

Experimental Validation of the Active Noise Control Methodology Based on Difference Potentials

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To achieve active noise cancellation over a large area, it is often necessary to get a measure of the physical properties of the noise source to devise a counter measure. This, however, is not practical in many cases. A mathematical approach, the difference potential method, can provide an alternative solution for active shielding over a large area. In this approach, the cancellation of unwanted noise requires only measurements near the boundary surface but not at the source itself, and no other information is required. Moreover, the solution based on difference potentials applies to bounded domains in the presence of acoustic sources inside the domain to be shielded. This paper reports on the results of experimental validation for the methodology. It has been demonstrated that while preserving the wanted sound, the developed approach can cancel out the unwanted noise. The volumetric noise cancellation offered by the proposed methodology, along with leaving the wanted sound unchanged, is a unique feature compared with other techniques available in the literature. It can be most useful in the context of applications related to civil aviation, in particular, for eliminating the exterior noise inside the passenger compartments for both current and future generations of commercial aircraft.

Nomenclature

b_{vol}	=	force per unit volume
C_d	=	loudspeaker dipole calibration constant
C_m	=	loudspeaker monopole calibration constant
c	=	speed of sound
D	=	subdomain of D_0 ($D \subset D_0$)
\bar{D}	=	domain with the boundary
D_0	=	domain
G	=	additional source

H	=	transfer function of the system
h	=	space step
k	=	wave number
M^0	=	grid counterpart of D_0
M^+	=	grid counterpart of D
M^-	=	grid counterpart of $D_0 \setminus D$
N^0	=	extended set of M^0
N^+	=	extended set of M^+
N^-	=	extended set of M^-
p	=	sound pressure
p_0	=	acoustic pressure used for active shielding
q_{vol}	=	volume velocity per unit volume
S	=	acoustic source
S_a	=	source of adverse noise
S_f	=	source of wanted sound
supp	=	support
U	=	total sound field from primary source
\tilde{U}	=	total sound field from primary and secondary sources
U_f	=	wanted sound
$U_{D_0}^{(h)}$	=	discrete counterpart of U_{D_0}
$U_g^{(h)}$	=	discrete counterpart of \tilde{U}
$U_\gamma^{(h)}$	=	total field on the grid boundary γ
u	=	particle velocity
u_0	=	particle velocity used for active shielding
γ	=	grid boundary
Δ	=	size of a source
∂D	=	the boundary of D
$\theta(x)$	=	Heaviside function

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ρ = air density

Subscripts

a = adverse sound (noise)

f = wanted sound

t = total

Superscripts

h = discrete function

$|m$ = value at node m

Introduction

ACTIVE control of sound is a rich and lively research area in acoustics. A typical problem formulation in this area involves a given region of space (bounded or unbounded) to be shielded from the unwanted noise by introducing special secondary sound sources called controls. The controls render active shielding (AS) of the protected region, which is a key distinction of this approach compared with passive shielding strategies that are based on acoustic insulation. In practice, active and passive noise control strategies could often be combined, because passive insulation is more efficient for higher frequencies, whereas active shielding is better suited for lower frequencies. The overall problem becomes more complicated if, along with the unwanted noise, a wanted sound component is present inside the protected region.

The focus of the current paper is to experimentally validate the design features and the performance of the AS technique that was introduced and studied theoretically in our previous publications. Its key characteristics include the capability to cancel out the unwanted noise on a large region, while requiring no detailed knowledge of either the sound-conducting properties of the medium or the noise sources. The only input data needed by the methodology are the acoustic quantities at the perimeter of the protected region (in practice, they can be measured). Moreover, these quantities pertain to the overall acoustic field composed of both the unwanted and wanted components, and the methodology automatically differentiates between the two.

For completeness of the presentation, in the first part of the paper we summarize the relevant theoretical findings from our previous work. The earliest theoretical publications in the literature on the subject of AS are attributed to Jessel [1], Malyuzhinets [2], and Fedoryuk [3]. Yet the first realistic practical implementations appeared much later (see, for example, [4–8]). Some available noise abatement techniques provide for the cancellation of noise in the selected discrete [7–10] or directional [11] areas. Other techniques (in particular, those developed by Kincaid et al. [12] and Kincaid and Laba [13]) require a detailed knowledge of the sources and nature of noise. A number of publications are also devoted to optimization of the strength of the spatially distributed controls to minimize a quadratic pressure cost function [14,15]. The method proposed in [4–6,16–18], hereby called the JMC (Jessel–Mangiante–Canevet) method, which is based on the standard Huygens' construction in wave propagation,^{††} (see, for example, [20]), requires only information at the perimeter of the shielded domain, which specifically refers to the adverse component of the field. The AS source term in the JMC method is implemented via the monopole, dipole, and quadruple arrangements [21,22]. A remarkable broadband noise attenuation was achieved in [6] for a duct with anechoic termination. Yet the JMC method cannot be used when there is a wanted component of the field generated inside the protected region. In addition, the JMC method has only been applied to either unbounded domains or to domains with anechoic termination. For further information on the general theory and practice of active noise control, we refer the reader to comprehensive surveys presented in the monographs [23–25] and in the review paper [18].

^{††}Not to be confused with the Huygens' principle, which is the existence of the sharp aft fronts of the waves for time-dependent wave propagation in odd-dimension spaces [19].

A number of publications are available in the literature in which the formulation of the AS problem allows for the presence of wanted sound. Perhaps the most general formulation requires only the knowledge of the total acoustic field at the perimeter of the protected region. It is very important to emphasize that in doing so, the individual unwanted and wanted components do not need to be known separately. If the appropriate Green's function is available, then the general solution of the AS problem can be obtained in a closed form. For the Helmholtz equation with constant coefficients, this was done by Malyuzhinets [2] and Tsynkov [26]. However, if, for example, the parameters of the medium are not constant, then a more universal approach is needed. The corresponding procedure is based on the difference potential method (DPM) [27–29]; it allows one to obtain the general solution to the AS problem for arbitrary geometries, properties of the medium, or boundary conditions. There are only two principal mathematical limitations, but they are usually met in many practical implementations. The problem *must be linear and must have a unique solution*. In contrast to many other active noise control techniques, the DPM-based AS solution can naturally be obtained in a discrete form. From the standpoint of practical implementation, this is advantageous, because a realistic AS system would require a discrete collection of control sources anyway. The DPM-based approach has been thoroughly analyzed for the Helmholtz equation and its variable-coefficient counterparts (see [29–32]). A comprehensive analysis of the continuous and discrete surface potentials for the Helmholtz-type operators was carried out by Tsynkov [26]. Optimization of the control sources with respect to different criteria was performed by Loncaric and Tsynkov [30–32]. In [33], the DPM was employed to solve a one-dimensional AS problem for the linearized Euler equations (a system as opposed to a scalar equation). The performance of the DPM solution was revealed in detail for the case of a duct with termination. It was shown that in enclosures, the resulting AS solution attenuates the incoming noise while retaining the echo effect. In [34], the DPM-based solution was extended to a wide class of multidimensional hyperbolic systems. The sensitivity analysis to input errors was accomplished in [35]. It was also proven that the solution is applicable to resonance regimes.

