

# EVOLUTIONARY SEARCH IN LETHAL ENVIRONMENTS

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**Abstract:** In Natural evolution, a mutation may be *lethal*, causing an abrupt end to an evolving lineage. This fact has a tendency to cause evolution to “prefer” mutationally robust solutions (which can in turn slow innovation), an effect that has been studied previously, especially in the context of evolution on neutral plateaux. Here, we tackle related issues but from the perspective of a practical optimization scenario. We wish to evolve a finite population of entities quickly (i.e. improve them), but when a lethal solution (modelled here as one below a certain fitness threshold) is evaluated, it is immediately removed from the population and the population size is reduced by one. This models certain closed-loop evolution scenarios that may be encountered, for example, when evolving nano-technologies or autonomous robots. We motivate this scenario, and find that evolutionary search performs best in a lethal environment when limiting randomness in the solution generation process, e.g. by using elitism, above-average selection pressure, a less random mutating operator, and no or little crossover. For  $NK\alpha$  landscapes, these strategies turn out to be particularly important on rugged and non-homogeneous landscapes (i.e. for large  $K$  and  $\alpha$ ).

## 1 INTRODUCTION

In this paper, we consider the use of evolutionary algorithms (EAs) for optimization in a setting where the notions of an individual and of a population size, are slightly different from standard, which forces us to reconsider how to configure an EA appropriately. The general setting is closed-loop optimization (Klockgether and Schwefel, 1970; Harvey et al., 1996; Rechenberg, 2000; Shir et al., 2007; Calzolari et al., 2008; Allmendinger and Knowles, 2011), which is an area in resurgence, and likely to expand further in future. In these type of problems, candidate solutions (genotypes) to an optimization problem are planned on a computer but their phenotypes are realized or prototyped using experiments and/or real hardware; the process of measuring the fitness (or quality) of the phenotype may also involve conducting an experiment and/or the use of hardware. There are several ways in which this kind of setup may cause difficulties, and there has been recent work (Allmendinger and Knowles, 2011) considering some aspects of closed-loop evolution related to dynamic constraints and the dynamic availability of resources. Here, our concern is for a particular sort of closed-loop setting where the hardware on which

individuals are tested are reconfigurable, destructible and non-replaceable.

Consider the following scenario. We are developing, by evolution, the control software for an autonomous endoscopic robot (similar to Moglia et al. (2007); Ciuti et al. (2010)), which is ultimately intended to be swallowed by a patient in capsule form, and then used within the body for screening, diagnosis and therapeutic procedures. Before robotic capsules are used on humans, however, their reliability and effectiveness typically needs to be first validated through in vivo animal trials. Imagine we have available  $\mu$  prototypes of a robotic capsule, and we can “radio in” new control software to each of these individual capsules and evaluate them. However, our control software may cause a capsule to malfunction in a lethal (for it) way, in which case it is no longer available (and falls out of the evolving population). In such a scenario, our population size for the remainder of the evolution is now at most  $\mu - 1$ , in terms of how many pieces of software can be tested each generation. If we continue to explore too aggressively, we may upload other pieces of software that cause the loss of individuals, which will cost evolution in terms of the extent of parallelism (and hence efficiency in time) we enjoy, and also perhaps in the loss of expen-

sive hardware. So whilst we wish to innovate by testing new control policies, this must be balanced carefully against considerations of maintaining a reasonable population size.

This setting may seem far-fetched to some readers. However, as we see evolution being applied more and more at the boundary between a full embodied type of evolution (Watson et al., 2002; Zykov et al., 2005) (where the hardware individuals may reproduce or exchange information) and a fully in silico simulation-only approach, especially as EAs are taken up in the experimental sciences more and more, it is likely that variants of the above scenario will emerge. It need not be robotic capsules; it could equally be nano-machines or drone planes we are dealing with; it could be robot swarms, or software released on the Internet, or anything that is reconfigurable remotely in some way, and also destructible.

Clearly, the principle of “survival of the fittest” is going to be somewhat dampened by the constraint that we wish all designs to survive. Indeed, no innovation at all will be possible if we enforce the evaluation of only safe designs, given the assumption that any unseen design could be a lethal one. So, as is usual with evolution, we should expect that some rate of loss of individuals is going to be desirable, but set against this is the fact that they cannot be replaced, and hence population size and parallelism will be compromised. So, the question is, how should one control the exploration/exploitation tradeoff in this optimization setup.

In more Natural settings than we consider here, the notion of robustness and the tendency of evolution to build in mutational robustness for free has been previously studied (Bullock, 2003; Schonfeld, 2007). Bullock’s work in particular shows that certain in-built biases toward mutational robustness can retard innovation (our primary concern here). He proposed methods that avoid the bias and hence explore neutral plateaux more rapidly. This work has a different purpose to ours, and involves different assumptions, but there are also obvious parallels which we hope will be apparent from our results.

Our more practical investigation of the effects on optimization performance of lethal mutations looks at some of the simple EA configuration parameters: degree-of-elitism, population size, selection pressure, mutation mode and strength, and crossover rate. We also vary aspects of the fitness landscape we are optimizing to observe which landscape topologies pose a particular challenge when optimizing in a lethal environment.

The rest of the paper is organized as follows. The next section describes the experimental setup including the search algorithms considered (Section 2.1)

and the family of binary test problems ( $NK\alpha$  landscapes) we use to analyze the impact of lethal solutions (Section 2.2). Experimental results are given in Section 3; we draw conclusions in Section 4. Finally, Section 5 outlines possible directions for future work.

## 2 EXPERIMENTAL SETUP

This section describes the family of binary test functions  $f$  we consider, the search algorithms for which we investigate the impact of a lethal optimization scenario, and the parameter settings as used in the subsequent experimental analysis.

