4

A General Equilibrium Model with Informal Financial Markets

The analytical model presented in the previous chapter was useful for understanding some basic implications of informal financial markets for the transmission mechanism of macroeconomic policy in developing countries. The model, however, to be tractable, required severe simplifying assumptions – most importantly, leakages between official and unofficial markets (a key aspect of these markets emphasized in chapter 1) had to be ignored. In this chapter, following Agénor, Haque and Montiel (1992), a more detailed analysis is provided, and a more complete, macroeconomic framework is developed. Because of its complexity, however, the expanded model cannot be solved analytically. We therefore resort to a numerical procedure. The structure of the model and the parameter estimates used in the simulations are described in this chapter, while the simulations themselves are reported in chapter 5.

We again consider a small, open economy in which there are four types of agents: the private sector (households and producers), the government, the central bank, and the commercial banks. The exchange rate system continues to be described by an official market which coexists with an illegal (or quasi-illegal) parallel market for foreign exchange. Commercial transactions of the private sector, however, are now settled only partly in the official market, at the fixed exchange rate s. The rest of the commercial transactions and all capital transactions are settled in the parallel market at the free exchange rate b, which is determined by market forces. The central bank determines the total amount of foreign currency sales in the official market, using a rationing rule based on the composition of private consumption imports, after

provision has been made for imports of intermediate goods and government imports.

The economy is now assumed to produce two goods: a home good, Q_h, which is used only for final domestic consumption, and an export good, Q_x, for the sole purpose of exports. The capital stock in each sector remains fixed over the time frame of the analysis, and labor is perfectly mobile across sectors. As in the previous chapter, a consumer good is imported which is not produced at home and is an imperfect substitute for the home good. Due to the existence of tariffs, foreign exchange controls, and a positive premium between the official and parallel exchange rates, exports are in part smuggled out, and consumer imports are in part smuggled in. Producers also import an intermediate input which is not produced domestically. The central bank provides unlimited access to foreign exchange through the official market to producers for the purchase of imported intermediate inputs.[1]

The financial system remains as before – i.e., it is characterized by the absence of organized markets for securities and equities, by exchange controls, and by legal ceilings on bank borrowing and lending rates, a situation which leads to the emergence of a parallel market for foreign exchange and an informal – or "curb" – loan market. However, we now allow households to hold four categories of assets: domestic currency notes, deposits with the banking system, foreign currency-denominated assets, and loans extended through the informal credit market.[2] Domestic currency notes bear no interest, and are held only for transaction purposes. Interest rates charged by commercial banks on their loans are controlled by the monetary authority. As a result, interest rates on bank deposits are also given under the zero-profit condition. The rate of return on foreign currency-denominated assets depends on the world interest rate and exchange rate expectations. Holdings of all assets are financed from agents' net worth or by borrowing through the banking system or through the informal loan market.[3]

Bank credit and curb market loans continue to be taken as perfect substitutes in the private sector's portfolio. While interest rates on assets and liabilities of the banking system are fixed by applicable legal norms, the interest rate on curb market loans, as well as the price of foreign exchange in the parallel market,

are determined by prevailing market conditions.

Under the small country assumption, the domestic price of intermediate imports, P_n, is exogenously determined by world prices, P_n^*. The price of the home good, P_h, is determined endogenously in the home market and is explained below. The domestic price of final imports, P_I, as well as the domestic price of exports, P_x, are also determined endogenously, as a result of smuggling activities.

1 Aggregate Supply

Departing from the model of chapter 3, we endow the present model with a fully specified supply side. Consider first the production of home goods. Technology for the production of the home good is assumed to be characterized by a separable, Cobb-Douglas function:

$$Q_h = \phi(\bar{K}_h)F(L_h, N_h) = \phi(\bar{K}_h)L_h^{\alpha_L}N_h^{\alpha_N} \tag{4.1}$$

where Q_h denotes gross output, $\bar{K}_h$ the stock of capital (assumed constant), L_h labor, N_h the imported intermediate input, and α_L and α_N the elasticities of output with respect to labor and intermediate inputs, respectively. Under the assumption of profit maximization and perfect competition, the following supply equation can be derived:[4]

$$\log Q_h = -\frac{1}{1-\alpha}[\alpha_L \log(\omega/P_h) + \alpha_N \log(P_n/P_h)], \tag{4.1'}$$

where ω denotes the nominal wage rate, $P_n \equiv sP_n^*$, and $\alpha = \alpha_L + \alpha_N < 1$.