This paper is contemplated as an experimental extension of our previous theoretical work. To the best of our knowledge, it is the first publication in the literature in which an AS methodology is experimentally tested and validated for cases in which the wanted sound component is present and is not separated from noise ahead of time and in which the protected domain has an arbitrary nonrigid termination. The experimental results that we have obtained (see the next sections) demonstrate an excellent agreement with the predictions of the DPM theory.

A unique feature of the proposed methodology is its capability to cancel the unwanted noise across the volume and keep the wanted sound unaffected. This capability can be very useful for the applications related to civil aviation, as it enables protection of the passenger compartment of a commercial airliner from the engine and airframe noise coming from the outside. In doing so, the active noise controls will not interfere with the ability of the passengers to enjoy the in-flight entertainment system or to merely converse.

Mathematical Formulation of the Active Shielding Problem

Assume that the propagation of sound is governed by a linear partial differential equation or system on the domain D_0 (bounded or unbounded):

$$LU = S \quad (1)$$

where L is the appropriate operator, and its solution U is subject to some additional conditions (e.g., homogeneous boundary conditions) that we formulate as the inclusion:

$$U \in U_{D_0} \quad (2)$$

In formula (2), U_{D_0} is a linear space of functions defined on \bar{D}_0 such that the inclusion (2) guarantees existence and uniqueness of the

solution to problems (1) and (2). The operator L in formula (1) can, in particular, correspond to the linearized Euler equations (acoustics system).

Next, consider a subdomain $D \subset D_0$. The acoustic sources S on the right-hand side of Eq. (1) can either belong to D or to its exterior:

$$S = S_f + S_a, \quad \text{supp } S_f \subset D, \quad \text{supp } S_a \subset D_0 \setminus D \quad (3)$$

In formula (2), S_f is inside D (see Fig. 1), whereas S_a is outside D . Hence and further, $\text{supp } g$ means the domain in which a function g is not zero. Accordingly, we can write $U = U_f + U_a$, where $LU_f = S_f$ and $LU_a = S_a$, and both U_f and U_a are defined on D_0 and satisfy formula (2). Now we can formulate the AS problem. It consists of finding such additional sources G that the solution \tilde{U} of the modified boundary-value problem [cf. formulas (1) and (2)]

$$L\tilde{U} = S + G, \quad \tilde{U} \in U_{D_0}, \quad \text{supp } G \subset D_0 \setminus D \quad (4)$$

would coincide with the wanted sound U_f alone, $\tilde{U} \equiv U_f$, on the subdomain D .

As shown in [36], for example, to obtain the general solution of the foregoing AS problem, it is sufficient to know only the trace $U_{\partial D}$ of the total acoustic field U on the boundary ∂D of the domain D . In particular, no knowledge of the actual sources S on the right-hand side of Eq. (1) is required. This indicates that other solutions for active controls can be constructed beyond the obvious first choice: $G = -S_a$, whereas this first choice may, in turn, be difficult to implement even if the adverse sources S_a were explicitly available.

General Solution of the Finite Difference Active Shielding Problem

Following [28,37], let us introduce some grid M^0 on D_0 . Then we introduce subsets of the grid M^0 as follows: $M^+ = M^0 \cap \bar{D}$ and $M^- = M^0 \setminus M^+$. Assume that the differential Eq. (1) is approximated on some stencil by the difference equation:

$$L_h U_{|m}^{(h)} \equiv \sum_n a_{mn} U_n^{(h)} = S_{|m}^{(h)}, \quad m \in M^0 \quad (5)$$

Equation (5) is supplemented by the discrete boundary condition that approximates the continuous boundary condition (2):

$$U^{(h)} \in U_{D^0}^{(h)} \quad (6)$$

In formula (6), $U_{D^0}^{(h)}$ is a linear space of grid functions such that the inclusion (6) guarantees existence and uniqueness of the solution to the boundary-value problems (5) and (6).

When the stencil of the scheme (5) is applied to the nodes from M^0 , M^+ , and M^- , it sweeps the extended sets N^0 , N^+ , and N^- , respectively. The intersection of the sets N^+ and N^- is, generally speaking, not empty, and we call it the grid boundary γ of the domain D : $\gamma = N^+ \cap N^-$. In its own turn, the grid boundary γ is split into two nonintersecting subsets: $\gamma = \gamma^+ \cup \gamma^-$, where $\gamma^+ = \gamma \cap M^+$,

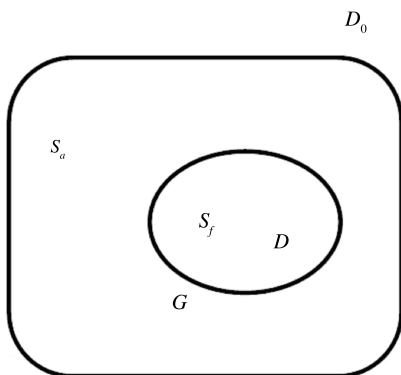


Fig. 1 Problem sketch.

and $\gamma^- = \gamma \cap M^-$. The finite difference AS problem is then formulated as follows. Consider problems (5) and (6), where

$$S_{|m}^{(h)} = S_{f|m}^{(h)} + S_{a|m}^{(h)}, \quad \text{supp } S_f^{(h)} \subset M^+, \quad \text{supp } S_a^{(h)} \subset M^- \quad (7)$$

We would like to find such control sources (additional terms on the right-hand side)

$$G^{(h)}: \text{supp } G^{(h)} \subset M^- \quad (8)$$

that the solution of the modified finite difference problem [cf. formulas (5) and (6)]

$$L_h U_{g|m}^{(h)} = S_{|m}^{(h)} + G^{(h)}, \quad m \in M^0, \quad U_g^{(h)} \in U_{D_0}^{(h)} \quad (9)$$

would coincide on $N^+ \subset N^0$ with only the wanted sound. The latter is the solution of problems (5–7) with the noise sources artificially removed (i.e., obtained for $S_a^{(h)} \equiv 0$):

$$L_h U_{f|m}^{(h)} = S_{f|m}^{(h)}, \quad m \in M^0, \quad U_f^{(h)} \in U_{D_0}^{(h)} \quad (10)$$

The general solution of the foregoing finite difference AS problem can then be obtained via the theory of difference potentials:

$$G^{(h)} = \begin{cases} -L_h V_{|M^-}^{(h)}, & \text{if } m \in M^- \\ 0, & \text{if } m \in M^+ \end{cases} \quad (11)$$

In formula (11), $V^{(h)}$ is an arbitrary function such that

$$V^{(h)} \in U_{D_0}^{(h)}, \quad V_\gamma^{(h)} = U_\gamma^{(h)} \quad (12)$$

One can prove (see [28,37]) that if the source term $G^{(h)}$ of formula (11) is used in formula (9), then $U_{g|N^+}^{(h)} = U_{f|N^+}^{(h)}$. Moreover, from formula (12), we see that the input data needed to construct the controls (11) reduce only to the trace of the overall field on the grid boundary γ : $U_\gamma^{(h)}$. In practice, the grid function $U_\gamma^{(h)}$ can be measured.