### 2.1 Search Algorithms

We consider three types of search algorithms in this work: a tournament selection based genetic algorithm (TGA), a modified version of it which we are calling RBS (standing for *reproduction of best solutions*), and a population of stochastic hill-climbers (PHC). Similar types of algorithms have been considered in previous studies related to mutational robustness (see e.g. Bullock (2003); Schonfeld (2007)).

Before we describe each algorithm, we first set out the procedures common to all of them. These are the generation process of the initial population of candidate solutions, the setting of the fitness threshold below which solutions are deemed lethal, the mutation operators, and the handling of duplicate solutions. Regarding the initialization, our aim is to simulate the scenario where the initial population, whose size we denote by  $\mu_0$ , consists of evolving entities (e.g. robotic capsules, robot swarms or software) that are of a certain (high or state-of-the-art) quality. We achieve this by first generating a sample set of  $S$  random and non-identical candidate solutions, and then selecting the fittest  $\mu_0$  solutions from this set to form the initial population. The lethal fitness threshold  $f_{LFT}$  is set to the fitness value of the  $q$ th fittest solution of the sample set. This threshold is kept constant throughout an algorithmic run.

For mutation, we will investigate two modes (similar to Barnett (2001)): (i) *Poisson mutation*, where each bit is flipped independently with a mutation rate of  $p_m$ , and (ii) *constant mutation*, where exactly  $d$  randomly selected bits are flipped. Furthermore, all search algorithms ensure that a solution is not evaluated multiple times. That is, all evaluated solutions are cached and compared against a new solution (offspring) before performing an evaluation; preliminary experimentation showed that avoiding duplication improves the performance significantly. If a solution

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**Algorithm 1** Tournament selection based GA (TGA)

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**Require:**  $f$  (objective function),  $G$  (maximal number of generations),  $\mu_0$  (initial parent population size),  $\lambda_0$  (initial offspring population size),  $L$  (maximal number of regeneration trials)  
 $g = 0$  (generation counter),  $Pop = \emptyset$  (current population),  $OffPop = \emptyset$  (offspring population),  $AllEvalSols = \emptyset$  (set of solutions evaluated so far),  $trials = 0$  (regeneration trials counter),  $counter = 0$  (auxiliary variable indicating the number of solutions evaluated during a generation)

Initialize  $Pop$  and set lethal fitness threshold  $f_{LFT}$ ; copy all solutions of  $Pop$  also to  $AllEvalSols$ , and set  $\mu_g = \mu_0, \lambda_g = \lambda_0$

```
while  $g < G \wedge \mu_g > 0$  do
   $OffPop = \emptyset, trials = 0, counter = 0$ 
  Set mutation rate to initial value
  while  $counter < \lambda_g$  do
    Generate two offspring  $\vec{x}^{(1)}$  and  $\vec{x}^{(2)}$  by selecting
    two parents from  $Pop$ , and then recombining and
    mutating them
    for  $i = 1$  to  $2$  do
      if  $\vec{x}^{(i)} \notin AllEvalSols \wedge counter < \lambda_g$  then
        Evaluate  $\vec{x}^{(i)}$  using  $f$ ,  $counter++$ ,  $trials = 0$ 
         $AllEvalSols = AllEvalSols \cup \vec{x}^{(i)}$ 
        if  $f(\vec{x}^{(i)}) \geq f_{LFT}$  then
           $OffPop = OffPop \cup \vec{x}^{(i)}$ 
        else  $trials++$ 
      if  $trials = L$  then
        Depending on the mutation operator, reset
        mutation rate to  $p_m = p_m + 0.5/N$  (Poisson
        mutation) or  $d = d + 1$  (constant mutation)
     $g++$ 
  Reset new population sizes to  $\mu_g = \lambda_g = |OffPop|$ , and
  form new population  $Pop$  by selecting the best  $\mu_g$  so-
  lutions from the union population of  $Pop \cup OffPop$ 
```

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has been evaluated previously, then the GA iteratively generates new solutions (offspring solutions) until it generates one that is evaluable, i.e. has not been evaluated previously, or until  $L$  trials have passed without success. In the latter case, we reset the mutation rate to  $p_m = p_m + 0.5/N$  ( $N$  is the total number of bits) and  $d = d + 1$  in the case of Poisson mutation and constant mutation, respectively; note, due to its deterministic nature, it is more likely that this reset step is applied with constant mutation. For TGA and RBS we set the mutation rates back to their initial values at the beginning of each new generation; for PHC it makes more sense to set the mutation rates back whenever a hill-climber of the current population is considered the first time for mutation (see pseudocode of PHC).

### 2.1.1 Tournament Selection Based GA (TGA)

The algorithm uses tournament selection with replacement for parental selection ( $TS$  shall denote the

tournament size) and uniform crossover (Syswerda, 1989) ( $p_c$  shall denote the crossover rate). Our TGA uses a  $(\mu + \lambda)$ -ES reproduction scheme, an elitist approach, which performed significantly better than a standard generational reproduction scheme in preliminary experimentation, and which we believe is generally applicable in this domain. We set the number of offspring solutions to be identical to the population size (or the number of parent solutions), i.e.  $\mu = \lambda$ . As the population size may decrease during the optimization process (due to evaluated lethal solutions), in future, we will denote the population size at generation  $g$  ( $0 \leq g \leq G$ ) by  $\mu_g = \lambda_g$  with  $g = G$  being the maximum number of generations. Clearly, the maximum number of generations can only be reached if the population does not “die out” beforehand. Algorithm 1 shows the pseudocode of TGA.