The index of domestic consumer prices, P, is defined as a geometric average of the price of the home good and the price of imported consumer goods, P_I:[5]

$$P = P_h^{\delta}P_I^{1-\delta}, \qquad 0 < \delta < 1, \tag{4.2}$$

where δ denotes the value share of the home good in final domestic consumption. The domestic currency price of imported consumer goods depends on the world price of imported goods, P_I^*, which is

exogenously determined, and on a weighted average of the official and parallel exchange rates:

$$P_I = (s^{\sigma}b^{1-\sigma})P_I^*, \qquad 0 \le \sigma \le 1 \tag{4.3}$$

where σ denotes the proportion of imports carried through official channels. The coefficient σ is related below to the rationing rule followed by the authorities in the official market for foreign exchange.[6]

Labor contracts last one period and are negotiated one period in advance, so that the wage rate is known by firms at the beginning of the period, that is, before output and home good prices are realized. Nominal wages adjust slowly to their equilibrium value, and are determined by:

$$\Delta \log \omega = \Psi(\log \bar{\omega} - \log \omega_{-1}), \qquad 0 \le \Psi \le 1 \tag{4.4}$$

where $\bar{\omega}$ denotes the equilibrium level of the nominal wage, and Ψ is the speed of adjustment of the nominal wage to its market-clearing level. The market-clearing wage is derived from the labor-market equilibrium condition, with labor demand being derived from the first-order conditions for profit maximization. This yields:[7]

$$\bar{\omega} = \alpha_L(P_hQ_h + P_xQ_x)/\bar{L}_s, \tag{4.5}$$

where $\bar{L}_s$ denotes the (exogenous) level of labor supply.

From the first-order conditions for profit maximization, the demand function for imported intermediate inputs can be derived as:

$$N_h = \alpha_N P_h Q_h / P_n. \tag{4.6}$$

Consider now the supply of exports. Since the economy being considered is that of a small country where the export price is fixed in the world market, the foreign demand for exports is infinitely elastic and the volume of exports is supply-determined. For simplicity, a production function identical to (4.1) is assumed for the export good:

$$Q_x = \phi(\bar{K}_x)F(L_x, N_x), \tag{4.7}$$

where the quantities $\bar{K}_x$, L_x, N_x are appropriately defined. Following the same reasoning as above, the following supply equation can be obtained:

$$\log Q_x = -\frac{1}{1-\alpha}[\alpha_L \log(\omega/P_x)] + \alpha_N \log(P_n/P_x)], \quad (4.7')$$

where P_x, the supply price of exports, is defined as:

$$P_x = [b\Phi + s(1-\tau_x)(1-\Phi)]P_x^*. \quad (4.8)$$

In equation (4.8), $0 < \tau_x < 1$ measures the *ad valorem* rate of taxation of exports, and Φ denotes the proportion of undeclared exports. The above expression shows that the relevant supply price for producers of exported goods (in domestic currency terms) is an average of the price of smuggled exports, bP_x^*, weighted by the proportion of smuggled exports, Φ, and the after-tax price of official exports, $s(1-\tau_x)P_x^*$, weighted by the proportion of exports that are legally sold abroad, $(1-\Phi)$. The determination of Φ is explained below.

From (4.7'), the demand function for imported inputs by the export sector is given by:

$$N_x = \alpha_N P_x Q_x / P_n. \quad (4.9)$$

The officially recorded level of domestic output must exclude that part of the domestic production for exports that is illegally smuggled out. Thus, recorded real gross aggregate output at factor cost Z_g can be defined as:

$$\begin{aligned} Z_g &= [P_h Q_h + (P_x - b\Phi P_x^*)Q_x]/P \\ &= [P_h Q_h + (1-\tau_x)(1-\Phi)sP_x^* Q_x]/P, \end{aligned} \quad (4.10)$$

where P is used as the deflator. To obtain *net* aggregate output at factor cost, Z, real imports of intermediate goods must be subtracted from Z_g:

$$Z = Z_g - (P_n N/P) = [P_h Q_h + (1-\tau_x)(1-\Phi)sP_x^* Q_x - P_n N]/P, \quad (4.11)$$

where $N = N_h + N_x$.

