
37 the domain. An obvious question in the case with wanted  
38 sound is how to obtain such separate cancellation of noise  
39 only from the total field measured at the boundary surface.  
40 Some available noise abatement techniques, for example,  
41 those developed by Kincaid *et al.*,<sup>1,2</sup> require a detailed  
42 knowledge of the sources and nature of noise. A number of  
43 publications are also devoted to the optimization of the  
44 strength of the spatially distributed controls in order to mini-  
45 mize a quadratic pressure cost function.<sup>3,4</sup>

46 In recent years, different approaches have been sug-  
47 gested to realize real-time active noise control (see, e.g.,  
48 Refs. 5–10). Most of them exploit the least mean square

(LMS) algorithm. Its application becomes problematic if the  
49 wanted sound component is present. In this case the use of  
50 LMS requires additional information on the wanted sound.  
51 For some applications it might be achieved via directional  
52 measurements.<sup>5,10</sup> There have also been a few attempts to  
53 apply the virtual sensing and surface integral control to  
54 tackle this problem.<sup>8,9</sup> All of them are based on trying to pre-  
55 dict the wanted sound component and, therefore, are quite  
56 limited because the wanted ingredient cannot completely be  
57 separated from the total acoustic field.  
58

59 The potential-based approach proposed can provide a  
60 convenient universal algorithm for the ASC problem in a  
61 quite general formulation associated with the unknown  
62 wanted sound and also unknown boundary conditions. The  
63 method requires no detailed knowledge of either the sound  
64 sources or boundary conditions, including reflection coeffi-  
65 cients that characterize the domain termination, to cancel out  
66 only the unwanted component. If the shape of the domain is  
67 complicated, the solution based on the developed technique  
68 allows us to choose a convenient boundary surface. The only  
69 input data needed for the control are the acoustic quantities of  
70 the field measured on the perimeter of the boundary surface.  
71 The measured quantities can pertain to be the overall field  
72 composed of both the adverse noise and wanted sound, and  
73 the methodology will automatically distinguish between the  
74 two.<sup>11,12</sup> In the current stage of the theoretical development,

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75 the potential-based approach allows one to obtain the general  
 76 solution to the ASC problem for arbitrary geometries, proper-  
 77 ties of the medium, or boundary conditions.<sup>13,14</sup>

78 The method developed by Jessel and Mangiante,<sup>15,16</sup>  
 79 and Canevet,<sup>17</sup> hereby called the JMC method, also requires  
 80 only information at the perimeter of the shielded domain for  
 81 global noise absorption when only the unwanted noise is  
 82 present in the protected domain. The main difference  
 83 between the approaches based on the potential-based method  
 84 and JMC is that only the former provides the advantages of  
 85 preservation of the wanted sound and volumetric noise can-  
 86 cellation through an entire shielded domain when the total  
 87 field composed of both the wanted sound and noise is meas-  
 88 ured at the boundary. One should note here that apart from  
 89 the JMC, there are a number of other noise abatement techni-  
 90 ques, which provide for the cancellation of noise in selected  
 91 discrete<sup>18,19</sup> or directional areas.<sup>20</sup> In contrast to many other  
 92 active noise control techniques, the potential-based ASC can  
 93 naturally be realized in a discrete form<sup>12</sup> via the Difference  
 94 Potential Method (DPM) formalism. From the standpoint of  
 95 practical implementation, this is a clear advantageous  
 96 because a realistic ASC system would require a discrete col-  
 97 lection of control sources.

98 In Ref. 21, the Difference Potential Method (DPM) was  
 99 employed to solve a one-dimensional ASC problem for the  
 100 linearized Euler equations. It was shown that the resulting  
 101 ASC attenuates the incoming noise while retaining the natu-  
 102 ral reverberation within an enclosure. The sensitivity analy-  
 103 sis to input errors was accomplished in Refs. 22 and 23. It  
 104 was also proven that the solution is applicable to resonance  
 105 regimes. Recently, the potential-based ASC technique has  
 106 experimentally been applied to multi-domain tests with  
 107 broadband signals in a one-dimensional enclosure (Refs. 24,  
 108 25, and 26). However, a three-dimensional implementation  
 109 is much more interesting from a practical point of view. This  
 110 issue is the primary objective of the current paper. The  
 111 unique feature of the proposed methodology to retain the  
 112 wanted sound unaffected is numerically demonstrated. The  
 113 capacity of the methodology to cancel unwanted noise across  
 114 a volume is realized in a series of laboratory experiments.  
 115 These results are another step toward developing the  
 116 approach for real applications, such as eliminating the exte-  
 117 rior engine and airframe noise inside the passenger compart-  
 118 ments of commercial aircraft, and the protection of a  
 119 predefined space against urban noise coming from the out-  
 120 side. In doing so, the controls will not interfere with the  
 121 wanted sound, such as communication among speakers in  
 122 the room. As we are in a stage of experimental investigation  
 123 of the method, a real-time control system has not been  
 124 implemented. The overall system is assumed to be linear  
 125 time-invariant and exactly repeatable. In addition, the con-  
 126 trol outputs are supposed to be accurately separable from the  
 127 input data. The experiments confirm that the potential-based  
 128 ASC method, validated in one-dimensional conditions,<sup>22,25</sup>  
 129 can be extended to cover full three-dimensional acoustic  
 130 conditions and achieve global noise cancellation while pre-  
 131 serving the wanted sound.

132 For completeness of the presentation, the relevant theo-  
 133 retical findings from our previous work are summarized in

the first part of the paper. The practical limitations of the  
 method used for ASC are clarified and the current difficulties  
 which require further work for real-time applications are  
 also discussed.

## II. POTENTIAL-BASED ACTIVE SOUND CONTROL TECHNIQUE

The approach to ASC is based on surface potentials  
 which can be considered in discrete and continuous formula-  
 tions.<sup>22,27</sup> In contrast to standard techniques, this approach  
 allows the existence of wanted sound in the protected domain.

Assume that the propagation of sound is governed by  
 the following equation:

$$LU = S, \tag{1}$$

considered on the domain  $D_0$ . In particular, Eq. (1) can rep-  
 resent the Helmholtz equation or acoustics equations. The  
 boundary conditions for Eq. (1) are formulated implicitly as  
 the inclusion

$$U \in U_{D_0}. \tag{2}$$

Here,  $U_{D_0}$  is a linear space of functions such that the solution  
 to problem (1), (2) exists and unique.