RBS is based on TGA and motivated by the *netcrawler* (Barnett, 2001), an algorithm that reproduces only a single best solution at each generation. RBS selects parents for reproduction exclusively and at random from the set of solutions of the current population with the highest fitness. Hence, it can be seen as a tournament selection based GA with  $TS = \mu_g$  and where the tournament participants are selected *without replacement*. Apart from the selection procedure, RBS will use the same algorithmic setup as TGA.

### 2.1.2 Population of Hill-Climbers (PHC)

This search algorithm maintains a population of stochastic hill-climbers, which explore the landscape independently of each other. Each hill-climber undergoes mutation and, if the mutant or offspring is not lethal, it replaces the original solution if it is at least as fit. Algorithm 2 shows the pseudocode of PHC. In essence, PHC with constant mutation is identical to running a set of random-mutation hill-climbers (Forrest and Mitchell, 1993) in parallel with the difference that, as in the case of TGA, the mutation strength is temporarily increased if a previously unseen solution cannot be generated within  $L$  trials. PHC has also similarities to the *netcrawler* (Barnett, 2001) and the *plateau crawler* (Bullock, 2003). These approaches have in common that they guarantee that each parent potentially leaves a copy or a slightly modified version of itself in the population. This ensures that offspring, parents, or other ancestors, are equally likely to be selected as future parents. For fitness landscapes featuring neutral networks, this feature can cause a population to be unaffected by mutational robustness as it allows the population to spend at each point on a neutral network an equal amount of time (Hughes, 1995; Bullock, 2003).

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**Algorithm 2** Population of hill-climbers (PHC)

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**Require:**  $f$  (objective function),  $G$  (maximal number of generations),  $\mu_0$  (initial parent population size),  $L$  (maximal number of regeneration trials)  
 $g = 0$  (generation counter),  $Pop = \emptyset$  (current population),  $OffPop = \emptyset$  (offspring population),  $AllEvalSols = \emptyset$  (set of solutions evaluated so far),  $trials = 0$  (regeneration trials counter),  $lethal = \text{false}$  (auxiliary boolean variable indicating whether a hill-climber is lethal)