2 Aggregate Demand, Income and Wealth

Total expenditure by the private sector, E^p, is given by:

$$\log E^p = h_1 \log Z^d + h_2 \log(A_{-1}/P) - h_3(i_L - E\pi_{+1}), \quad 0 < h_1 < 1 \quad (4.12)$$

where A denotes total wealth of the private sector, Z^d real disposable income, and i_L the nominal interest rate in the informal credit market. $E\pi_{+1}$ denotes the expected rate of inflation for period $t+1$, formed at t. Equation (4.12) indicates that total private spending is a positive function of income and wealth, and a negative function of the real rate of interest - defined as the nominal interest rate in the informal credit market corrected for the expected change in consumer prices. Real disposable income, Z^d, is defined as:

$$PZ^d = (1-\tau)PZ + \Phi bP_x^* Q_x + (i_L - i_c)L^p_{-1} - [(1-\mu)i_L - i_d]D^p_{-1}, \quad (4.13)$$

where τ denotes the income tax rate, L^p the stock of domestic bank credit held by the private sector, D^p the stock of deposits held by the private sector in the banking system, i_c the interest rate on bank loans, and i_d the interest rate on bank deposits. The first term in equation (4.13) represents after-tax factor income derived from legally recorded production activities. The second term represents income derived from the smuggling of exports - which is therefore not taxed - and is equal to illegal exports in foreign currency terms, $\Phi P_x^* Q_x$, valued at the parallel exchange rate, b. Finally, the last two terms represent the implicit subsidies and taxes from the existence of interest rate controls.[8]

To understand how the last two expressions are derived, recall that, as explained in chapter 3, the existence of interest rate ceilings imposes implicit taxes and subsidies on private agents, as creditors and debtors with the banking system. In the general case $i_L > i_c$, so that the interest rate ceiling provides an implicit subsidy to debtors equal to $(i_L - i_c)L^p_{-1}$. Similarly, if μ denotes the reserve requirement on bank deposits, interest rate regulation imposes an implicit tax on creditors equal to $[(1-\mu)i_L - i_d]D^p_{-1}$.[9] The net effect depends therefore on $(i_L - i_c)L^p_{-1} - [(1-\mu)i_L - i_d]D^p_{-1}$. Since i_c will typically be greater than i_d, controls will provide a net implicit subsidy if agents are sufficiently indebted with respect to commercial banks, that is, if:

$$L^p_{-1}/D^p_{-1} > [(1-\mu)i_L - i_d]/(i_L - i_c).$$

This notion of a "financial repression tax" (or subsidy) provides a general way of formulating the impact of interest rate controls. As shown in the previous chapter, an index of financial repression, ρ, can be formally defined as:

$$\rho = (i_L - i_c)/i_L, \qquad 0 \leq \rho \leq 1 \tag{4.14}$$

where ρ measures the present value of the subsidy, per unit of bank credit, which is implied by the prevailing interest rate ceilings. When interest rate ceilings are not binding, $i_c = i_L$ and $\rho = 0$. As the curb loan interest rate rises relative to the controlled rate i_c, the constraints become more and more binding and ρ approaches unity. Equation (4.14) can be explicitly introduced in (4.13) by replacing $(i_L - i_c)$ by ρi_L.[10]

The nominal value of agents' financial wealth, A, is defined as the sum of holdings of domestic currency notes CC, bank deposits D^p, bank credit (which is a liability) L^p, and foreign exchange bF^p:[11]

$$A = CC + D^p + bF^p - L^p. \tag{4.15}$$

The budget constraint of the private sector, which determines flow changes in financial wealth, takes the form:

$$\Delta A = P(Z^d - E^p) + i_d D^p_{-1} + i^*_f bF^p_{-1} - i_c L^p_{-1} + \Delta bF^p_{-1}, \tag{4.16}$$

where i^*_f denotes the world interest rate. The first term in equation (4.16) represents the portion of disposable income that is not spent (domestic savings). The second and third terms measure interest income derived, respectively, from bank deposits and holdings of foreign-currency denominated assets. Interest payments on foreign assets are assumed to be repatriated through the parallel market, and are therefore valued at the unofficial exchange rate. The fourth term represents interest payments on bank credit. Finally, the last term captures valuation changes due to fluctuations in the parallel exchange rate.