In order to consider the discrete formulation of the ASC  
 problem, some grid in the entire space is introduced. The  
 nodes belonging to the domain to be shielded form set  $M^+$ ,  
 while the other nodes represent set  $M^-$  (see Fig. 1). The total  
 combination of the nodes gives us the set  $M^0$ . The primary  
 acoustic sources can either belong to  $M^+$  or to its exterior  
 $M^-$ . In this formulation, wanted sound sources  $S_f$  are inside  
 $M^+$ , while sources  $S_a$  situated outside  $M^-$ , are considered as  
 "unwanted."

In the discrete formulation of the ASC problem it is  
 required to find such additional sources that the total field  
 from the primary and secondary sources coincides with the  
 wanted sound on grid set  $M^+$ .

The boundary value problem (1), (2) is assumed to be  
 approximated by the following:

$$\begin{aligned} L_h U|_m &= S|_m, \\ U^{(h)} &\in U_D^{(h)}. \end{aligned} \tag{3}$$

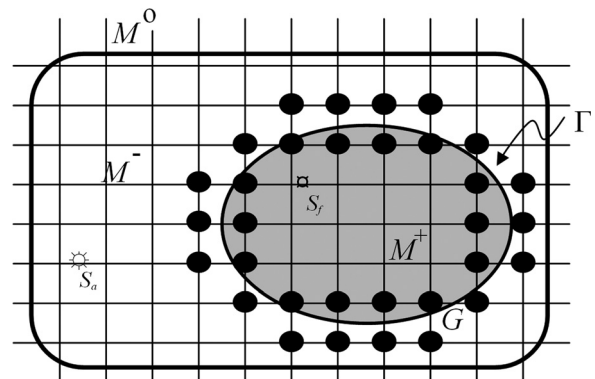


FIG. 1. Finite difference ASC problem,  $\Gamma$ : boundary,  $M^+$ : discrete counter-  
 part of shielded domain, and  $M^-$ :  $M^0 \setminus M^+$ .

167 Suppose that the right-hand side in Eq. (3) consists of wanted  
 168 and unwanted primary sources  $S_f^{(h)}$  and  $S_a^{(h)}$  as well as con-  
 169 trols  $G^{(h)}$ ,

$$S^{(h)} = S_f^{(h)} + S_a^{(h)} + G^{(h)}.$$

170 The general solution to the foregoing finite-difference AS prob-  
 171 lem can be obtained via the theory of difference potentials

$$G^{(h)} = -\theta(M)L_h V^{(h)}. \tag{4}$$

172 Here,  $\theta(M)$  is the indicator function equal to 1 on the set  $M$   
 173 which includes the grid boundary, and equal to 0 anywhere  
 174 else.

175 In formula (4),  $V^{(h)}$  is an arbitrary function such that

$$V_\Gamma^{(h)} = U_\Gamma^{(h)} \tag{5}$$

176 on the boundary  $\Gamma$ , where  $V \in U_D^{(h)}$ . In practice, the grid  
 177 function  $U_\Gamma^{(h)}$  can be measured.

178 As shown, e.g., in Ref. 14 for the ASC solution, it is suffi-  
 179 cient to have an access only to the trace of the total acoustic  
 180 field on the boundary  $\Gamma$ . In other words, no knowledge of the  
 181 actual sources (wanted and unwanted) is required. Thus, such  
 182 active controls are more practical than controls determined by  
 183 only unwanted field, which may not be separable from the  
 184 wanted sound. This capability is potentially very useful for  
 185 applications related to noise control and room acoustics, as it  
 186 enables protection of the predefined space against the noise  
 187 coming from the outside, while at the same time not interfer-  
 188 ing with the ability of the listener to listen to wanted sound  
 189 from different domains or communicate across the rooms.

190 To demonstrate the meaning of controls (4), assume that  
 191 the governing equation in (1) is represented by the Euler  
 192 acoustics equations with

$$\begin{aligned} \frac{\partial p}{\partial t} + \rho c^2 \nabla u &= \rho c^2 q_{vol} + f_p, \\ \frac{\partial u}{\partial t} + \frac{\nabla p}{\rho} &= \frac{b_{vol}}{\rho} + f_u. \end{aligned} \tag{6}$$

193 Here,  $f_p$  and  $f_u$  are source functions for the continuity and  
 194 momentum equations, respectively.

195 In the continuous space, the counterpart of control (4) is  
 196 given by (see Refs. 21 and 28)

$$\begin{aligned} q_{vol} &= u_n(\Gamma)\delta(\Gamma), \\ \vec{b}_{vol} &= \vec{n}p(\Gamma)\delta(\Gamma). \end{aligned} \tag{7}$$

197 Here,  $\vec{n}$  is the external normal to the boundary  $\Gamma$  of the pro-  
 198 tected domain,  $\delta(\Gamma)$  is the delta-function assigned to the sur-  
 199 face  $\Gamma$ ,  $u_n$  is a normal component of particle velocity to  $\Gamma$ ,  
 200  $p(\Gamma)$  is acoustic pressure. The values of both  $u_n(\Gamma)$  and  $p(\Gamma)$   
 201 can be obtained from measurements on the boundary, and  
 202 they normally correspond to the total sound field composed  
 203 of both the unwanted and wanted components.

204 Note that if the wanted sound is absent, then the ASC  
 205 solution will be equivalent to that given by the JMC  
 206 method.<sup>16,29</sup> It appears that the JMC solution applies to a

broader range of conditions than the one under which it was  
 originally derived (see Refs. 15 and 30). In particular, it is  
 not limited by unbounded domains without wanted sources.  
 However, if the wanted sound is present then the JMC-based  
 approach cannot be applicable if the controls operate on the  
 basis of the total field from both primary and secondary  
 sources.

Finally, it is worth noting that even though we have ex-  
 plicitly obtained the control sources, their subsequent opti-  
 mization or due allowance for diffraction effects may  
 require the solution of an additional problem (see Ref. 31). If  
 the shape of the protected region is complicated, then the  
 unique capability of the DPM to efficiently resolve the geo-  
 metric attributes becomes very important.

### III. NUMERICAL SIMULATION

#### A. Noise shielding

The general solution (7) is applicable in the general case  
 of full 3D flow field in theory. As shown in Ref. 31, to obtain  
 the ASC solution based on difference potentials in bounded  
 or unbounded domains, one needs to know only the normal  
 component of the particle velocity at the control boundary of  
 the shielded domain.