Initialize  $Pop$  and set lethal fitness threshold  $f_{LFT}$ ; copy all solutions of  $Pop$  also to  $AllEvalSols$ , and set  $\mu_g = \mu_0$

```
while  $g < G \wedge \mu_g > 0$  do
   $OffPop = \emptyset, trials = 0$ 
  for  $i = 1$  to  $\mu_g$  do
    Set mutation rate to initial value,  $lethal = \text{false}$ 
    repeat
      Mutate solution  $\vec{x}_i$  of  $Pop$  to obtain mutant  $\vec{x}'_i$ 
      if  $\vec{x}'_i \notin AllEvalSols$  then
        Evaluate  $\vec{x}'_i$  using  $f$ ,  $trials = 0$ 
         $AllEvalSols = AllEvalSols \cup \vec{x}'_i$ 
        if  $f(\vec{x}'_i) \geq f_{LFT}$  then
          if  $f(\vec{x}'_i) \geq f(\vec{x}_i)$  then
             $OffPop = OffPop \cup \vec{x}'_i$ 
          else  $OffPop = OffPop \cup \vec{x}_i$ 
          else  $lethal = \text{true}$ 
        else  $trials++$ 
      if  $trials = L$  then
        Depending on the mutation operator, reset
        mutation rate to  $p_m = p_m + 0.5/N$  (Poisson
        mutation) or  $d = d + 1$  (constant mutation)
      until  $\vec{x}'_i$  is evaluable  $\vee lethal = \text{true}$ 
     $g++$ 
  Reset new population size to  $\mu_g = |OffPop|$ , and form
  new population  $Pop$  by copying all solutions from
   $OffPop$  to  $Pop$ 
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## 2.2 Test Functions $f$

Our aim in this study is to understand the effect of lethal solutions on EA performance on real closed-loop experimental problems (ultimately). Hence, it might be considered ideal to use, for testing, some set of real-world closed-loop optimization problems subject to lethal solutions, that is: real experimental problems featuring real resources that may potentially be fatally damaged. That way we could see the effects of EA design choices directly on a real-world problem of interest. But even granting this to be an ideal approach, it would be very difficult to achieve in practice due to the inherent cost of conducting closed-loop experiments and the difficulty of repeating them to obtain any statistical confidence in results seen. For this reason, our study will use more familiar *artificial* test problems augmented with the possibility of encountering lethal candidate solutions.

The family of test functions we consider here

Table 1: Default parameter settings of search algorithms.

Algorithm	Parameter	Setting
TGA, PHC, RBS	Parent population size $\mu_0$	30
	Constant mutation rate $d$	1
	Regeneration trials $L$	1000
	Number of generations $G$	200
	Sampling size $S$	1000
	Solution rank $q$ for lethal fitness threshold setting	250
TGA	Offspring population size $\lambda_0$	30
	Tournament size $TS$ (Selection with replacement)	4
	Crossover rate $p_c$	0.0
RBS	Offspring population size $\lambda_0$	30
	Tournament size $TS$ (Selection without replacement)	$\mu_g$
	Crossover rate $p_c$	0.0

is a variant of  $NK$  landscapes called  $NK\alpha$  landscapes (Hebbron et al., 2008). The use of this test problem should help us to gain initial insights into the effects of lethal solutions on fitness landscapes featuring various degrees of epistasis and ruggedness.

### 2.2.1 $NK\alpha$ Landscapes

The general idea of the  $NK\alpha$  model (Hebbron et al., 2008) is to extend Kauffman’s original  $NK$  model (Kauffman, 1989) to model epistatic network topologies that are more realistic in mapping the epistatic connectivity between genes in natural genomes. The  $NK\alpha$  model achieves this by affecting the distribution of influences of genes in the network in terms of their connectivity, through a preferential attachment scheme. The model uses a parameter  $\alpha$  to control the positive feedback in the preferential attachment so that larger  $\alpha$  result in a more non-uniform distribution of gene connectivity. There are three tunable parameters involved in the generation of an  $NK\alpha$  landscape: the total number of variables  $N$ , the number of variables that interact epistatically at each of the  $N$  loci,  $K$ , and the model parameter  $\alpha$  that allows us to specify how influential some variables may be compared to others. As  $\alpha$  increases, an increasing influence is given to a minority of variables, while, for  $\alpha = 0$ , the  $NK\alpha$  model reduces to Kauffman’s original  $NK$  model with neighbors being selected at random; this model has already been used previously to analyze certain aspects of real-world closed-loop problems (see e.g. Thompson (1996)).

## 2.3 Parameter Settings

The experimental study will investigate different settings of the parameters involved in the search algo-

rithms. However, if not otherwise stated, we use the default settings as given in Table 1 with constant mutation being the default mutation operator. Remember that RBS uses the same default parameter settings as TGA with the difference that reproductive selection is done with  $TS = \mu_g$  and tournament participants are chosen without replacement.