Consider now the determination of the composition of households' financial portfolio. Desired holdings of foreign currency-denominated assets are defined as:

$$\log[bF^p/(A - CC)] = \lambda_0 - \lambda_1 i_L - \lambda_2 i_d + \lambda_3 i_f, \tag{4.17}$$

where i_f denotes the perceived rate of return on foreign assets, defined as:

$$i_f = (1 + i^*_f)(\mathrm{E}b_{+1}/b) - 1 \cong i^*_f + \mathrm{E}\tilde{b}_{+1}, \tag{4.18}$$

where $\mathrm{E}\tilde{b}_{+1} \equiv \mathrm{E}\Delta \log b_{+1}$ denotes the (one-period ahead) expected rate of depreciation of the parallel exchange rate. Equation (4.17) indicates that a rise in the interest rate on curb market loans or bank deposits induces a fall in desired holdings of foreign assets, while a rise in the perceived interest rate on foreign interest-bearing assets (due to a rise in the world interest rate *per se* or to a higher expected rate of depreciation of the parallel market exchange rate) raises the desired proportion of interest-bearing financial wealth that households would like to hold in the form of foreign assets in their portfolios.

Desired holdings of bank deposits, which consist only of interest-bearing deposits, are given by:

$$\log(D^p/(A - CC)] = \alpha_0 - \alpha_1 i_L + \alpha_2 i_d - \alpha_3 i_f. \tag{4.19}$$

Equation (4.19) indicates that desired holdings of bank deposits rise as a proportion of interest-bearing financial wealth with an increase in the deposit rate or a fall in the interest rate on curb market loans or in the rate of return on foreign assets.

The demand for domestic currency notes reflects a pure transaction motive, and is given by:

$$\log(CC/P) = \varphi_1 \log(Z) + \varphi_2 \log(\Phi q P_x Q_x/P), \qquad \varphi_2 > \varphi_1 \tag{4.20}$$

where the assumption $\varphi_2 > \varphi_1$ is taken to capture the view that illegal activities require a more intensive use of currency notes than legally recorded ones.[12,13]

Finally, with private expenditure given by (4.12), aggregate demand for the home good can be written as:

$$Q_h = G_h + \delta PE^p/P_h, \tag{4.21}$$

where G_h denotes exogenous real government spending on home goods.

3 The Current Account and the Balance of Payments

To derive the current account of the economy in the presence of fraudulent trade transactions, it is important to distinguish between *reported* trade flows, that is, officially measured transactions with the rest of the world, and *unreported* flows.

Given private expenditure as outlined above, the demand by the private sector for imported consumer goods can be determined as a constant share of total expenditure:[14]

$$I^p = (1 - \delta) PE^p / P_I. \tag{4.22}$$

The government imports $P_I^* G_I$ of final goods, in foreign currency terms, and the central bank provides unlimited access for these imports. Producers also have unlimited access to the official foreign exchange market for imports of intermediate inputs. By contrast, the central bank satisfies only a portion σ of total private sector demand for imported final goods.

Formally, the central bank sets the total amount of foreign currency sales in the official exchange market equal to a minimum quantity $P_n^* N + P_I^* G_I$ - which corresponds to the demand by domestic producers for imported inputs plus the demand for final imports by the public sector - plus a fraction σ of private sector demand for consumer imports, given by equation (4.22).[15] Total sales of foreign exchange (expressed in foreign currency), F_S, are therefore equal to:

$$F_S = P_n^* N + P_I^* G_I + \sigma P_I^* I^p. \qquad 0 \leq \sigma \leq 1 \tag{4.23}$$