It is important to emphasize that the control sources (11) are obtained for the general case and do not require knowledge of the specific Green’s function. It is also clear that the function $V^{(h)}$ in formula (12) is not unique. A particular simple choice of this function corresponds to $V_{|M^0 \setminus \gamma}^{(h)} = 0$.

Implementation of the general solution (11) for specific acoustic settings, as well as its subsequent experimental validation, may require additional discussion. To better understand the structure of the solution (11), let us consider a 1-D case. Assume that the primary noise sources S_a are located in the area $x > 0$, whereas the secondary source (control) G is to be placed at $x = 0$ to shield the area $x < 0$. It is noted that the locations of the primary sources are not known. Fig. 2 illustrates the case.

Let us write the governing system as

$$p_t + \rho c^2 u_x = \rho c^2 q_{\text{vol}}^{(h)} + f_p, \quad u_t + \frac{p_x}{\rho} = \frac{b_{\text{vol}}^{(h)}}{\rho} + f_u \quad (13)$$

where f_p and f_u are source functions for the continuity and momentum equations, respectively.

Subject to the assumption $kh \ll 1$, in [33] the controls were obtained as

$$q_{\text{vol}}^{(h)}(x) = \frac{\Theta_h(x)}{h} u_0, \quad b_{\text{vol}}^{(h)}(x) = \frac{\Theta_h(x)}{h} p_0 \quad (14)$$

In formulas (14),

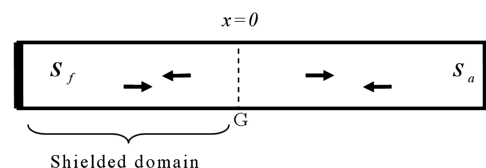


Fig. 2 AS in a finite duct.

$$\Theta_h(x) \equiv \theta(h/2 - x)\theta(h/2 + x)$$

and u_0 and p_0 are the particle velocity and acoustic pressure, respectively. The values of both u_0 and p_0 can be obtained from measurements, and they normally correspond to the total sound field at $x = 0$.

The controls are $q_{\text{vol}}^{(h)}$ and $b_{\text{vol}}^{(h)}$ of formulas (14). The former alters the balance of mass in the system, and the latter alters the balance of momentum (see [30] or [23]). In an experimental setting (see the next section), the control $q_{\text{vol}}^{(h)}$ is implemented as an acoustic monopole and the control $b_{\text{vol}}^{(h)}$ is implemented as a dipole. Likewise, from the standpoint of mathematics, if we were to write a second-order equation for the pressure instead of the first-order system for both pressure and velocity, then the source $q_{\text{vol}}^{(h)}$ would be a monopole (i.e., a delta function on the right-hand side), whereas $b_{\text{vol}}^{(h)}$ would be equivalent to a dipole (i.e., the first derivative of a delta function). The directionality of the dipole is a key factor that enables the method to distinguish the wanted sound generated in the protected domain as compared with its external, unwanted, counterpart [33].

The AS control sources (14) depend on the parameter h , which, according to [33], should be set equal to the thickness of the AS sources Δ . Note that if the wanted sound is absent ($S_f \equiv 0$), then our AS solution will be equivalent to that given by the JMC method [1]. At the same time, if the wanted sound is present and the controls are generated based on the total field, then the JMC-based solution can be formally extended to include this case as well. Hence, our analysis shows that the JMC solution applies to a broader range of conditions than the one under which it was originally derived (see [1,4]). In particular, it is not limited by unbounded domains without wanted sources. However, applicability of the JMC method to this more general setting cannot be demonstrated within the JMC framework itself.

Finally, let us mention that even though we have explicitly obtained the control sources, their subsequent optimization or due allowance for diffraction effects may require the solution of an additional problem (see [34]). If the shape of the protected region is complicated, then the unique capability of the DPM to efficiently resolve the geometric attributes becomes very important.

Experimental Setup

In an experimental setup for active control of sound, there are certain limitations on the frequency of the acoustic waves. Namely, the size (thickness) of the AS system should be much shorter than the typical wavelength considered [33]. On the other hand, the frequency cannot be too low either, due to the low frequency limit of the loudspeaker source used in the experiment [38]. In our experiments, we set the lower frequency limit at 40 Hz to prevent a substantial drop in the power output. For frequencies lower than 40 Hz, the performance of the dipole deteriorates considerably as the two monopoles with opposite phases start to noticeably damp one another. Another important consideration pertains to the dimension of the problem. As shown in [34], to obtain the AS solution in a multidimensional setting, one needs to know only the normal component of the particle velocity at the boundary of the shielded domain. Hence, if the acoustic waves propagate normally to the boundary, then the AS solution (14) also remains applicable in a full 3-D case.

In view of the previous comment regarding the validation of the DPM-based AS solution, it is natural to start with the analysis of a one-dimensional linear problem. This is done in a cylindrical duct manufactured of polypropylene tubing, which is sufficiently rigid to allow losses through the duct walls to be neglected initially. The duct is 4.42 m in length. Its inner diameter is 0.17 m, which allows it to be approximated acoustically as one-dimensional up to a frequency of about 1 kHz. As illustrated in Fig. 3, the noise source is mounted at the right endpoint of the duct, and the shielded domain stretches from its middle all the way to the left endpoint. The theoretical shielded domain shown in Fig. 3 is 2.12 m in length. As the sound field near the control sources could have distorted the assumed plane wave

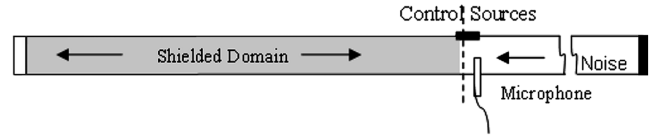


Fig. 3 Experimental setup.

behavior, the effective shielded domain length is taken to be two duct diameters less than the theoretical shielded domain (i.e., 1.78 m). Through all of the experiments the sound pressure is measured along the axis of the duct.

The control sources obtained in the previous section contain both an acoustic monopole and a dipole. In our experiment, these sources are implemented using special combinations of loudspeakers placed in rigid wooden enclosures. The acoustic dipole for the experiment is designed according to [6] and consists of two loudspeakers mounted back to back in an enclosure.

The dipole setup is shown in Fig. 4. The distance d between the two loudspeakers forming the dipole must be small compared with the wavelength: that is, $kd \ll 1$ (see [39]).

The total volume velocity generated by the monopole is given by

$$q = q_{\text{vol}}^{(h)} S h \quad (15)$$

where h is already taken to be equal to the thickness of the source Δ ($\Delta \ll \lambda$), and S denotes its cross section. Consequently, the source strength of the control monopole from Eq. (14) is as follows:

$$q = u_0 S \quad (16)$$

To actually obtain the strength of the monopole q in Eq. (16), one needs to know the particle velocity u_0 . This can be measured using the well-known two-microphone technique [36], according to which,

$$u_0 \approx \frac{i}{\rho\omega} \left[\frac{\Delta p}{\Delta x} \right]$$

where $\Delta x < \lambda/6$ (see [40]).