For  $NK\alpha$  landscapes, we analyze the impact of lethal solutions using different settings of the neighborhood size  $K$  and the model parameter  $\alpha$ . We fix the search space size to  $N = 50$ , which corresponds to a typical search space size we have seen in related closed-loop problems (e.g. a drug discovery problem with a library of 30–60 drugs (Small et al., 2011)).

Any results shown are average results across 100 independent algorithm runs. We will use a different randomly generated problem instance for each run but, of course, the same instances for all algorithms. Furthermore, to allow for a fair comparison, we also use the same initial population and (hence) lethal fitness threshold for all algorithms in any particular run.

### 3 EXPERIMENTAL RESULTS

Before we analyze the effect of lethal solutions on evolutionary search, we first give some indication of the properties of the  $NK\alpha$  landscapes, and the effect of varying  $K$  and  $\alpha$  for ( $N=50$ ) over the ranges used in our experiments. For this, we follow the experimental methods employing *adaptive walks* already used by Hebborn et al. (2008). Starting from a randomly generated solution, an adaptive walk calculates the fitness of all neighbors of the solution that can be generated with a single mutation step, and selects one of the fitter neighbors at random to move to. If there is no fitter neighbor, then the walk has reached a local optima and terminates. We have performed 10000 adaptive walks for different values of  $K$  and  $\alpha$ , and recorded the length of each walk and the genotype of the local optima where each walk terminated. We have repeated this process 10 times for different values of  $K$  and  $\alpha$  using always a different randomly generated landscape. Figure 1 shows the average adaptive walk length (top plot) and the average number of unique (local) optimal solutions (bottom plot). In essence, this figure shows that increasing  $K$  reduces the average length of an adaptive walk, i.e. the landscapes becomes more rugged, and at the same time increases the probability that the EA gets trapped at a new local optimal solution. On the other hand, increasing  $\alpha$  reduces the ruggedness of the fitness landscapes and decreases the number of unique local optimal solutions. As is apparent from the figure, the parameters  $K$  and

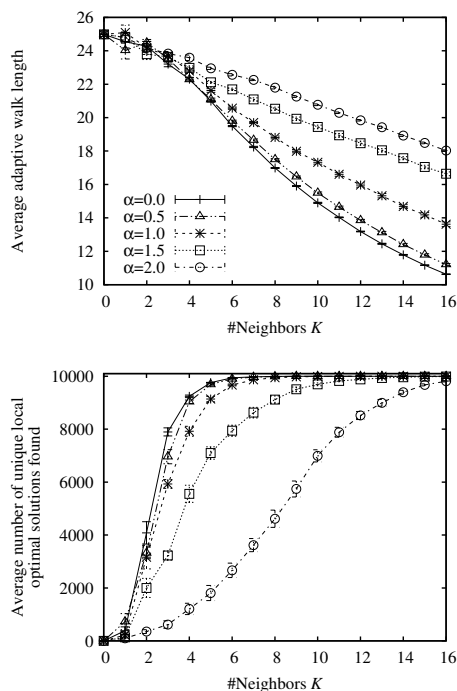


Figure 1: Plots showing the average length (top) and the average number of unique local optimal solutions found (bottom) across 10 repetitions of 10000 adaptive walks for different neighborhood sizes  $K$  and model parameter values  $\alpha$ .

$\alpha$  allow us to cover a reasonable mixture of fitness landscapes exhibiting different topologies or levels of ruggedness and epistasis.

Figure 2 shows how a lethal optimization scenario affects the best solution fitness found (top plots) and the (remaining) population size (bottom left plot) for TGA with a rather traditional setup ( $TS = 2$ ,  $p_c = 0.0$  and Poisson mutation with  $p_m = 1/N$ ) as a function of the parameters  $K$  and  $\alpha$ ; the bottom right plot indicates the scope available for optimization after the initialization of the population. From the top plots we can see that the performance is most affected for fitness landscapes that are rugged (large  $K$ ) but contain some structure in the sense that some solution bits are more important than others (large  $\alpha$ ). As is apparent from the bottom left plot, the reason for this pattern is that the population size at the end of the optimization decreases as  $K$  and/or  $\alpha$  increase, i.e. lethal solutions are here more likely to be encountered. In fact, for large  $K$  and  $\alpha$ , the optimization terminates on average after only about 25 generations (out of 200). The reason that the population size reduces for large  $K$  is that the fitness landscape becomes more rugged, which increases the risk of evaluating lethal solutions despite them being located close to high-quality solutions. Although an increase in  $\alpha$  introduces more structure into the fitness landscape, it has also the effect that so-

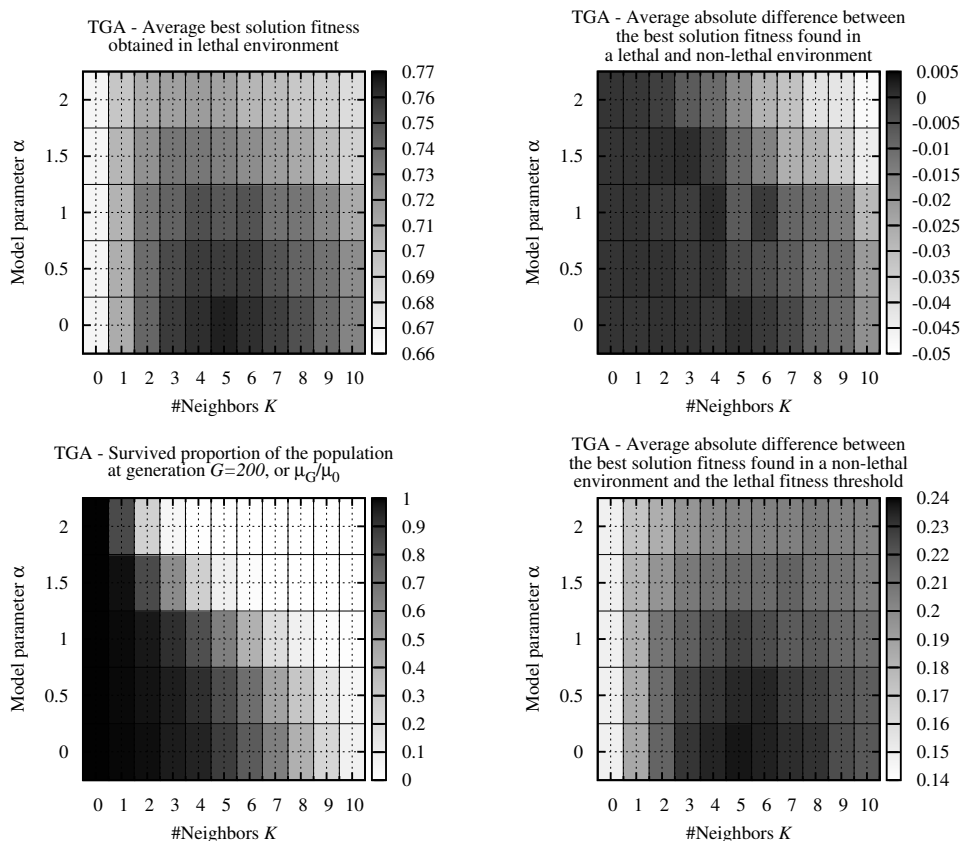


Figure 2: Plots showing the average best solution fitness found by TGA (using  $TS = 2$ ,  $p_c = 0.0$  and Poisson mutation with  $p_m = 1/N$ ) in a lethal environment (top left), the average absolute difference between this fitness and the fitness achieved in a non-lethal environment, i.e.  $f_{LFT} = 0$  (top right), the average proportion of the population that survived at generation  $G = 200$ , or  $\mu_G/\mu_0$  (bottom left), and the average absolute difference between the best solution fitness found and the lethal fitness threshold in a non-lethal environment (bottom right), on  $NK\alpha$  landscapes as a function of the neighborhood size  $K$  and the model parameter  $\alpha$ .