The officially recorded current account, C, is equal to the value of recorded exports minus the value of recorded imports of final goods by the private sector and the government authorized by the central bank, plus imports of intermediate goods and interest income on net foreign assets of the public sector and the central bank. Measured in foreign currency terms, C is therefore given by:

$$C = (1 - \Phi) P_x^* Q_x + i_f^* (F^g_{-1} + R_{-1}) - F_S, \tag{4.24}$$

where Φ measures the fraction of exports that are smuggled out of the country, so that officially recorded exports amount only to $(1 - \Phi) P_x^* Q_x$. Equation (4.24) also assumes, as previously mentioned, that interest income on private foreign currency-denominated assets is repatriated through the parallel market for foreign exchange.[16] Note that since intermediate imports are not rationed and there is no under-invoicing by producers, their full value is recorded in the official trade statistics. Foreign exchange demand by the private sector for final imports not eligible for the official market, that is $(1 - \sigma) P_I^* I^p$, spills over to the parallel market.

The coefficient Φ is assumed to depend positively on the exchange rate ratio $q \equiv b/s$. The particular functional form adopted here is given as:

$$\Phi(q, \tau_x) = 1 - [(1 - \tau_x)/q]^\nu. \qquad \nu > 0 \tag{4.25}$$

The function $\Phi(q, \tau_x)$ can be shown to satisfy the following properties:

$$\partial\Phi/\partial q > 0, \ \partial\Phi/\partial\tau_x > 0, \ \partial^2\Phi/\partial q^2 < 0, \qquad \lim \Phi = 1 \text{ for } q \to \infty.$$

The first two properties indicate that the higher the tax rate on exports, or the higher the premium, the higher will be the under-invoicing share. The magnitude of $\partial\Phi/\partial q$ and $\partial\Phi/\partial\tau_x$ can in general be expected to be inversely related to the perceived implicit and explicit costs of engaging in illegal transactions and to the degree of enforcement of exchange restrictions. The third property indicates that there are rising costs to engaging in illegal activities (related perhaps to a higher probability of being detected). Finally, the fourth property indicates that when the premium is high enough, relative to the tax rate on exports, the under-invoicing share tends to unity. Note also that even if the premium $(q - 1)$ is zero (that is, if $b = s$), the existence of an export tax still provides an incentive to fake invoices.

Consider now the capital account. Net private capital flows through the official foreign exchange market are prohibited, and public capital inflows are treated as exogenous. The capital account surplus is therefore equal to minus the change in net holdings of external assets of the public sector:

$$\Delta F = -\Delta F^g \tag{4.26}$$

Using (4.24) and (4.26), the change in net foreign assets of the central bank - measured in foreign currency terms - is given by:

$$\Delta R = C - \Delta F^g. \tag{4.27}$$

The unreported current account, C_U, which determines the rate of change of the private stock of foreign currency-denominated assets, is in turn given by:

$$C_U \equiv \Delta F^p = \Phi P_x^* Q_x + i_f^* F_{-1}^p - (1-\sigma) P_I^* I^p. \tag{4.28}$$

4 Banks, Budget Deficits and the Money Supply

Bank assets consist of reserves held at the central bank, denoted RR, and credit extended to households and producers, L^p. Their liabilities are the deposits held by the public, D^p, and credit received from the central bank, L^b. The balance sheet of the banking system is therefore given by:

$$RR + L^p = D^p + L^b. \tag{4.29}$$

Banks are assumed to hold no excess reserves. Given a required reserve ratio of μ, reserve holdings are given by:

$$RR = \mu D^p. \qquad 0 < \mu < 1 \tag{4.30}$$

Reserves held at the central bank pay no interest, but credit extended to commercial banks by the monetary authorities carries an interest charge which, for convenience, is set equal to the interest rate which banks charge their customers for loans, i_c. Under these conditions, the zero-profit condition for the banking system is given by:

$$i_c = i_d/(1-\mu), \tag{4.31}$$

which determines the interest rate paid on bank deposits, i_d. The banks' willingness to accept deposits is infinitely elastic, implying that the actual level of deposits held in the banking system is determined on the demand side, that is, households' demand. Equation (4.31) fixes the relative price between the lending and deposit rates. Since $\mu > 0$, in general $i_d < i_c$.

