Linear and nonlinear problems of active sound control

Sergei V. Utyuzhnikov*1, Victor S. Ryaben’kii**2, and Ali Turan***1

1 School of Mechanical Aerospace and Civil Engineering, University of Manchester, P.O. Box 88, Manchester, M60 1QD, U.K.
2 Keldysh Institute of Applied Mathematics, Miusskaya Sq. 4, Moscow, 125047, Russia.

The problem of active shielding of some domain from the effect of the field generated in another domain is considered. The active shielding is implemented via the placement of additional sources in such a way that the total contribution of all sources leads to the desirable effect. The obtained solution does not require either the knowledge of the particular Green’s function or any other information on source distribution and the surrounding medium. It is also important that along with undesirable field to be shielded, a desirable field can be admitted in the analysis. The solution of the problem requires only knowledge of the total field on the perimeter of the shielded domain. The active shielding sources are obtained for both the linear and nonlinear formulations of the problem.

S. V. Utyuzhnikov, V. S. Ryaben’kii, A. Turan

1 Introduction

The active shielding (AS) of some domain from the effect of the field (noise) generated in another domain is realized via the implementation of additional sources in such a way that the total contribution of all sources leads to the desirable effect. The comprehensive reviews of the theoretical and experimental methods related to these subjects can be found in [1], [2]. Most theoretical approaches assume some quite detailed information about the undesirable sources and the properties of the medium.

The solution of the inverse source problem of AS was obtained in a discrete formulation via the Difference Potential Method in [3]. The solution requires only the knowledge of the total field (both desirable and undesirable) at the grid boundary of the shielded domain. This solution was extended to arbitrary hyperbolic systems of equations in [4]. The general solution for the Helmholtz equation with constant and variable coefficients is derived in [5],[6]. The optimization of the solution was studied in detail by Lončarić and Tsynkov (see, e.g., [7]). In [8] the problem of AS in composite domains is formulated for the first time and its general finite–difference solution is provided.

In [9] the solution of the AS problem is obtained in the continuous space for linear boundary value problems with constant and variable coefficients. The performance of the AS solution is also demonstrated on the example of the Maxwell equations in [10]. In the current paper a nonlinear formulation is considered. With respect to the active noise shielding, the obtained solution can be a theoretical foundation for the nonlinear AS from noise induced by, for example, turbulent flows.

2 Formulation of the AS problem

The generalized mathematical formulation of the AS problem is presented in the following form. Let us assume that some field (sound) \( W \) is described by the following correct boundary value problem (BVP) in a domain \( D \subseteq \mathbb{R}^m \):

\[
L(W) = f, \quad W \in \Xi(D).
\]

Here, operator \( L \) can be, generally speaking, nonlinear, \( \Xi(D) \) is some space of functions such that the solution of BVP (1), (2) is unique. In addition, we suppose that the solution of the homogeneous equation (1) is only trivial.

Consider some bounded domain \( D^+ : \overline{D^+} \subseteq D \) having a smooth boundary. The sources on the right–hand side can be situated both in \( D^+ \) and outside \( D^+ \):

\[
f = f^+ + f^-, \quad \text{supp } f^+ \subset D^+, \text{ supp } f^- \subset D^- \equiv D \setminus \overline{D^+}.
\]

Here, \( f^+ \) is the source of a "friendly" field (sound), while \( f^- \) is the source of an "adverse" field (noise). It is supposed that \( f \) is a regular function in some vicinity of the boundary \( \Gamma \).

Suppose that we know the distribution \( W_\Gamma \equiv W|_\Gamma \) of the function \( W \) on the boundary \( \Gamma \) of \( D^+ \). It is to be noted that only this information is assumed to be available.

* Corresponding author: e-mail: s.utyuzhnikov@manchester.ac.uk, Phone: +0044 161 306 3707, Fax: +0044 161 306 3723
** ryab@keldysh.ru
*** a.turan@manchester.ac.uk
3 Solution of the inverse source problem

Suppose that the field $W$ is described in $\mathbb{R}^m$ by the following correct BVP set in $D = D^+ \cup D^-$:

$$L(W) \equiv \sum_{i=1}^m \frac{\partial F^i}{\partial x^i} = f,$$

$$W \in \mathcal{D}(D),$$

where $\{x^i\}$ $(i = 1, \ldots, m)$ is a Cartesian coordinate system; $W$ and $f$ are vector–functions with the dimension of $m$; $F^i(W) \in C^\infty(D)$ $(i = 1, \ldots, m)$. Assume that $L(U) = 0 \Leftrightarrow U = 0$ and either $f^+ \equiv 0$ or the operator $L$ is linear in $D^-$ along with the boundary conditions at the boundary $\partial D$.

Then, the solution of AS problem is as follows:

$$G = G_0 \equiv F_n(W_T)\delta(\Gamma).$$

In particular linear case of $F^i = A^i(x)W$, $A^i(x) \in C^\infty(D)$, then we obtain the AS source term for the linear case [9]

$$G_0 = A_nW_T\delta(\Gamma),$$

where $A_n = \sum n_iA^i\delta(\Gamma)$.

In the case of the Euler equations for gas dynamics:

$$L(W) = W + \sum_{i=1}^3 F^i(W)x_i,$$

where

$$W = (\rho, \rho u_1, \rho u_2, \rho u_3, e)^T, \quad F^i(W) = u_iW + p(0, \delta_{1,i}, \delta_{2,i}, \delta_{3,i}, u_i)^T,$$

$\rho$ is the density; $u_1, u_2, u_3$ are the velocity coordinates in some Cartesian coordinate system $\{x_i\}$ $(i = 1, 2, 3)$; $e$ is the internal energy; $p$ is the pressure; $\delta_{ij} = 1$ if $i = j$, $\delta_{ij} = 0$ if $i \neq j$.

The AS solution is then as follows

$$G_0 = (\rho u_n, \rho u_1u_n + \rho n_1, \rho u_2u_n + \rho n_2, \rho u_3u_n + \rho n_3, (e + p)u_n)^T\delta(\Gamma),$$

where $u_n = \sum n_i u_i$ is the component of the velocity normal to the boundary $\Gamma$.

In contrast to the acoustics equations, solution (8) depends on the all components of the velocity, not only on the normal one, at the perimeter of the shielded domain.

4 Conclusion

The solution of the nonlinear AS problem has been obtained in the form of a simple–layer source term in some general formulation. The solution only requires the knowledge of the total field on the perimeter of the shielded domain.

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