lutions which have some of these important bits set incorrectly are more likely to be poor. A plausible alternative explanation for the poor performance at large  $\alpha$  may be that the lethal fitness threshold is very close to the best solution fitness that can be found, thus reducing the scope of optimization possible and causing many solutions to be lethal; the bottom right plot rules this explanation out. Regarding the (remaining) population size, a greater selection pressure, i.e. a larger tournament size, can help to maintain a large population size for longer, particularly for large  $K$  and small  $\alpha$ . On the other hand, being more random in the solution generation process by using, for example, a larger mutation and/or crossover rate, has the opposite effect, i.e. the population size decreases more quickly.

The effect on the (remaining) population size translates into an impact on the average best solution fitness. In fact, from Figure 3, we can see that, for rugged landscapes with structure (i.e. in the range  $K > 5, \alpha > 1$ ), TGA is outperformed by a GA with

a rather traditional setup and a PHC in a non-lethal optimization scenario (right plots) but performs better than both algorithms in a lethal optimization scenario (left plots). In fact, as we will also see later, in the range  $K > 5, \alpha > 1$ , PHC performs best among all the search algorithms considered in the non-lethal environment but worst in the lethal environment. The poor performance of PHC in this range is due to the fact that the hill-climbers in the population are searching the fitness landscape independently of each other. This may be an advantage in a non-lethal environment because it can help to maintain diversity in the population and prevent premature convergence. However, in a lethal environment, uncontrolled diversity is rather a drawback as it increases the probability of generating solutions with genotypes that are significantly different from the ones in the population, which in turn increases the probability of generating lethal solutions. The less diverse and random optimization of TGA is also the reason that it outperforms a GA with a more

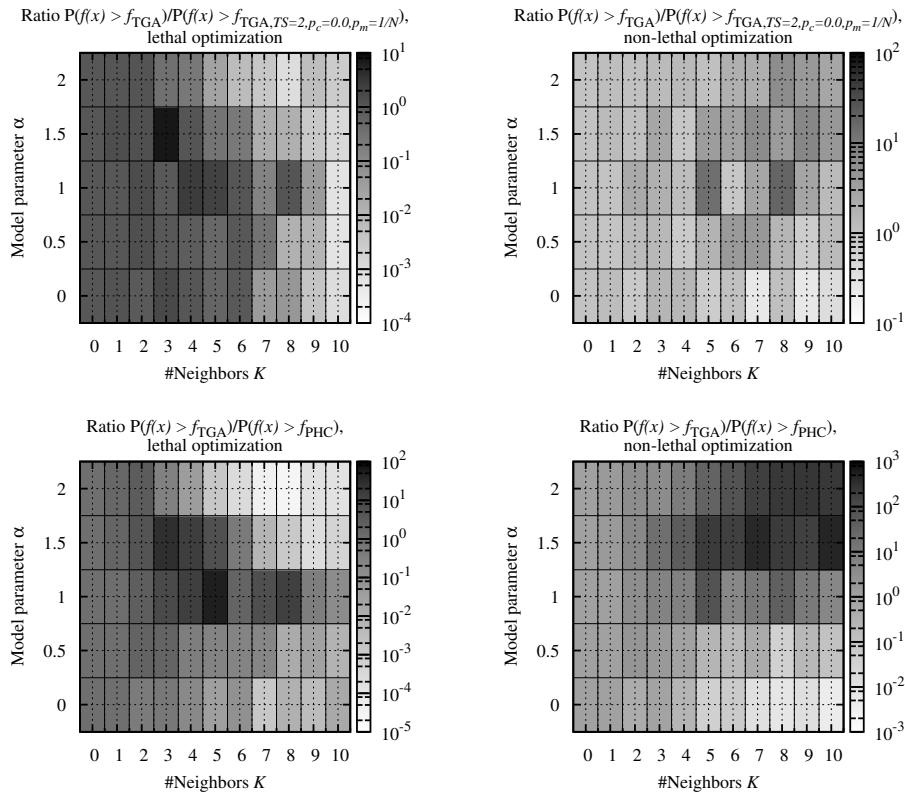


Figure 3: (Top) Plots comparing the relative performance of TGA (default parameters) and TGA with  $TS = 2, p_c = 0.0, p_m = 1/N$ , in a lethal (top left) and non-lethal environment (top right). Darker shades indicate that TGA with  $TS = 2, p_c = 0.0, p_m = 1/N$  performs better. (Bottom) Plots comparing the relative performance of TGA and PHC (both using default parameters) in a lethal (bottom left) and non-lethal environment (bottom right). Darker shades indicate that PHC performs better. Performance is plotted as a function of  $K$  and  $\alpha$  in all cases. Relative performance is calculated as  $P(f(x) > f_A) / P(f(x) > f_B)$  where  $P$  is a sample probability estimated from 10000 samples and  $f(x)$  is a random draw from the search space, while  $f_A$  and  $f_B$  represent the best fitness obtained by the two algorithms.

traditional setup on rugged landscapes with structure.

Let us now analyze how the search algorithms fare with different parameter settings. Table 2 shows the average best solution fitness obtained by different algorithm setups in a lethal and non-lethal environment (values in parenthesis) on  $NK\alpha$  landscapes with  $K = 4, \alpha = 2.0$  (top table) and  $K = 10, \alpha = 2.0$  (bottom table), respectively. Note that due to the nature of  $NK\alpha$  landscapes (particularly due to the absence of plateaus or neutral networks in the landscapes), there was only a single best solution in the population at each generation. For RBS, this has the effect that the performance is independent of the crossover rate because crossover is applied to identical solutions. The two tables largely confirm the observation made above: while a relatively high degree of population diversity and randomness in the solution generation process may be beneficial in a non-lethal environment (which is particularly true for rugged landscape as can be seen from the bottom table), it is rather a drawback in a lethal environment as it may increase the

risk of generating lethal solutions and subsequently limit the effectiveness of evolutionary search. More precisely, for a lethal environment, the tables indicate that one should use a GA instead of a PHC, and reduce randomness in the solution generation process by avoiding crossover (generally), and using constant mutation as well as a relatively large tournament size. The poor performance of Poisson mutation, which is a typical mutation mode for GAs, is related to situations where an over-averaged number of solution bits is flipped at once. Such variation steps lead to offspring that are located in the search space far away from their non-lethal parents, which can in turn increase the probability of generating lethal solutions.