The following simulation case is done in a square duct  
 which is perfectly rigid to allow no energy losses through  
 the duct walls. The duct is 4 m in length and 1 m in width  
 and height for the inner cross-section. The shielded domain  
 is defined to be three times longer in length than the height  
 of the square control surface, so that the measurement can  
 show clearly the effectiveness of the cancellation at positions  
 far away from the control sources. As illustrated in Fig. 2,  
 the noise source is situated outside of the duct at 1 m away  
 from the open inlet of the duct, whereas the shielded domain  
 stretches from the control surface ‘‘A’’ all the way to the left  
 end. The size of the domain is 3 m in length. The noise  
 source is placed off center outside the duct to generate a  
 three-dimensional sound field more effectively. The system  
 can be either with or without a wanted sound. On the control  
 surface four discrete control units each consisting of a dipole  
 and a monopole source, are used.<sup>32</sup> Theoretically, it has been

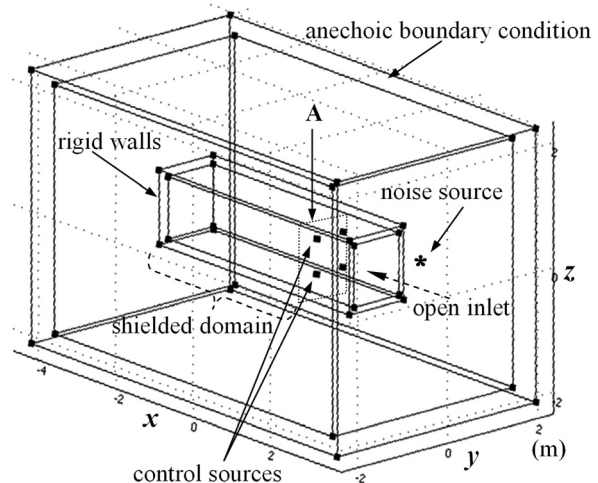


FIG. 2. Configuration of the numerical model for ASC in an anechoic space.



246 shown that at least four control units are required (three per  
 247 wavelength in each direction) for three-dimensional ASC to  
 248 achieve a level of 40 dB attenuation on a relatively simple  
 249 active boundary surface.<sup>33</sup>

250 For effective attenuation the distance between sources is  
 251 recommended to be less than  $\lambda/2$ .<sup>34</sup> That is, the wave length  
 252 should be longer than twice the diagonal distance of the  
 253 sources, which translates into

$$f < \frac{c}{2L},$$

254 here  $L$  is a diagonal distance of the sources, i.e.,  $\sqrt{2/3}$  in  
 255 Fig. 3. Hence, the range for the test frequency should be  
 256  $f < 210$  Hz in the simulation. In addition, the size of the dis-  
 257 crete surface element (effective surface area of each source  
 258 unit), i.e.,  $1/4$  m<sup>2</sup> in Fig. 3, should also be smaller than  
 259  $\lambda^2/4\pi$ .<sup>33,35</sup> This implies a further condition that  $f < 194$  Hz  
 260 in our particular test case. However, to ensure that a three-  
 261 dimensional sound field is produced in the simulation, a test  
 262 frequency of 250 Hz is chosen. This is higher than the cutoff  
 263 frequency of the (1,1) mode of the square duct, so that the  
 264 higher order (1,1), (1,0), and (0,1) modes as well as the fun-  
 265 damental mode will all be excited. The frequency is higher  
 266 than the upper bound frequency of 194 Hz that was derived  
 267 from the set-up of the control sources, which means that the  
 268 effectiveness of the control may be reduced. However, it is  
 269 more important here to use a higher frequency to demon-  
 270 strate the three-dimensional applicability of the method. The  
 271 one-dimensional effectiveness of the method has already  
 272 been demonstrated in our previous publication.<sup>22</sup> The num-  
 273 ber of control sources is kept to four in the numerical simula-  
 274 tions as that coincides with the number of controls used in  
 275 the experiment in Sec. IV.

276 In practice, to maximize the efficiency of attenuation in  
 277 3D space, the optimum distribution of the control sources on  
 278 boundary surfaces has to be defined. Optimization of the  
 279 control sources with respect to different criteria has been  
 280 studied by Loncaric and Tsynkov in Refs. 36 and 37. The

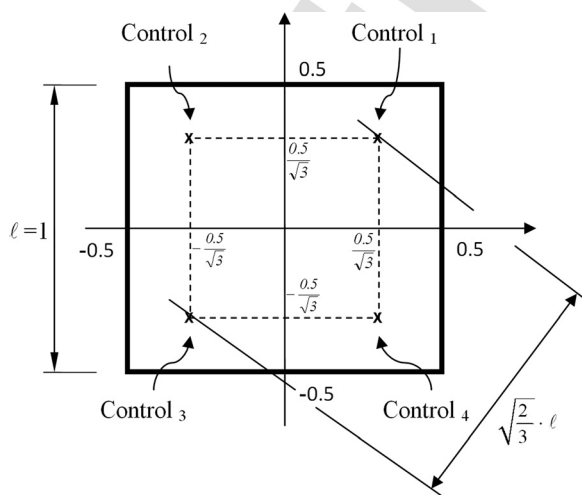


FIG. 3. Optimized positions for the distribution of the controls on a square plat boundary surface where the reference point is defined to be in the center of the square.

distribution of four sets of controls on a square boundary sur-  
 face can be optimized by putting each set at a position deter-  
 mined by the length of each edge times  $1/\sqrt{3}$  in Fig. 3.

At each control point on the boundary surface the total  
 sound pressure and particle velocity of the initial sound are  
 measured before calculating the ASC solution. Based on the  
 measurement the ASC solution, (7), defines the strength of  
 control sources which are also situated at the measuring  
 point. When the proposed solution is applied, the result  
 shown in Fig. 4(a) and 4(b) confirms that it is able to cancel  
 the unwanted noise through the entire shielded domain. In  
 the figure, the sound pressure is shown along the cross-  
 sectional x-y plane of the duct. The attenuation estimated by  
 the simulation is from 30 to 68 dB when the controls are acti-  
 vated on the control boundary at 250 Hz. In the simulation  
 the propagation of unwanted noise is clearly not unidirec-  
 tional because of the three-dimensional reverberation. The  
 simulation confirms that the method is applicable to such  
 reverberant cases in 3D space.

Figure 4(a) and 4(b) also shows the important point that  
 the initial sound field does not change outside of the shielded  
 domain, where  $x > 0.8$  or  $x < -3$  m, while the controls are  
 activated. This particular feature is potentially very useful  
 for real time realization of the control system, since the noise  
 field without the controls can be measured directly outside  
 the shielded domain but in the close neighborhood of the  
 boundary even when the controls are on. Moreover, this  
 shielding method can be seen as a safer method since the  
 sound field remains the same (and not increased by the con-  
 trols) outside the domain while the ASC solution is applied.

The result of the sound pressure distribution on the x-y  
 plane illustrated in Fig. 4 shows that the whole aimed do-  
 main is shielded when the controls are switched on. This  
 ability of global noise cancellation and preservation of  
 wanted sound based on the method has been theoretically  
 proven in Refs. 14 and 21.