Similarly, the fluctuating force f_d of the control dipole is obtained from Eq. (14) in the form

$$f_d = b_{\text{vol}}^{(h)} S h = p_0 S \quad (17)$$

It is important to emphasize, in particular, that only the total field near the boundary is available for measurements; however, no information beyond that is strictly needed to build the controls. Specifically, no knowledge of either the reflection factor or the distance from the noise source is required. Thus, the particle velocity u_0 and the total sound pressure p_0 can be measured at some convenient location h_1 very close to the boundary, as long as $h_1 \ll \lambda$.

Note that in addition to the frequency limits outlined previously, the test frequency range is also strictly limited by the requirement that all the waves propagating in the system can be considered plane. The upper bound for the frequency is set by the following condition for a plane wave [39]:

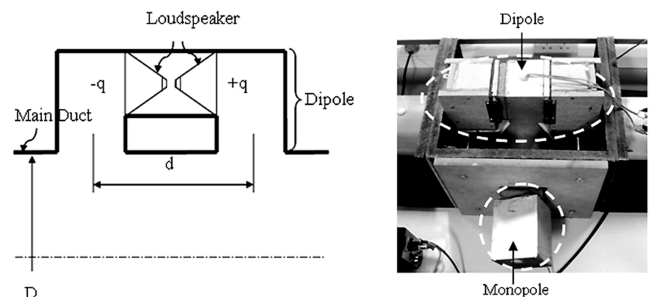


Fig. 4 Configuration of an acoustic dipole.

$$f < \frac{1.84c}{\pi D}$$

where D is the diameter of the duct.

In addition, to avoid potentially large measurement errors due to the onset of extreme sensitivity at tube resonances, the lower frequency bound is generally set at above the first resonance. To do that, the wave length is chosen to be shorter than twice the total length of the duct, which translates into

$$f > \frac{c}{2L_t}$$

This lower bound for the frequency appears very close to the lower frequency limit of the loudspeaker obtained from the power output considerations. Hence, the range for the test frequency f is taken as

$$40 \text{ Hz} < f < 1000 \text{ Hz} \quad (18)$$

where the upper bound is obtained from the previously given condition of a plane wave.

Four different test cases were used to verify the DPM-based AS solution. The simplest case corresponds to a duct with rigid termination at the endpoint opposite to the noise source. The corresponding test results can be compared against existing solutions. The second test case employs nonrigid termination achieved by putting an approximately 1-in.-thick randomly chosen fibrous sound-absorbing material next to the rigid plate. The actual surface impedance of the absorbing material is deliberately not evaluated, because the DPM-based AS automatically takes into account the properties of the sound-conducting medium inside the protected region. In the third experiment, an acoustically complex obstacle is placed inside the shielded domain. The obstacle may have a fairly complex shape and it is wrapped in a layer of an acoustically soft material. The experiment shows that unlike the many techniques available in the literature, the DPM-based noise control methodology is capable of achieving the abatement of noise even if the fundamental solution is not known. In the fourth experiment, a wanted sound component is introduced in the shielded domain by adding a loudspeaker at the end of the duct opposite to the noise source. This case demonstrates that the DPM-based controls can automatically split the total acoustic field into the adverse and wanted components and then cancel out only the unwanted noise while preserving the wanted sound.

The measurement and control modules are illustrated in Fig. 5. The measurement procedure first measures the total particle velocity and acoustic pressure at a location near the AS control sources, which are switched off at this stage of the measurement. The measured pressure and velocity are then incorporated into the DPM solution, together with the calibrated loudspeaker transfer functions to generate the desired control source signals offline using MATLAB, which are then saved as phase-synchronous .wav files that can be played back using any multichannel compatible wave editor. The sound-generation system consists of loudspeakers, amplifiers, and a

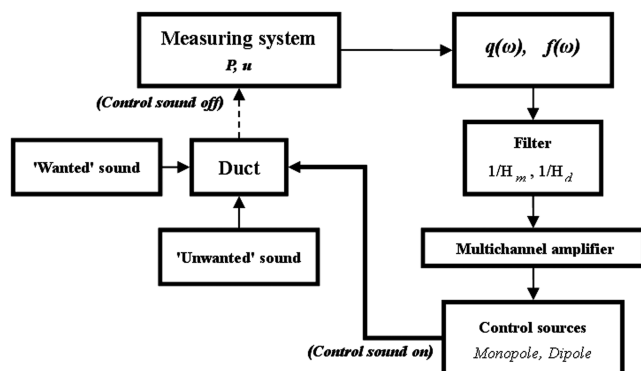


Fig. 5 Block diagram of measurement and control system.

PC with a five-channel sound card. The results reported subsequently correspond only to single-frequency measurements.

Calibration

Because the DPM-based AS approach requires fine-tuning of the control sources, it is necessary to have those sources accurately calibrated ahead of time. If one assumes that the medium in the entire domain is lossless, then at low frequencies, the waves can be considered as plane waves [39], and the effective volume velocity of the monopole is determined by the movement of the loudspeaker cone [36]. This effective volume velocity can be calculated from the sound pressure measured by a microphone located at a reference position in the tube with known termination impedances (at both endpoints) [23]. Quite independently, the normal surface velocity of the loudspeaker cone can be measured using an accelerometer mounted on the surface. From these two sets of measurements, a linear relationship between the effective volume velocity and the normal surface velocity of the loudspeaker cone can be derived:

$$q_{\text{eff}} = C_m U_m \quad (19)$$

In formula (19), q_{eff} is the effective volume velocity based on the sound pressure, and U_m is the surface velocity measured by the accelerometer.

Next, according to the definition of an acoustic dipole [39], this naturally comprises two loudspeakers of equal strength with a 180 deg phase shift. To tune the phase shift, a reference signal is used so that the relative phase angle between the loudspeakers can be monitored independently. The fluctuating force generated by the dipole is given as follows:

$$f_d = C_d U_d \quad (20)$$

where U_d is the surface velocity of the dipole loudspeakers. The loudspeakers are adjusted so that they both have the same U_d at the same location on the cones. Similar to the calibration of a monopole source, the left-hand side of Eq. (20) is calculated using the technique of [23], and the surface velocity U_d is directly measured by accelerometers. The calibration coefficients for the monopole and dipole, C_m and C_d , are obviously frequency-dependent. Figure 6 illustrates the relationship between the effective volume velocity q_{eff} determined by calibration [see formula (19)] and the apparent volume velocity q_{acc} given by the product of the measured surface normal velocity and the surface area of the cone [see formula (16)]. Figure 6a shows that the maximum phase difference is 10.91 deg if the frequency is below 350 Hz, but the phase difference increases dramatically at higher frequencies. The amplitude ratio $q_{\text{eff}}/q_{\text{acc}}$ stays near 1 at below 350 Hz in Fig. 6b, but again deviates substantially from 1 at higher frequencies. The results indicate that the loudspeaker cone is mostly pistonlike below 350 Hz.