Finally, Figure 4 analyzes whether evolving a large set of entities for a small number of generations yields better performance than evolving only a few entities for many generations; i.e. how is the trade-off between the initial population size  $\mu_0$  and the maximum number of generations  $G$ . For a non-lethal environment (right plots), we make two obser-

Table 2: The table shows the average best solution fitness found in a lethal environment after  $G = 200$  generations, and in parenthesis, the fitness found in a non-lethal environment, for different algorithm setups on  $K = 4, \alpha = 2.0$  (top) and  $K = 10, \alpha = 2.0$  (bottom); the results of *random sampling* were obtained by generating  $G \times \mu_0 = 200 \times 30 = 6000$  solutions per run at random and averaging over the best solution fitness values found. The numbers in the subscript indicate the rank of the top five algorithm setups within the respective environment and fitness landscape configuration. We highlighted all algorithm setups (among the top five) in bold face that are not significantly worse than any other setup. A Friedman test revealed a significant difference between the search algorithm setups in general, but differences among the individual setups were tested for in a post-hoc analysis using (paired) Wilcoxon tests (significance level of 5%) with Bonferroni correction.

$K = 4, \alpha = 2.0$						
		Constant mutation		Poisson mutation		
		$d = 1$	$d = 2$	$p_m = 0.5/N$	$p_m = 1.0/N$	
TGA	TS = 1	$p_c = 0.0$	<b>0.722<sub>4</sub></b> (0.7297 <sub>4</sub> )	0.6998 (0.7286)	0.7174 (0.7272)	0.7141 (0.7285)
		$p_c = 0.25$	0.7188 (0.7272)	0.6969 (0.7296 <sub>5</sub> )	0.7128 (0.7275)	0.7029 (0.7296)
		$p_c = 0.5$	0.713 (0.7279)	0.6901 (0.7287)	0.7091 (0.7276)	0.7078 (0.7277)
	TS = 2	$p_c = 0.0$	<b>0.7223<sub>3</sub></b> (0.7269)	0.7053 (0.7287)	0.7198 (0.7271)	0.7175 (0.7275)
		$p_c = 0.25$	0.7217 (0.7263)	0.7039 (0.7284)	0.7172 (0.7263)	0.7119 (0.7275)
		$p_c = 0.5$	0.7178 (0.7261)	0.7003 (0.728)	0.7176 (0.7268)	0.7135 (0.727)
	TS = 4	$p_c = 0.0$	<b>0.7218<sub>5</sub></b> (0.7266)	0.7107 (0.726)	0.7203 (0.7269)	0.7184 (0.7269)
		$p_c = 0.25$	<b>0.7238<sub>1</sub></b> (0.7257)	0.7085 (0.7254)	0.7196 (0.7273)	0.7169 (0.7278)
		$p_c = 0.5$	<b>0.7224<sub>2</sub></b> (0.7251)	0.7101 (0.7256)	0.7172 (0.7263)	0.7167 (0.726)
RBS		0.7179 (0.7226)	0.7129 (0.723)	0.7177 (0.7229)	0.7172 (0.7223)	
PHC		0.7117 ( <b>0.7334<sub>2</sub></b> )	0.6843 (0.7271)	0.7033 ( <b>0.7343<sub>1</sub></b> )	0.6912 ( <b>0.7329<sub>3</sub></b> )	
Random sampling		0.6197 (0.6358)				

$K = 10, \alpha = 2.0$						
		Constant mutation		Poisson mutation		
		$d = 1$	$d = 2$	$p_m = 0.5/N$	$p_m = 1/N$	
TGA	TS = 1	$p_c = 0.0$	0.6918 (0.7317)	0.6637 (0.7383 <sub>4</sub> )	0.6812 (0.7338)	0.6767 (0.7356 <sub>5</sub> )
		$p_c = 0.25$	0.6752 (0.7324)	0.6546 (0.7332)	0.6623 (0.7299)	0.6578 (0.7324)
		$p_c = 0.5$	0.6653 (0.7298)	0.6472 (0.7331)	0.6478 (0.7264)	0.6472 (0.7287)
	TS = 2	$p_c = 0.0$	0.702 <sub>4</sub> (0.7299)	0.6697 (0.7318)	0.6938 (0.7307)	0.6821 (0.7305)
		$p_c = 0.25$	0.6897 (0.7277)	0.662 (0.732)	0.6717 (0.7281)	0.6643 (0.7302)
		$p_c = 0.5$	0.6768 (0.7288)	0.6537 (0.7296)	0.6607 (0.7263)	0.659 (0.7274)
	TS = 4	$p_c = 0.0$	<b>0.7072<sub>2</sub></b> (0.7275)	0.677 (0.7294)	0.6981 <sub>5</sub> (0.7302)	0.6902 (0.7272)
		$p_c = 0.25$	0.6979 (0.7294)	0.6701 (0.7298)	0.6834 (0.7279)	0.6805 (0.7263)
		$p_c = 0.5$	0.6936 (0.7265)	0.6635 (0.7296)	0.6715 (0.7258)	0.67 (0.7255)
RBS		<b>0.7102<sub>1</sub></b> (0.7207)	0.6871 (0.7219)	0.7056 <sub>3</sub> (0.7217)	0.6987 (0.7215)	
PHC		0.6651 ( <b>0.7434<sub>2</sub></b> )	0.6476 (0.7345)	0.6568 ( <b>0.7438<sub>1</sub></b> )	0.6517 ( <b>0.7421<sub>3</sub></b> )	
Random sampling		0.632 (0.6524)				

vations: (i) best performance is achieved with PHC using a rather small population size of about  $\mu_0 = 30$ , and (ii) for population sizes  $\mu_0 > 50$  one should use a GA whereby the applied tournament size or selection pressure should increase with the population size. As the population size increases, the maximum number of generations  $G$  available for optimization is simply insufficient for PHC or a GA with low selection pressure and/or much randomness in the solution generation process to converge quickly to a high-quality part of the search space. However, on the other hand, a GA with too much selection pressure may cause the population to get stuck at local optima, especially on rugged fitness landscapes (see result of RBS in the bottom right plot). In a lethal optimization scenario

(left plots), a GA clearly outperforms PHC (regardless of the population size) due to the reasons mentioned above. Furthermore, unlike to the non-lethal case, we observe that a GA should use an above-average large tournament size already for small population sizes of  $\mu_0 > 30$ . Similar to the non-lethal case, however, there is a saturation point with RBS, beyond which a further increase in the population size has no effect (see e.g. bottom left plot).