The central bank does not lend directly to households. Its balance sheet equates base money (defined as the sum of currency notes in circulation plus commercial bank reserves) to the sum of international reserves valued at the official exchange rate and the stock of domestic credit extended to the government and commercial banks minus the central bank's net worth, denoted Ω:

$$CC + RR = sR + (L - L^p) - \Omega, \tag{4.32}$$

where total domestic credit L, consists of credit to the private sector by commercial banks L^p, and credit to the public sector L^g and to banks by the central bank, L^b:

$$L = L^p + (L^g + L^b). \tag{4.33}$$

Changes in the central bank's net worth are given by the difference between interest income on foreign reserves, lending to commercial banks and the government, valuation changes, and net transfers to the government budget, t^g:[17]

$$\Omega = \Omega_{-1} + i_f^* sR_{-1} + i_c(L_{-1}^b + L_{-1}^g) + \Delta sR_{-1} - t^g, \tag{4.34}$$

The model's dynamic specification is completed with a description of the behavior of the non-financial public sector. The government's revenue sources consist of income taxes on the private sector, export taxes, income from holdings of foreign-currency denominated assets, and transfers from the central bank. It consumes both home and imported goods, and pays interest on its domestic debt. It borrows on external markets as well as from the central bank. Consequently, the government deficit, denoted D, can be written as follows:

$$\begin{aligned} D = {} & \tau PZ + \tau_x(1-\Phi) sP_x^* Q_x + i_f^* sF_{-1}^g \\ & + t^g - (P_h G_h + sP_I^* G_I) - i_c L_{-1}^g \end{aligned} \tag{4.35}$$

The deficit is financed by borrowing either at home or abroad:

$$s\Delta F^g = D + \Delta L^g. \tag{4.36}$$

The model can be operated in several different modes, according to the deficit financing rule. In what follows, it will be assumed that government expenditure is exogenous and that a ceiling is imposed on foreign financing. Deficits are therefore financed by central bank credit – a typical situation in many developing countries. Equation (4.36) therefore determines the evolution of L^g.

Finally, the money supply can be defined as:

$$M = CC + D^p. \tag{4.37}$$

5 The Complete Framework

The equations of the model are presented in summary form in an appendix to this chapter; variable definitions are given in table 4.1. The basic innovations of the model are its integrated treatment of informal markets and government restrictions, as well as the treatment of expectations. Controls on interest rates and foreign exchange allocation lead to the emergence of informal credit and foreign currency markets, and these determine how various macroeconomic shocks affect prices, output and portfolio decisions. The dynamics of the model arise essentially from forward-looking expectations and asset accumulation in the presence of such markets, in contrast with early macroeconomic models for developing countries which rely on partial adjustment and adaptive expectations mechanisms. This implies, for example, that unlike in earlier models, anticipated future shocks will have immediate effects through flexible financial markets, and the current effects of current shocks will in general depend on their expected duration.

Table 4.1 Definition of variables

A	Nominal private financial wealth
b	Parallel market exchange rate
C	Reported current account
CC	Private holdings of domestic currency notes
C_U	Unreported current account
D	Fiscal deficit
D^p	Private sector deposits in commercial banks
Δ	First-difference operator
E^p	Private sector expenditure
$E\pi_{+1}$	Expected inflation rate for period $t+1$ formed at t
$E\tilde{b}_{+1}$	Expected rate of depreciation of the parallel exchange rate formed at t for period $t+1$
Φ	Under-invoicing share of exports

Table 4.1 *(Cont)*

F^p	Private holdings of foreign currency
F^g	Net foreign assets of the public sector (exogenous)
G_h	Real government spending on home goods
G_f	Real government spending on final imported goods (exogenous)
I^p	Private imports of final goods
i_c	Interest rate on bank loans (exogenous)
i_d	Interest rate on bank deposits
i_f	Rate of return on foreign-currency denominated assets
i_f^*	World interest rate (exogenous)
i_L	Interest rate on curb market loans
L	Total domestic credit
L^p	Bank credit to the private sector
L^b	Credit to commercial banks by the central bank (exogenous)
L^g	Bank credit to the public sector (exogenous)
$\bar{L}_s$	Labor supply (exogenous)
M	Domestic money stock
N	Total imports of intermediate inputs
P	Consumer price index
p	Parallel market premium
P_h	Price of home goods
P_f	Domestic price of final imports
P_f^*	World price of final imports (exogenous)
P_n^*	World price of imported intermediate inputs (exogenous)
P_x	Supply price of exports
P_x^*	World price of exports (exogenous)
q	Ratio of parallel to official exchange rate
Q_h	Production of home goods
Q_x	Production of export goods
R	Net foreign assets of the central bank
RR	Required reserves
ρ	Financial repression tax
s	Official exchange rate (exogenous)
τ	Income tax rate (exogenous)
τ_x	Tax rate on exports (exogenous)
t^g	Transfers from the central bank to the government
ω	Nominal contract wage
$\bar{\omega}$	Equilibrium nominal wage
Z_g	Officially recorded gross domestic product
Z	Officially recorded net domestic product