As the termination impedance of the tube changes, the source strength for a given driving signal can also change due to the coupling between the tube and the loudspeaker. The amount of change of q_{eff} depends on the frequency as well as the termination impedance. Because the impedance is generally not known in the experiment, this presents an uncertainty regarding the control of the active shielding sources. This uncertainty can be estimated by measuring the differences in the effective source strengths of the loudspeaker sources under different tube conditions. The maximum difference is found to be smaller than 2.3% for a range of sound pressure amplitudes and frequencies used in the experiments with both rigid and nonrigid (fibrous) termination used in the experiment. The relative difference D_q in the volume velocities between the cases of rigid and nonrigid termination is calculated as

$$D_q = \frac{(q_{\text{eff}}^{\text{nonrigid}} - q_{\text{eff}}^{\text{rigid}})}{q_{\text{eff}}^{\text{rigid}}} \cdot 100\%$$

The behavior of the quantity D_q is illustrated in Fig. 7. Based on this result, the average error in the volume velocity term used in the experiment is less than 1 dB at 200, 350, and 500 Hz.

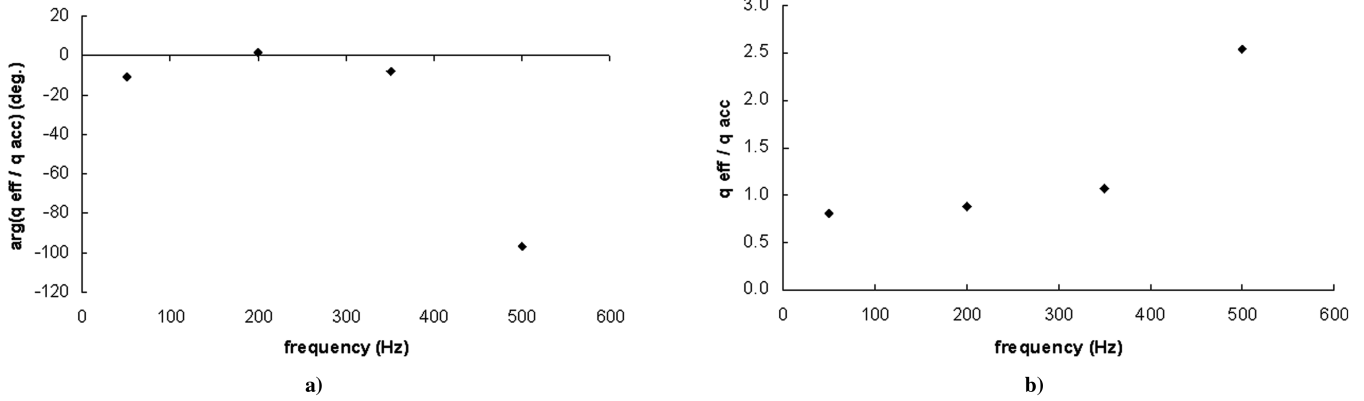


Fig. 6 Plots of a) phase difference between q_{eff} and q_{acc} and b) frequency response of q_{eff}/q_{acc} .

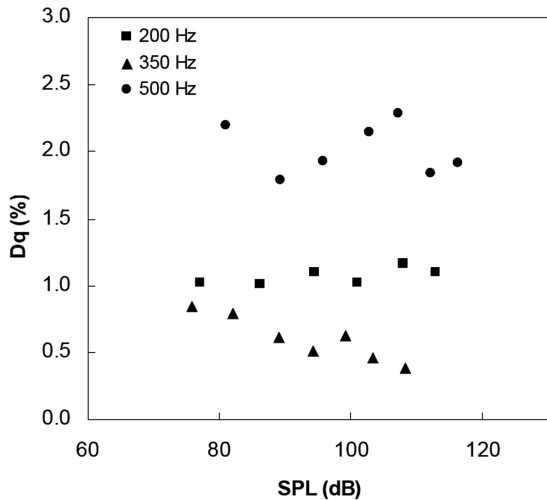


Fig. 7 Difference D_q in the volume velocities q_{eff} .

Figure 8 shows the theoretical accuracy required in the measurement and control system on the boundary to achieve various attenuation levels. For instance, an attenuation of more than 20 dB requires the total amplitude error alone to be below 1 dB or the total phase error alone to be below 6 deg.

To estimate the actual accuracy of the control system in the experiment, the amplitude error in decibels and phase error in degrees between the sound pressure at a monitoring-microphone position measured with the AS sources operating on their own and the target sound pressure required by the AS solution are determined through a set of preliminary measurements. Figure 9 shows these errors measured over a range of frequencies. Swept sine excitation was used in the measurement. The results demonstrate

experimentally that the error in the control system at the frequencies chosen for the test, 200, 350, and 500 Hz, is largely below 0.5 dB in amplitude and below 3 deg in phase. Therefore, according to Fig. 8, an AS system with these errors should allow us to achieve about 20–25 dB attenuation. This is indeed consistent with the attenuation result we obtained in the experiments described in the following sections.

Experimental Results

Active Shielding for the Case of Rigid Termination

Figure 10 illustrates the propagation of noise from the right to the left in a one-dimensional rigid-walled enclosure with rigid termination at both ends. The shielded domain is the left half of the enclosure. The cancellation of noise is rendered by the control sources activated on the boundary of the shielded domain. We recall that the traditional approaches to active noise control require accurate description of the sources and medium information. In other words, distance factors, density of the medium, termination impedances, etc., must be known to implement a global AS solution. In distinct contrast to that, the DPM-based solution does not require any information regarding the original noise sources, impedance of the terminations, or the medium. All of the necessary data are, in fact, contained in the measured field quantities at the boundary of the protected region. As has been shown, the DPM-based control sources are composed of a monopole and a dipole [see formula (14)]. The volume velocity of the monopole is given by formula (16) using the total sound pressure measured at the boundary, and the dipole force is found by formula (17) using the particle velocity of the total field measured at the boundary.

Figure 11 shows the attenuation of noise inside the shielded domain as a function of frequency for the duct with a rigid termination. In this particular case of a 1-D tube with rigid terminations, a global cancellation solution can be derived easily [23] without using the DPM approach. The purpose of this first test case is

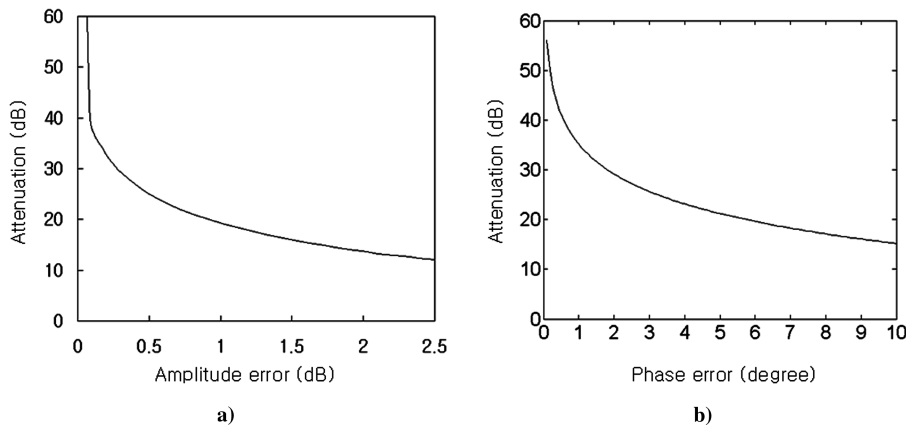


Fig. 8 Theoretically expected attenuation with amplitude error (a), and phase error (b).