## B. Preservation of the wanted sound

To demonstrate the distinct capabilities of the potential-  
 based noise control methodology further, an additional simu-  
 lation is carried out in which a wanted sound source is  
 placed inside the shielded domain. For the study the same  
 configuration of the numerical model illustrated in Fig. 2 is  
 used except the addition of a wanted sound source situated at  
 the position  $x = -3, y = -0.5, z = -0.5$  to generate a wanted  
 sound component inside the shielded domain as shown in  
 Fig. 5. Again, we assume that the noise, the wanted sound,  
 and the properties (e.g., reflection properties) of the walls are  
 unknown.

The control sources for ASC are placed on the boundary  
 surface of the protected volume. In order to determine the  
 strength of the control sources, the sound pressure and parti-  
 cle velocity of the total acoustic field (the sum of the adver-  
 se noise and wanted sound) are measured at the boundary.  
 Then, the strength of the acoustic monopole and dipole is  
 derived as shown in the above section using Eq. (7). The key  
 point is that there is no need to distinguish between the  
 wanted sound and the noise explicitly in the measurements.

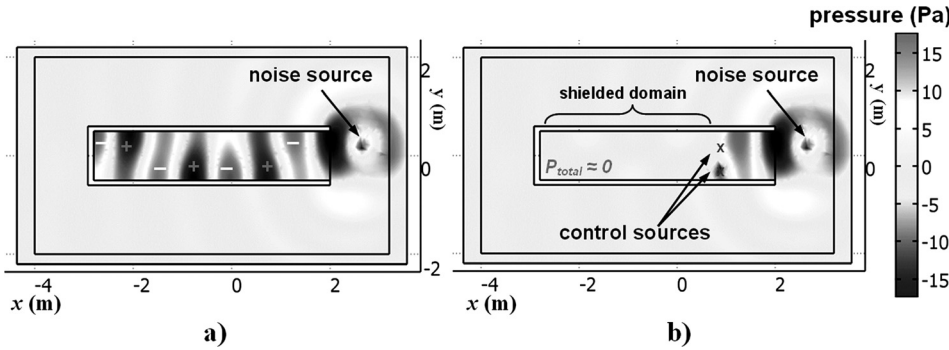


FIG. 4. Sound pressure distribution (a) of noise and (b) of the sum of noise and control output at 250 Hz on  $x$ - $y$  plane, where  $-4 < x < 3$ ,  $-2 < y < 2$ ,  $z = 0$  in 3D space.



338 This is possible because the sources of the wanted sound and  
 339 unwanted sound are on different sides of the boundary of the  
 340 shielded domain. The measurement of the particle velocity  
 341 at the boundary is able to capture this information inherently.  
 342 When the control devices are applied, the dipole source pro-  
 343 vides the necessary directional element that allows the can-  
 344 cellation of sound from outside the shielded domain (the  
 345 unwanted sound) but not from the inside (the wanted sound).  
 346 Figure 5 illustrates the general configuration of the simu-  
 347 lation model on the  $x$ - $y$  plane with sound pressure distribu-  
 348 tion when the controls are turned off.

349 Figure 6 illustrates the sound pressure distribution, at  
 350 250 Hz, in the case described above in Fig. 5. The light and  
 351 shade in Fig. 6(a) show the initial sound pressure when the  
 352 noise and wanted sounds are both switched on, while the con-  
 353 trol sources are still off. Figure 6(b) represents the net sound  
 354 pressure field when the noise is canceled out after the activa-  
 355 tion of the AS control sources. For comparison, the original  
 356 wanted sound is separately measured at the same reference  
 357 position when both the AS control and unwanted noise sour-  
 358 ces have been turned off. This is shown in Fig. 6(c). The result  
 359 upon shielding and the original wanted sound along  $x$  axis  
 360 ( $y = 0$ ) in the shielded domain are overlaid in Fig. 6(d) to  
 361 give a clearer view. Obviously, when the unwanted noise be-  
 362 comes stronger relative to the wanted sound, the error between  
 363 them increases due to the decrease in signal to noise ratio.  
 364 However, even at a signal to noise ratio of  $-10$  dB, the ampli-  
 365 tude error has been reported to be theoretically less than 1 dB  
 366 in the authors' previous study on one-dimensional AS prob-  
 367 lems.<sup>22</sup> Fig. 6(d) shows the similarity between the original

wanted sound pressure—and the result • when the controls are  
 368 switched on. In the simulation, a challenging condition is set  
 369 up by introducing a significantly bigger unwanted sound pres-  
 370 sure than the wanted one (about 10 dB higher), so that the  
 371 results can give a reliable guidance of the attenuation that can  
 372 be achieved in practice when the wanted sound has been seri-  
 373 ously contaminated by strong unwanted noise. Figure 6(d)  
 374 also shows that, on the whole, the total sound field with the  
 375 potential-based control sources resembles closely the original  
 376 wanted sound field at each measuring position everywhere in  
 377 the shielded domain.

The similarity between the net sound field shielded by  
 379 the AS control sources and the original wanted sound field is  
 380 also evaluated by the cross-correlation of the two results.  
 381 When the AS control sources are switched on, the cross-  
 382 correlation of the wanted sound and the shielded total sound  
 383 pressure (unwanted noise, wanted sound, and sound field  
 384 with the controls) is 0.998. The ideal cross-correlation of  
 385 two identical signals is 1.0. This is almost achieved in the  
 386 simulation, which shows that the shielded net sound field  
 387 with the controls on matches the original wanted sound field  
 388 very well. The numerical simulation clearly proves that  
 389 wanted sound can be very effectively protected by the active  
 390 controls based on the proposed method even in a three-  
 391 dimensional problem where both wanted sound and  
 392 unwanted noise are unknown, while noise is significantly  
 393 suppressed by the AS control sources.