## 4 SUMMARY AND CONCLUSION

This paper has conducted an initial investigation of the impact of *lethal* solutions (here modeled as can-



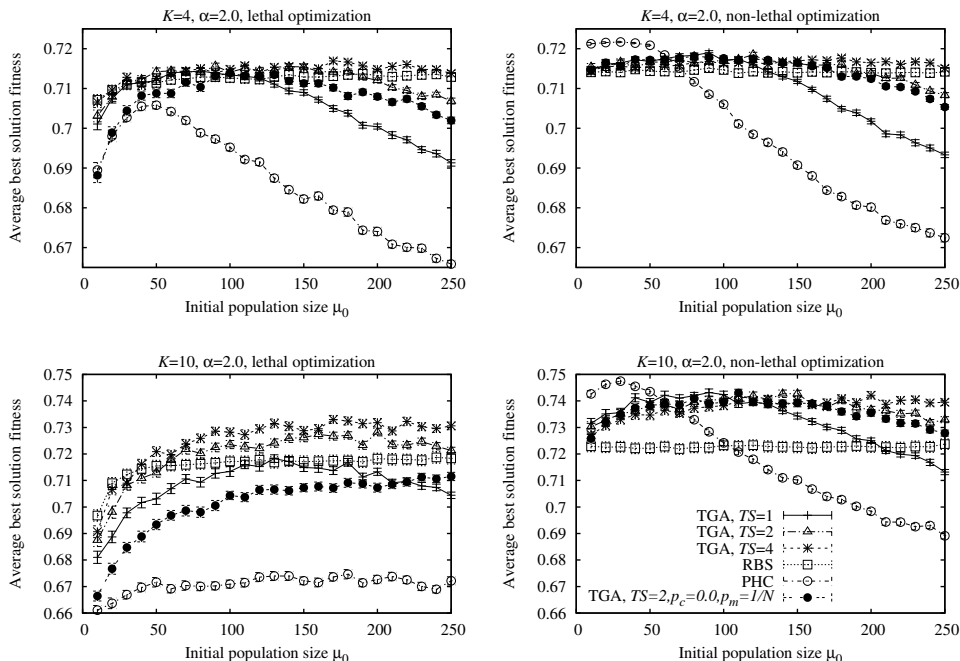


Figure 4: Plots showing the average best solution fitness obtained by different search algorithms in a lethal environment (left) and non-lethal environment (right) on  $NK\alpha$  landscapes with  $K = 4, \alpha = 2.0$  (top) and  $K = 10, \alpha = 2.0$  (bottom) as a function of the initial population size  $\mu_0$ ; the maximum number of fitness evaluations available for optimization was fixed at 6000 (corresponding to the default setting  $G = 200, \mu_0 = 30$ ), i.e.  $G = \lfloor 6000/\mu_0 \rfloor$ . If not otherwise stated, an algorithm used a constant mutation mode with  $d = 1$  and no crossover, i.e.  $p_c = 0.0$ .

didate solutions below a certain fitness threshold) on evolutionary search. In essence, the effect of evaluating a lethal solution is that the solution is immediately removed from the population and the population size is reduced by one. This kind of scenario can be found in Natural evolution, where the presence of lethal mutants may promote robustness over evolvability, but it is also a characteristic of certain closed-loop evolution applications, for example, in autonomous robots or nano-technologies. When faced with this kind of scenario in optimization, the challenge is to discover innovative and fit candidate solutions without reducing the population too rapidly.

Our analysis has been focused on the following three main aspects: (i) analyzing the impact of lethal solutions on a standard evolutionary algorithm (EA) framework, (ii) determining challenging fitness landscape topologies in a lethal environment, and (iii) tuning EAs to perform well within a lethal optimization scenario. Generally, the presence of lethal solutions can affect that performance of EAs, but the largest (negative) impact was observed for fitness landscapes that are rugged and possess some structure in the sense that some solution bits are more important than others; in terms of our test functions, which were  $NK\alpha$  landscapes, this type of landscape corresponds to large values of  $K$  and  $\alpha$ . For this

fitness landscape topology, we obtained best performance in a non-lethal environment using a small population of stochastic hill-climbers. In a lethal environment, however, significantly better results were obtained using an EA that limits randomness in the solution generation process by employing elitism, a relatively large selection pressure, a constant mutation mode (i.e. flipping exactly one solution bit as opposed to flipping each bit independently with some low probability), and no or a small crossover rate. Also, in a lethal optimization scenario, we observed that an EA that evolves a large set of entities (e.g. autonomous robots or software) for a small number of generations performs better than one that evolves a few entities for many generations. The practical implication of this is that, if possible, a larger budget should be allocated for the production of the entities in the first place rather than for attempting to prolong the testing or optimization phase.

## 5 FUTURE WORK

Our study has of course been very limited, and there remains much else to learn about optimization problems subject to lethal solutions and search policies for dealing with these problems. Our current re-

search is looking at the design and tuning of intelligent search policies. We have experimented with an EA that learns offline, e.g. using reinforcement learning, when to switch between different selection and variation operators settings (online) during the optimization (here we used an approach similar to Allmendinger and Knowles (2011)); this EA can yield better performance than a static or non-learning EA. It might be worth mentioning that a well-performing policy learnt offline by this EA is one that increases randomness in the solution generation process if and only if the optimization is in the final stages and the remaining population of reasonable size.

Alternatively to a learning approach, an EA may be augmented with a strategy that uses assumptions of local fitness correlation to pre-screen the designs and forbids the upload of potential lethals. Such a strategy is similar to brood selection with repair and/or some fitness approximation schemes used in EAs to filter solutions before evaluation (Walters, 1998; Jin, 2005).

Finally, analyzing the effect of lethal solutions and search policies on different and perhaps more realistic fitness landscapes than  $NK\alpha$  landscapes, e.g. ones with neutral plateaux, is another avenue we are pursuing. In the further future, we might enjoy trying out our strategies on real lethal environments in autonomous robots, nano-technologies or similar.

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