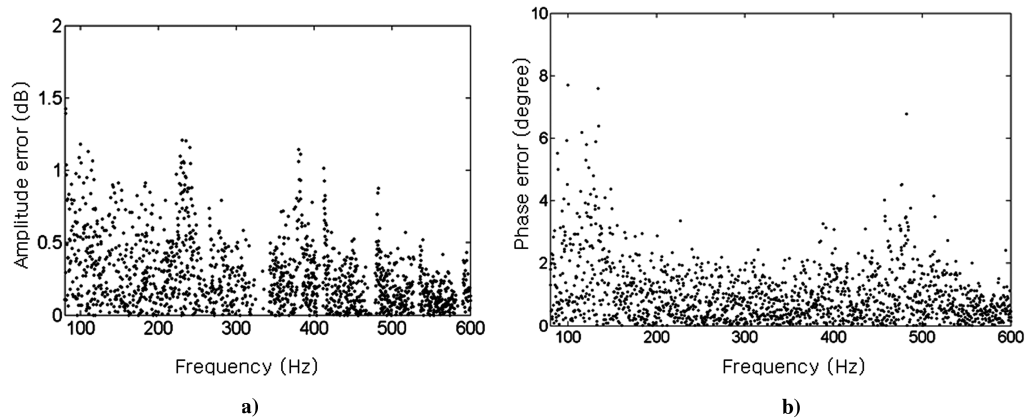


Fig. 9 Plots of a) total amplitude error and b) total phase error of AS control sound pressure measured in the shielded domain.

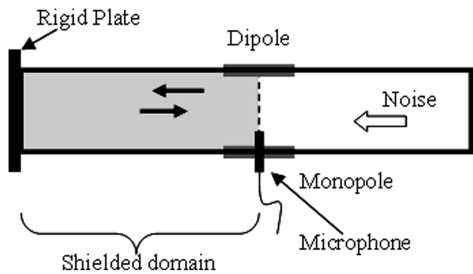


Fig. 10 Experimental setup for the test with a duct terminated by rigid plates.

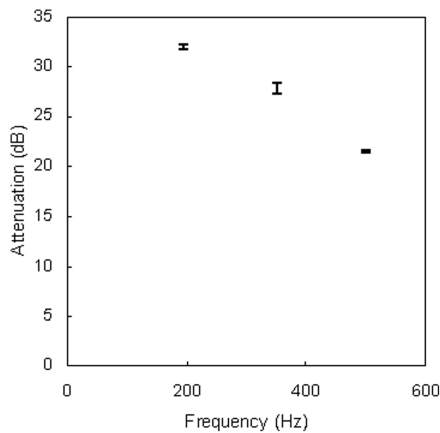


Fig. 11 Attenuation of noise with error bars: 32 ~ 33 dB at 200 Hz, 27 ~ 29 dB at 350 Hz, and 23 ~ 24 dB at 500 Hz.

therefore just to confirm that the DPM approach can at least match existing solutions in this simple case. Attenuation of about 30 dB is attained for the frequencies of 200 and 350 Hz. The efficiency of attenuation drops to about 20 dB as the test frequency increases to 500 Hz. This is partially because the approximation that interprets the loudspeaker cone as a piston breaks down (see Fig. 6). This breakdown makes the precise calibration of the control sources difficult at frequencies above 350 Hz. In addition, the active cancellation of noise may generally become less efficient at higher frequencies, due to the increased error content in the measured phase data, which are necessary to define the control sources.

Figure 12 shows that the DPM-based AS solution provides for a global cancellation of unwanted noise in the targeted area. After the AS control sources are turned on, the distribution of an acoustic pressure is measured at 8 different points along the axis of the duct in the shielded domain. The measuring positions are -1.855 , -1.805 , -1.705 , -1.405 , -1.105 , -0.705 , and -0.455 m along the tube, with the reference position at the AS control source. The length of the shielded domain, 10 times the diameter of the duct, is so defined that

the measurement shows clearly the effectiveness of the cancellation at positions far away from the control sources. Within two diameters of the duct from the control sources, noise cancellation is not fully accomplished, due to the complex nature of the sound field near the sources, which renders the assumption of simple plane waves invalid. However, in most of the protected region, the cancellation of noise is high and consistent over a large area. The \square symbols represent the distribution of SPL in the shielded domain before the AS control sources are turned on, and the \blacklozenge symbols represent the SPL after the control sources are turned on. The microphone positions are marked by the vertical bars in the diagram of the tube.

Active Shielding for the Case of Nonrigid Termination

The following three sections describe test cases that are designed to demonstrate the advantages of the DPM-based active shielding approach over traditional approaches. The experiment described in this section is aimed at showing that the DPM-based AS approach can produce global noise cancellation in cases with unknown nonrigid terminations. In so doing, it does not require knowledge of either the actual reflection conditions at the endpoints of the duct or the nature of the noise sources. In the experiment, the nonrigid termination is achieved by putting an approximately 1-in.-thick generic fibrous sound-absorbing material on the rigid plate. The properties of the fibrous material are not known and are not needed in the DPM approach. This is a particularly important distinction between many existing traditional methods and the DPM-based approach. Our experiment convincingly demonstrates that the DPM-based AS automatically extracts all the necessary information about the impedance and the noise itself from the measurements performed at the boundary. The source strength of the control monopole and dipole is obtained by substituting the measured quantities u_0 and p_0 into Eq. (17) for the dipole and into Eqs. (15) and (16) for the monopole.

When the control sources are activated, a reduction of the total sound pressure level in the shielded domain is observed. For nonrigid termination, the extent of this reduction is around 21 dB at a frequency of 200 Hz. This is slightly less than the attenuation achieved in the previously analyzed case with rigid termination (see the previous section), which may be partially due to the fact that the control sources were calibrated using rigid termination (see the discussion on the control source uncertainty in the Calibration section). Figure 7 has already shown that there is a small error if the calibration is applied to nonrigid cases. Hence, a more efficient cancellation should be expected with better calibration. However, the readily available cancellation of 21 dB is, in any event, sufficient to demonstrate the unique capability of the DPM-based AS solution to automatically take into account the internal and external environments of the shielded domain.