IV. EXPERIMENT 395

The performance of the active shielding technique in  
 396 three-dimensional is tested in an experiment. The solution  
 397 for the ASC problems either with or without the wanted  
 398 sounds has previously been experimentally validated in a  
 399 one-dimensional duct, and the results were reported in Refs.  
 400 22 and 25. Following those works, this experiment extends  
 401 the methodology to a three-dimensional problem. In the  
 402 experiment we concentrate our effort in a case without  
 403 wanted sound in a three-dimensional space. For the three-  
 404 dimensional case, two-dimensional arrays of actuators and  
 405 microphones are required on the boundary surfaces to realize  
 406 the shielding of a given volume. The key factors investigated  
 407 in this realization of the three-dimensional ASC are the  
 408 physical size, number, and positioning of the control sources  
 409 (actuators) and monitoring microphones. In the experiment  
 410 they are optimized in order to achieve the best noise cancel-  
 411 lation in the shielded domain. The experimental model stated  
 412

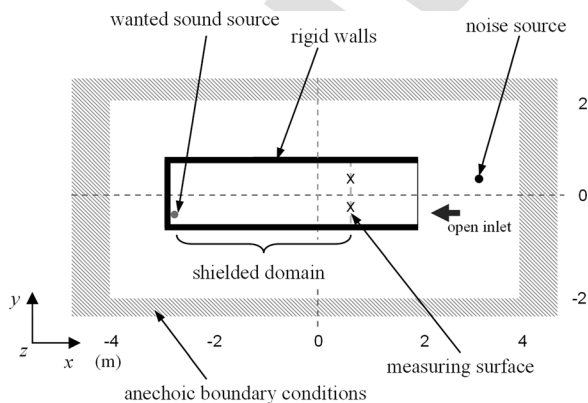


FIG. 5. Configuration with unwanted and wanted sound sources in a 3D space on  $x$ - $y$  plane.

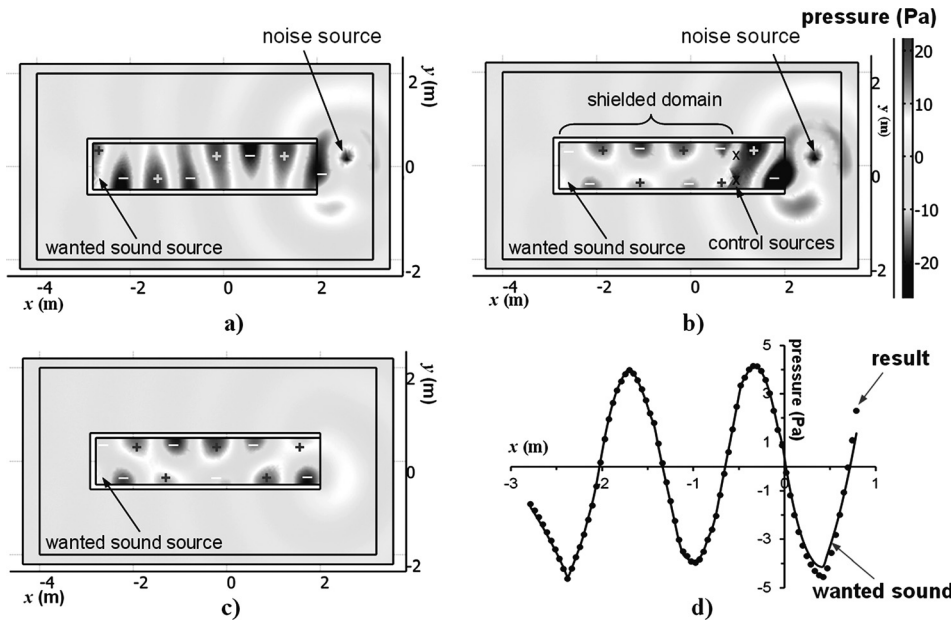


FIG. 6. Sound pressure distribution in a space, where  $-4 < x < 3$ ,  $-2 < y < 2$ ,  $z = 0$  at 250 Hz: (a) the sound pressure of noise and wanted sound without control, (b) shielded total sound pressure (the sum of noise, wanted sound and control output), (c) wanted sound pressure, and (d) —: wanted sound pressure, and ● shielded total sound pressure along  $y = 0$  in the shielded domain.

413 in this section is accurately designed and tested. It is based  
 414 on the difference potential theory which is studied in Sec.  
 415 III. For example, source positions, measuring method, and  
 416 the number of controls are strictly defined using the original  
 417 theory.

418 The sound generation system consists of loudspeakers,  
 419 power amplifiers, digital signal processing (DSP) modules,  
 420 and a PC with multi-channel sound cards. The audio data  
 421 measured on the active surfaces are fed into the control system  
 422 through an Alesis Digital Audio Tape Protocol (ADAT)  
 423 converter first. The converted data are sent to a Multi-  
 424 channel Audio Digital Interface (MADI) converter. After all  
 425 these conversions, the resulting data are stored in a computer  
 426 through MADI card and can be used for further DSP manip-  
 427 ulation. The data received in the computer are then incorpo-  
 428 rated into the ASC algorithm together with the calibrated  
 429 loudspeaker transfer functions and directivity to generate the  
 430 desired sound signals, which are then saved as phase-  
 431 synchronous audio data files, which can be played back  
 432 using a multi-channel audio editor. In the system, after the  
 433 filtering process the signals are led to the ADAT matrix  
 434 which splits them to provide each input channel of a render  
 435 with an output signal. After played back by the render, the

436 separate audio signals are converted to MADI and sent to the  
 437 MADI-ADAT converter via RME HDSP sound cards with  
 438 64 channel outputs in MADI format and then ADAT-audio  
 439 converter successively.

440 To estimate the actual accuracy of the control system in  
 441 the experiment, the phase error in degrees between the input  
 442 signal and the DSP apparatus is determined through a set of  
 443 preliminary measurements. Figures 7(a) and 7(b) show such  
 444 an error measured over a range of frequencies up to 1.5 kHz  
 445 with swept sine excitation. The results shown in Fig. 7(a)  
 446 demonstrate experimentally that the error in the control sys-  
 447 tem at the frequencies chosen for the test, i.e., above 90 Hz,  
 448 is largely below 0.15 degrees in phase. Therefore, according  
 449 to the theoretical sensitivity analysis reported in the earlier  
 450 publication Ref. 23, an AS system with these errors should  
 451 allow us to achieve about 50–55 dB attenuation.<sup>25</sup> This is  
 452 indeed consistent with the attenuation we obtained in the nu-  
 453 merical analysis. The corresponding time delay error, which  
 454 can be caused by the DSP apparatus, is below 8  $\mu$ sec. if the  
 455 frequency is above 60 Hz in Fig. 7(b). This has been meas-  
 456 ured at a sampling frequency of 44.1 kHz.

457 The sound generating system consists of loudspeaker  
 458 arrays and power amplifiers. A driver is chosen to guarantee

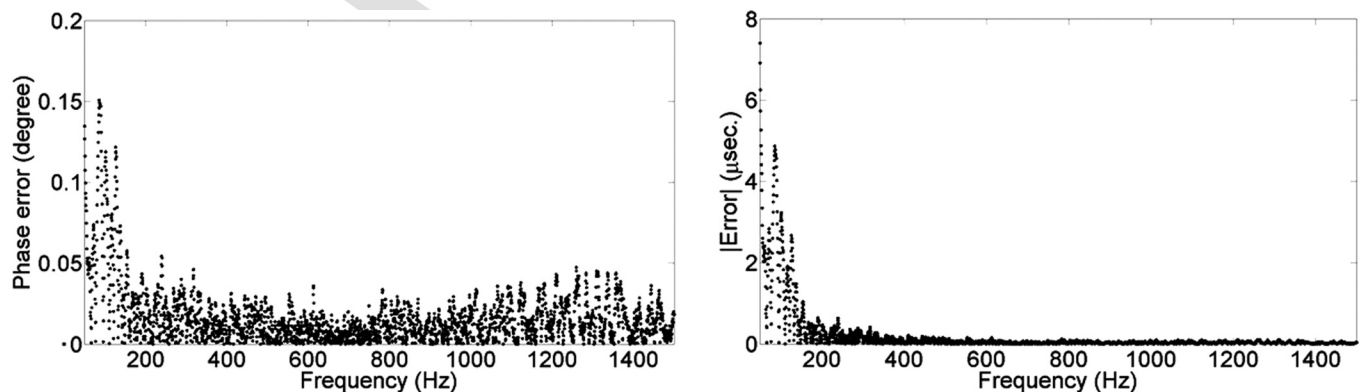
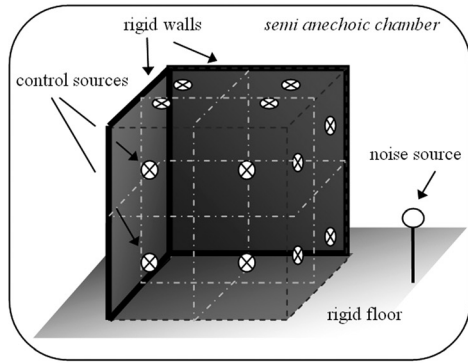
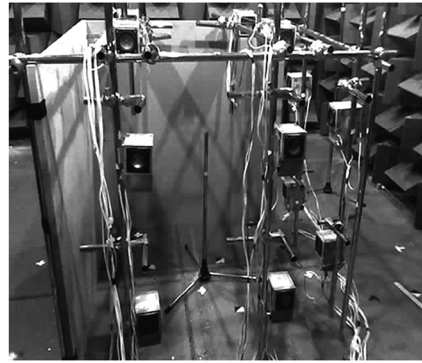


Fig. 7. (a) Phase error and (b) time delay error of DSP apparatus.





a)



b)

FIG. 8. Experimental setup 

459 good stiffness, dynamic stability, and low distortion degree. 504  
 460 The driver shows a quite stable linear frequency response 505  
 461 especially in the test frequency range, i.e., below 500 Hz. 506  
 462 The sensitivity errors of each driver are about 1 dB in the 507  
 463 range between 90 and 250 Hz and about 3 dB between 80 508  
 464 and 1500 Hz. A  $-3$  dB cut-off frequency and the resonance 509  
 465 frequency appear at 80 Hz. The sensitivity of a driver is 510  
 466 82 dB at 2.83 v/1 m. The effective piston area of the driver is 511  
 467  $0.003 \text{ m}^2$ . Each set of the control is designed with a dipole 512  
 468 and monopole source. In addition, a loudspeaker is also used 513  
 469 as an external noise source. Each secondary source set is 514  
 470 constructed with thick medium-density fiberboard (MDF) 515  
 471 enclosures and clamped directly on the supporting metal bar. 516

472 A shielded domain is defined in a cube with 1.5 m 517  
 473 in each side length. The three sides of the domain are terminated 518  
 474 by two rigid walls and a floor. The other sides are 519  
 475 acoustically transparent and allow propagation of three- 520  
 476 dimensional sound fields through them. The cube sits on the 521  
 477 floor of a semi-anechoic chamber. In this setup the effect of 522  
 478 reflection on the walls does not need to be considered separately 523  
 479 as it is considered automatically.<sup>14,31</sup> This capability 524  
 480 belongs to the original nature of the method. Therefore, we 525  
 481 believe that the method is practically applicable in a wide 526  
 482 range of applications even with randomly incoming reflected 527  
 483 sound. 528

484 To make the experimental model more general, and to 529  
 485 take advantage of the potential-based method's ability to 530  
 486 work without precise knowledge of system conditions, the 531  
 487 acoustic properties of the walls and floor are not known in 532  
 488 the experiment, and are not needed in the potential-based 533  
 489 approach. To generalize the experiment further, the position 534  
 490 of the noise source is supposed to be unknown. The noise is 535  
 491 generated by a broadband white noise signal containing an 536  
 492 equal amount of all frequencies in the range between 50 and 537  
 493 250 Hz. Figure 8(a) illustrates the positions of an unwanted 538  
 494 noise source outside of the domain and control sources on the 539  
 495 boundary surfaces. Figure 8(b) shows the general configuration 540  
 496 of a two-dimensional active boundary surface consisting 12 541  
 497 control sets arranged at the control points in the realization. 542

498 The direction of the dipole source mounted on the bound- 543  
 499 ary defines the inside and outside of a shielded domain. For 544  
 500 this reason the direction of the dipole source must be perpen- 545  
 501 dicular to the boundary and pointed out from the shielded do- 546  
 502 main. The sound pressure and particle velocity are measured 547  
 503

on the perimeter of each control source set. The distribution 504  
 of four sets of controls on a square boundary surface can be 505  
 optimized by putting each set at a position determined by the 506  
 length of each edge times  $1/\sqrt{3}$  (see Fig. 3). The measured 507  
 values, adjusted for the transfer-function of the signal genera- 508  
 tor, are used to calculate offline the control source signals 509  
 based on the difference potential theory. 510

In the measuring process, before obtaining the ASC 511  
 solutions for a given problem, directional and non-directional 512  
 components of the sound field are measured using a B&K 513  
 PULSE Sound & Vibration analyzer with the control sources 514  
 off. The former is the normal component of the particle velocity 515  
 $u_o$ , and the latter is the acoustic pressure  $p_o$  of the total 516  
 field at the boundary. Then, the directional component measured 517  
 defines a non-directional control source which is a 518  
 monopole. The non-directional component measured is used 519  
 to define a dipole control source which is directional. 520

The source strengths of the controls  $b$  and  $q$  normalized 521  
 to the reference signal  $V_{ref}$  are 522

$$\hat{b} = \frac{\hat{p}_o A_s}{H_d}, \quad \hat{q} = \frac{(\hat{u}_o \vec{n}) A_s}{H_m}. \quad (8)$$