The classical approach developed in [23] for the case of a termination with finite impedance (see Fig. 13) exploits the analytical solution [23] based on the knowledge of the Green's function. The porous material used is a common glass fiber that is 25 cm thick with

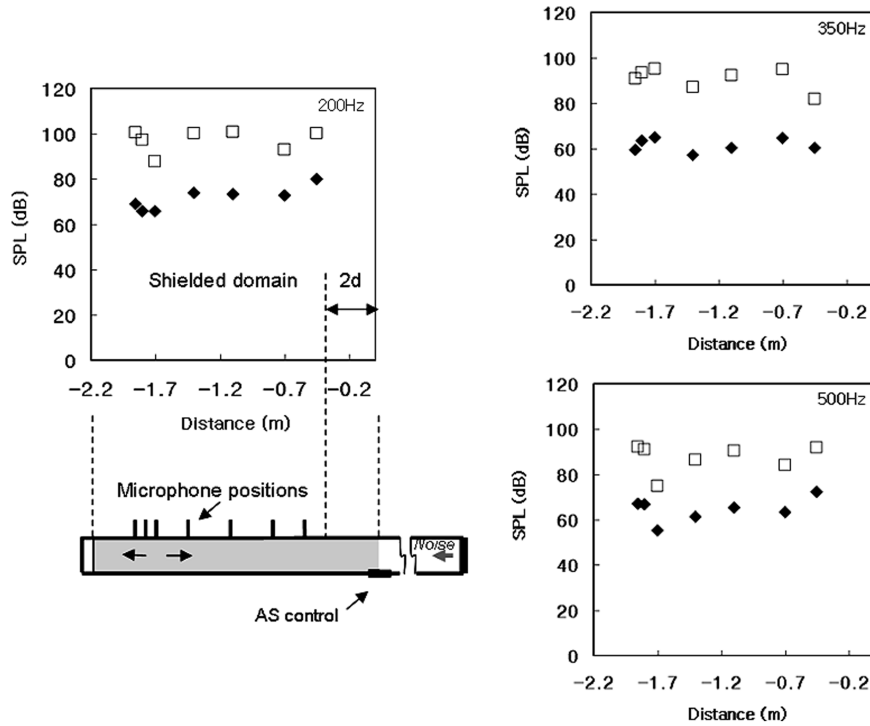


Fig. 12 Sound pressure distribution along the axis of a duct terminated by rigid plates at 200, 350, and 500 Hz.

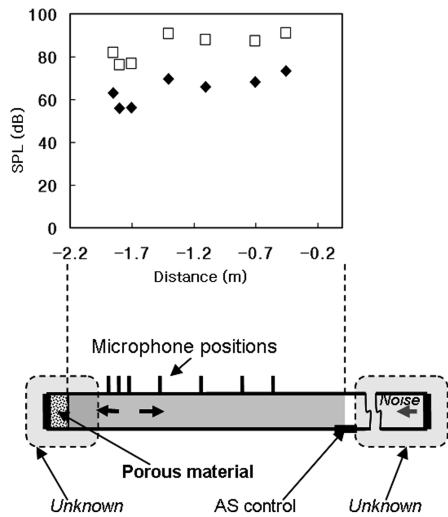


Fig. 13 Sound pressure distribution along the axis of a duct terminated by absorptive material at one end at 200 Hz.

rigid backing. If this is implemented via a monopole, then it requires knowledge of the reflection coefficient at the right endpoint of the duct, as well as knowledge of the distance to this point. Moreover, if either the position of the noise source or the impedance change, then all the corresponding quantities must be measured again. In many practical cases, however, the distance factors, the end impedances, and the noise sources may not be known.

Active Shielding with an Obstacle Inside the Duct

To demonstrate the unique capabilities of the DPM-based noise control methodology, an additional experiment is carried out in which an obstacle (scatterer) is placed inside the shielded domain. The setup for the test is shown in Fig. 14, and we assume that neither the shape nor composition (i.e., reflection properties) of the scatterer are known. We emphasize that as the DPM does not require knowledge of the fundamental solution or the Green’s function of the boundary-value problem, it does not require knowledge of the nature

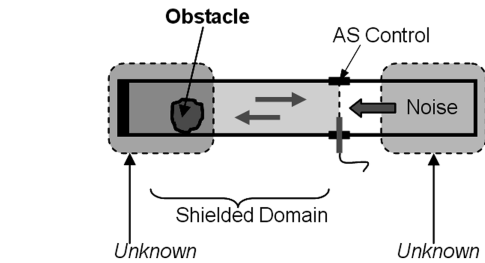


Fig. 14 Configuration of the test devices with an obstacle inside the duct.

of the obstacle either. On the other hand, other existing methods would require this knowledge to enable the cancellation of noise when the sound field inside the protected region is altered by the scatterer.

Figure 15 shows the sound pressure distribution inside the shielded domain. The ■ symbols denote the sound pressure amplitude before the control sources are turned on, and the △ symbols

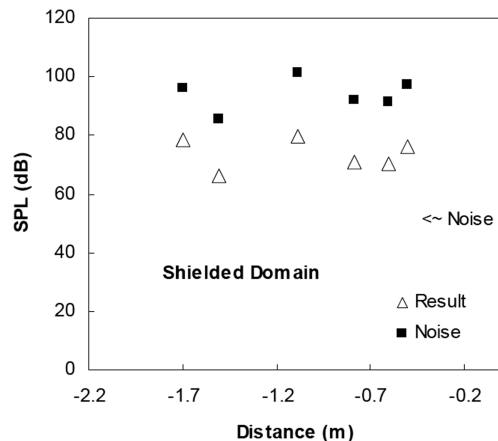


Fig. 15 Sound pressure distribution measured at 350 Hz along the axis of the duct including an obstacle inside the shielded domain.

show the reduced sound pressure over a large area inside the shielded domain when the control sources are switched on. We see that the DPM-based controls enable the abatement of noise at the level of approximately 21 dB at 350 Hz.

Preserving the Wanted Sound

One of the most important distinctions between the DPM-based AS approach and the conventional methods available in the literature is that the DPM can keep the wanted sound inside the protected region unchanged and cancel out only the noise coming from outside the shielded domain. Recall that the wanted sound is defined as the component of the overall acoustic field attributed to the sources inside the shielded domain. The main objective of the experiment described in this section is to validate the premise that the DPM-based controls are indeed capable of automatically preserving the wanted sound component.

As shown in Fig. 16, the unwanted noise propagates from the right to the left in the duct. In addition, a wanted sound component is generated inside the shielded domain. The precise nature and location of the sources of wanted sound do not need to be known for building the DPM-based controls, and as a matter of convenience, we have this sound generated by a loudspeaker mounted at the left endpoint of the tube. The control sources for AS are placed at the boundary of the protected region. Experiments are carried out at 200 and 500 Hz. To determine the strength of the control sources, the sound pressure and particle velocity of the total acoustic field (the sum of the adverse noise and wanted sound) are measured at the boundary. Then the strength of the acoustic monopole and dipole is derived as shown in the preceding sections, using Eqs. (14), (16), and (17). The key point is that there is no need to distinguish between the wanted sound and the noise explicitly in the measurements. This is possible because the sources of the wanted sound and unwanted sound are on different sides of the boundary of the shielded domain. The measurement of the particle velocity at the boundary is able to capture this directional information inherently. When the control devices are applied, the dipole source provides the necessary directional element that allows the cancellation of sound from one direction (the unwanted sound) but not the other (the wanted sound).

In the experiment, the wanted sound pressure is 110.85 dB at 500 Hz at a reference position 0.5 m away from the termination of the left-hand side in the shielded domain when the noise and the control sources are switched off. The total sound pressure at the same position, when both the wanted sound and the noise are on, is 114.92 dB. When the control sources are activated, the total sound pressure measured is reduced back to 110.92 dB at the same reference position inside the shielded domain. These results show that the original wanted sound is kept practically unaltered, whereas the unwanted noise is cancelled out inside the protected region by the DPM-based active controls.