Here  $\hat{b} = b/V_{ref}$ ,  $\hat{q} = q/V_{ref}$ ,  $\hat{p}_o = p_o/V_{ref}$ , and  $\hat{u}_o = u_o/V_{ref}$ .  $A_s$  523  
 is a surface area element,  $H_d$  is the transfer-function of the 524  
 dipole source signal generator,  $H_m$  is the transfer-function of 525  
 the monopole source signal generator, and  $\vec{n}$  is a unit normal 526  
 vector on the boundary surface in Eq. (7). Then, the control 527  
 source signals are saved as phase-synchronous.wav files 528  
 which can be played back using a multi-channel signal 529  
 generator. 530

A typical example of the ASC results based on difference 531  
 potentials in a three-dimensional space is shown in Fig. 9. To 532  
 test the capability of the method in practical cases, a white 533  
 noise source is used in a room to generate a three-dimensional 534  
 sound field in the experiment. 535

The listening position is located at the middle of the 536  
 shielded domain surrounded by the three active surfaces and 537  
 three hard walls. The distance between each control source 538  
 on a surface is  $2/3 \times 1.5$  m. The frequency range is limited to 539  
 below 250 Hz in the experiment due to this distance between 540  
 control sources on each side of the cube. 541

The rigid line in Fig. 9 shows the initial sound pressure 542  
 distribution when the noise is activated, while the control 543  
 sources are still off. The dotted line represents the 544



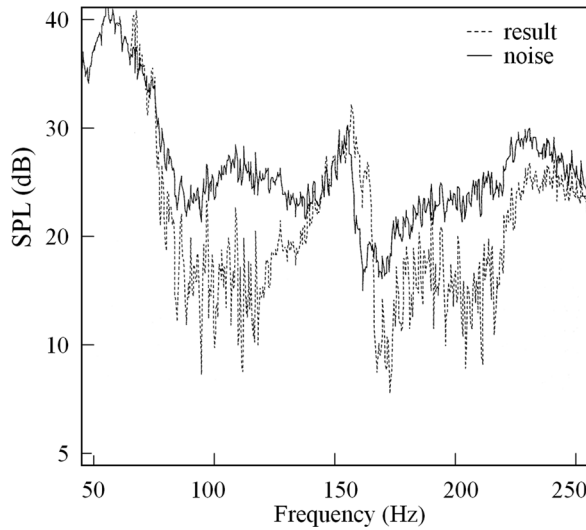


FIG. 9. Plots of the result.

545 distribution of the net sound pressure when the noise is sup-  
 546 pressed by the controls. When the control sources are acti-  
 547 vated, the control system attains attenuations of around 5 to  
 548 13 dB in the middle of the shielded domains at the frequency  
 549 range of 80 to 220 Hz.

550 Because of the difficulty in dealing with a number of  
 551 bulky control sources and the complexity of three dimen-  
 552 sional sound fields, the result in this section shows lower ef-  
 553 ficiency in the overall attenuation, when compared with the  
 554 result achieved in a one-dimensional experiment and  
 555 reported in the previous publication, which was around 15 to  
 556 20 dB.<sup>22</sup> One of the main reasons can be found in the design  
 557 of the experimental model. That is, the control sources them-  
 558 selves cause disturbances to the sound fields. These distur-  
 559 bances near the active boundary surface were not considered  
 560 in the design of this experimental model.

561 Near 150 Hz, the control sources are about multiples of  
 562 a 3/4 wavelength from the hard surfaces where the sound  
 563 pressure is low. As a result, the output of the controls  
 564 becomes very small and noise shielding is not effective near  
 565 this particular frequency (see Fig. 9).

566 The experiment demonstrates that the potential-based  
 567 ASC automatically extracts all the necessary information  
 568 about the system and the unwanted noise itself from the  
 569 measurements performed at the boundary surface. The  
 570 experiment proves the potential possibilities of suppression  
 571 of unwanted noise by the active controls based on the

proposed method even in a three-dimensional space,  
 although significant challenges remain in how to account for  
 the presence of the control sources.

V. CONTROL OUTPUTS

The proposed approach in this paper can be realized provided that the contribution of the control sources to the input data can be separated. A natural question to follow up is if the solution can still be obtained without such separation in practice. Two simulated cases are examined to answer this.

Figure 10 shows the case when there is no wanted sound. The result shows that the contribution of the controls vanishes everywhere outside the domain in Fig. 10(b). The key factor is the direction of the dipole source defining the inside or outside of a domain. The output of the dipole source at the control point exactly cancels any monopole source contribution outside the shielded domain. However, inside the domain the sign of the dipole source is reversed, and the combination of the dipole source and monopole source produces the sound field that is 180 degrees out of phase with the noise and cancels the noise inside the domain [compare Figs. 10(a) and 10(b)].

Therefore, the contribution of the dipole and monopole sources based on solution (7) in the absence of any wanted sound is summarized as follows:

$$p_m + p_d(q_-) = -p_a, \text{ inside shielded domain}$$

and

$$p_m + p_d(q_+) = 0, \text{ outside the domain.}$$

Here  $p_a$  is the pressure of adverse noise.

In cases where there is no wanted sound to be preserved, as in Fig. 10(b), the simulation result shows that the controls do not make any additional sound field anywhere outside of the shielded domain. Hence in this case, it could be possible to measure the sound field without the controls near the outside boundary of the shielded domain even when the controls are on.

In a further simulation, wanted sound pressure with magnitude twice that of the unwanted noise is introduced into the space, so that the output of the controls can be investigated to show its relationship with either the wanted sound, noise or none of them in each domain (inside, or outside the shielded domain). In the shielded domain the same

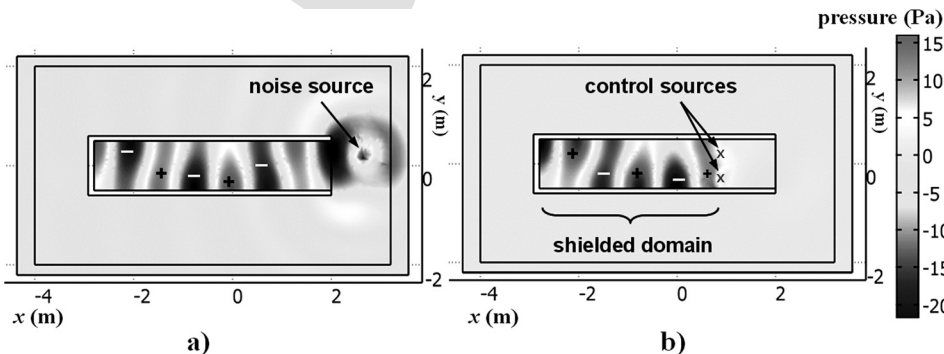


FIG. 10. Sound pressure distribution of (a) noise and (b) control output without noise in a space, where  $-4 < x < 3$ ,  $-2 < y < 2$ ,  $z = 0$  at 250 Hz.