A more thorough experiment to verify that the wanted sound does not get changed by the DPM-based active controls is performed at a lower frequency of 200 Hz. Unlike the previous case, in this experiment we measure the field at several locations inside the shielded domain and present the results in Fig. 17. The total sound pressure of the noise and wanted sound at the boundary of the shielded domain is 97.44 dB. The strength of the control sources is determined through the measurements at the boundary, as before. For the same reference position as in the previous experiment, 0.5 m from the left boundary of the shielded domain, the total sound pressure

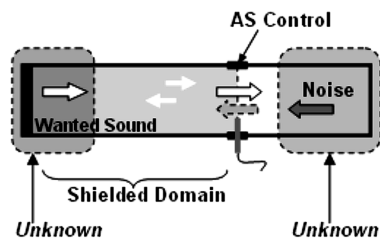


Fig. 16 Noise cancellation and preservation of wanted sound.

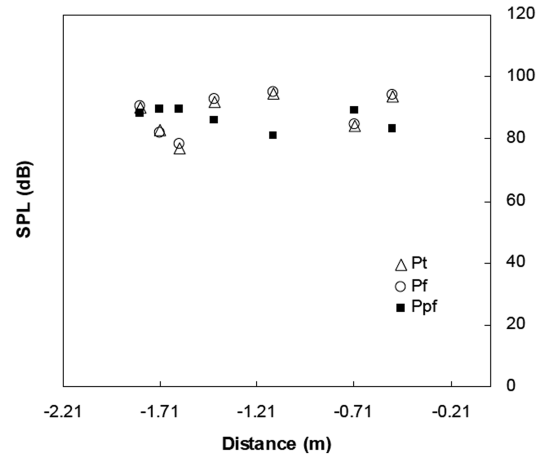


Fig. 17 Preservation of wanted sound at 200 Hz in the shielded domain; ○: wanted sound pressure, △: total sound pressure (the sum of noise, wanted sound, and control output), and ■: the sound pressure of noise and wanted sound without control.

becomes 90.08 dB once the control sources are activated. With no noise and no control, the wanted sound measured at the same position is 90.63 dB. The overall distribution of sound pressure at multiple locations inside the shielded domain for the case with wanted sound is shown in Fig. 17.

The AS control sources are mounted at $x = 0$ and the duct ends are at $x = -2.2$ m with rigid termination. The ■ symbols in Fig. 17 show the initial sound pressure distribution when the noise and wanted sound are both on, whereas the control sources are still off. The △ symbols represent the distribution of the net sound pressure when the noise is canceled out by the AS control sources. The net sound pressure △ can be compared with the wanted sound alone, shown by the ○ symbols in Fig. 17. The net sound pressure after the DPM-based control coincides almost exactly with the wanted sound pressure at each of the measurement positions in the shielded domain. This result proves that the wanted sound is left unaltered, whereas the noise is entirely cancelled out by the AS control sources. Note that the experiment was performed at a single frequency. Interferences between noise and the wanted sound field can cause the SPL of the combined sound field to increase or decrease depending on position. Therefore, the SPL of the total field of noise and wanted sound combined can be lower or higher than the wanted sound alone at the various measuring positions shown in the figure.

In our last test, we consider the case when there is no noise and only the wanted sound is present. The idea is to demonstrate that the DPM-based controls will change nothing in the acoustic field inside the protected region. It is clear that formulas (14) for the control sources still apply in this case. The experimental setting is similar to that used in the previous test (see Fig. 16). In the experiment, the total pressure of the control and wanted sound adds up to 112.39 dB at the reference position (see Fig. 18) if the AS control sources are turned on. The wanted sound measured separately at the same position inside the shielded domain equals 112.18 dB. The result of the experiment shows that about 97% of the wanted sound is preserved by the DPM-based AS solution. Note that even though there is no noise, the AS system does not get turned off and is still working in

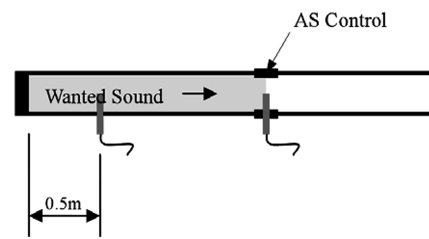


Fig. 18 Preservation of wanted sound without noise.

this case, because the input for the controls is provided by the total field. Because of the special structure of the DPM-based sources, they do not affect the acoustic field inside the shielded domain.

Current Limitations and Future Work

Similar to other active control technologies, the method suffers from the usual limitations of sound cancellation, such as those arising from phase errors at very low and very high frequencies and errors introduced by the finite sizes and positioning of transducers. Moreover, the current set of experiments is also limited to single-frequency measurements in a one-dimensional sound field in a tube. The frequencies were chosen to avoid the resonance frequencies of the tube. The reason for choosing such a simple configuration is to allow unambiguous validation of the fundamental properties of the DPM-based active shielding approach and to allow a clear demonstration of its unique capability over existing methods in cases of unknown boundary conditions and in the presence of wanted sound. The simple configurations tested are, however, far from the intended eventual applications: for example, real aircraft cabin noise. Theoretically, the DPM-based approach has been shown to be able to deal with these real problems if they are linear. To prove this in the laboratory, we will need to extend the experiments to cover the following challenging aspects: 1) broadband sound fields, including multiple resonance regions; 2) 3-dimensional sound fields, including the design, installation, and positioning of control sources for 3-dimensional spaces; 3) sensitivity of control on installation errors; and 4) design and implementation of real-time adaptive controllers.

Currently, we are concentrating our effort on the first two aspects: that is, broadband control in a 3-D sound field. For the 3-D case, 2-D arrays of actuators and microphones are required on the boundary surfaces to realize the shielding of a 3-D volume. The key factors in the realization of the 3-D AS are the physical size, number, calibration, and positioning of the control sources (actuators) and monitoring microphones. They must be optimized to achieve the maximally close agreement with the theoretical predictions. We are currently finalizing our experiment for a broadband 3-D case. The results and full discussion will be presented in a follow-up paper in the near future. Points 3 and 4 will be investigated later.

Conclusions

The main focus of our study was the experimental validation of the design properties of the active noise control technique based on the method of difference potentials. In the paper, we have addressed the case of noise abatement on bounded regions. For a given region to be protected, the technique based on the difference potential method allows for the exact volumetric cancellation of noise generated outside of this region, leaving the sound generated inside unaffected. The only inputs required by the control system are the acoustic quantities measured on the perimeter of the protected domain. These quantities can pertain to the overall field composed of both the adverse noise and the wanted sound, and the methodology will automatically distinguish between the two. Furthermore, no information is required regarding either the properties of the sound-conducting medium (including reflection coefficients that characterize the domain termination) or the structure and characteristics of the original noise sources. The series of experiments that we have planned and carried out unambiguously corroborate all the theoretical design features of the methodology based on the difference potential method. In particular, it has been shown that even though only the total field is measured at the boundary and its adverse component alone remains unknown, the methodology can automatically discriminate between the sound and noise and subsequently tackle the noise only, leaving the sound unaltered. This feature is unparalleled by other approaches to active noise control available in the literature.

Future research will focus on the development and study of the active shielding solution under resonance conditions and on the extension of the method to the case of broadband spectra of frequencies.

Acknowledgments

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