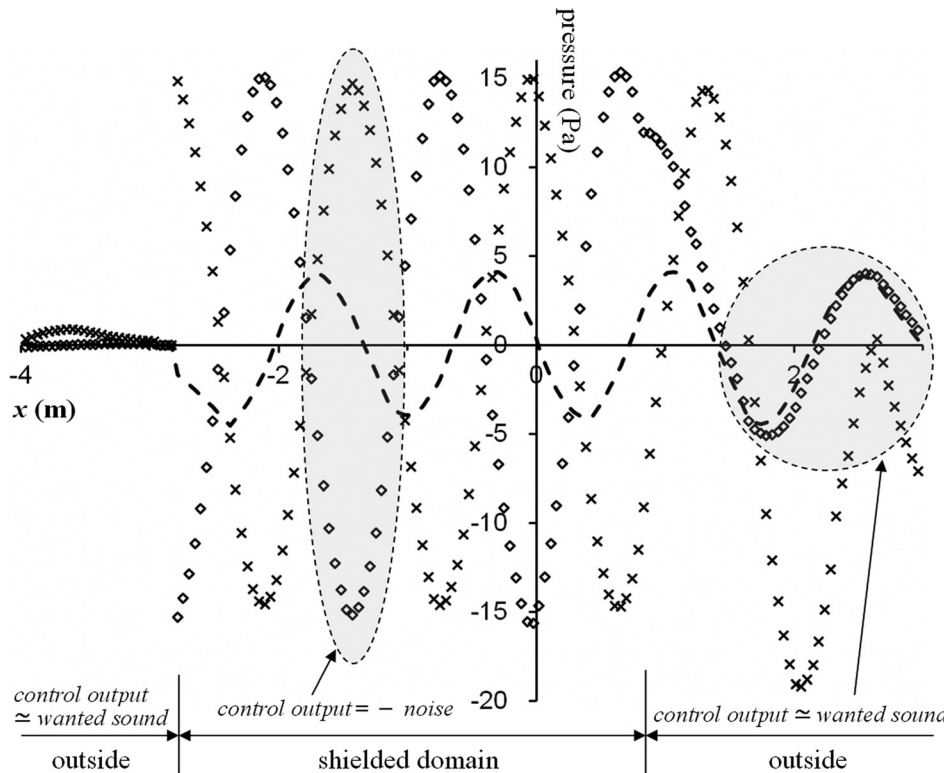


FIG. 11. Plots of sound pressure distribution along  $x$  axis in a 3D space. Sound pressure  $\times$ : wanted sound, and  $\diamond$ : without noise and wanted sound at 250 Hz.

611 conclusion of Fig. 10 is applicable to the case with wanted  
 612 sound, as illustrated in Fig. 11. The plots shown in Fig. 11  
 613 are brought from the result described in Sec. III B. Figure 11  
 614 shows that the output of the controls produces the sound field  
 615 180 degrees out of phase with the noise inside the domain either  
 616 with or without a wanted sound. However outside  
 617 shielded domain, unlike the conclusion of the case without a  
 618 wanted sound, the output of the controls now duplicates the  
 619 wanted sound field. It makes the sound outside the shielded  
 620 domain to be 6 dB louder after the controls are switched on.  
 621 Hence, even in the case with wanted sound, it may still be  
 622 feasible to deduce the sound field without contribution from  
 623 the controls by having the value of the measured sound field  
 624 near the outside boundary of the shielded domain when the  
 625 controls are on.

626 The results above show that it may be possible to deter-  
 627 mine the original sound field without switching off the con-  
 628 trol sources, which could then lead to a real time realization  
 629 of a practical adaptive active shielding methodology. A  
 630 proper mathematical framework for this will be developed in  
 631 a further work.

## 632 VI. CONCLUSIONS

633 The practicality of active shielding based on the method  
 634 of difference potentials has been demonstrated and validated  
 635 with broadband acoustic sources in a three-dimensional  
 636 space. It has been shown that attenuation of around 12 dB  
 637 has been achieved in the experiment in a large volume of a  
 638 shielded domain. Apart from the practical difficulties associ-  
 639 ated with the realization of control source arrays on the  
 640 boundary surface, the results of the experiment and numeri-  
 641 cal analysis show that the method can provide an effective

solution in a three-dimensional space through a broadband  
 spectrum of low frequencies.

642  
 643  
 644 The physical size of the control sources has been considered  
 645 as one of the reasons that limit the performance of the system.  
 646 This is a common problem in most existing active control meth-  
 647 ods. The size of a control source is still a factor restricting the  
 648 effective frequency range for suppression of noise. In addition to  
 649 the suppression of noise, the proposed method has been shown  
 650 through numerical simulations to effectively preserve the  
 651 wanted sound separately from the total fields composed of noise  
 652 and wanted sound, in three-dimensional spaces where the sys-  
 653 tem characteristics are not known. The results clearly demon-  
 654 strate the potential advantages of the method under these  
 655 extended experimental conditions. All the current set of experi-  
 656 ments has been limited to a non-real-time control system. The  
 657 proposed approach has only been tested in experiments where  
 658 the contribution of the control sources can be completely sepa-  
 659 rated. However, the numerical simulation and theoretical studies  
 660 have shown that, in cases where there is no wanted sound, the  
 661 proposed approach in its present form can be applicable in real-  
 662 time system since the noise field without the control outputs can  
 663 be measured directly in the close neighborhood of the boundary.  
 664 In cases with wanted sound, additional on-line calculations will  
 665 be required for the separation of control outputs from input data.  
 666 Future research will focus on the development and study of the  
 667 real-time active control, and on the extension of the method to  
 668 the case with three-dimensional wanted sound field.

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 672 project codes, GR/T26825 and GR/T26832/01. 673

674 **NOMENCLATURE**

- 676  $b_{vol}$  Force per unit volume
- 677  $c$  Speed of sound
- 678  $L$  Operator
- 679  $q_{vol}$  Volume velocity per unit volume
- 680  $t$  Time
- 681  $u$  Particle velocity
- 682  $\rho$  Air density

683 **SUBSCRIPTS**

- 686  $a$  Adverse sound (noise)
- 687  $d$  Dipole
- 688  $m$  Monopole
- 689  $|m$  Value at node  $m$
- 690  $D$  Value in a domain  $D$
- 691  $h$  Discrete counterpart

692 **SUPERSCRIPTS**

- 695 <sup>(h)</sup> Discrete function

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