In this setting, the central bank retains five instruments of policy – the level of controlled bank loan interest rates i_c, the required reserve ratio μ, the amount of credit it extends to the commercial banking system L^b, the proportion of private final imports satisfied in the official foreign exchange market σ, and the official exchange rate s. The fiscal authority determines the income tax rate τ and the rate of taxation of exports, τ_x, as well as the levels of spending on home and imported goods, G_h and G_f.

6 Calibration of the Model

The estimation of a model as complex as the one presented above represents a difficult task under rational expectations. In view of this complexity and the lack of reliable and detailed data of adequate length for a sufficient number of countries, as well as the complete absence of data for certain variables, a useful strategy is to adopt a calibration procedure.[18] This approach permits the analysis of the model without undue restrictions on its structure. The parameters used are based on available econometric estimates where possible, but do not pertain to any individual real-world economy. Rather, they reflect conditions underlying a wide variety of possible systems. In adopting this procedure, some key parameters were subjected to sensitivity analysis, in order to determine the robustness of observed patterns in the behavior of the model. This view of simulation is thus basically a compromise between analytical modeling and a more "empirical" simulation approach, such as econometric simulation.[19]

Available estimates of key macroeconomic parameters differ greatly with regard to countries and periods covered, specification of estimated equations, and econometric methodology. The parameter set used here is based partly on available evidence, in particular on the econometric estimates derived by Haque, Lahiri and Montiel (1990), using pooled time-series cross-section data for 31 developing countries.[20]

Table 4.2 presents the key parameters of the model (excluding all constant terms, which are determined residually), as well as initial values of the exogenous variables. The initial values of endogenous variables are derived by an iterative method which

Table 4.2 Parameters and initial values

	Parameters	
$\alpha_L = 0.3$	$\alpha_N = 0.2$	$\gamma = 0.0$
$\delta = 0.8$	$\sigma = 0.2$	$h_1 = 0.7$
$h_2 = 0.1$	$h_3 = -0.5$	$\mu = 0.4$
$\lambda_1 = 0.5$	$\lambda_2 = 0.6$	$\lambda_3 = 0.9$
$\varphi_1 = 0.4$	$\varphi_2 = 0.7$	$\alpha_1 = 0.6$
$\alpha_2 = 1.1$	$\alpha_3 = 0.5$	$\nu = 0.1$
$\Psi = 0.0^*$	$\Phi = 0.4^{**}$	$\rho = 0.25^{**}$
	Exogenous variables	
$P_n^* = 1.0$	$P_x^* = 1.0$	$P_f^* = 1.0$
$s = 1.0$	$\tau = 0.3$	$\tau_x = 0.1$
$i_c = 0.05$	$i_f^* = 0.06$	$G_f = 120$
$G_h = 340$	$F^0 = 2800$	$L^b = 1400$

* Set to 0.8 for the exchange rate experiment.
** Initial baseline value.

approximates the steady state solution, that is, the static, long-run solution of the model. The normalization rule used to solve the model enforces all macro-equilibrium conditions. For instance, equilibrium in the market for the home good is given by equating the supply of home goods with total demand; for solution purposes, the demand equation is used to determine the price of home goods.

Some features of the parameter set are noteworthy. The elasticity of output with respect to labor and imported inputs is consistent with the results reported elsewhere in the literature. The speed of adjustment of the nominal wage to its market-clearing level Ψ is set to 0 throughout, except for the exchange rate experiment where $\Psi = 0.8$. The interest elasticity of private sector expenditure is close to the elasticity for consumption derived by Rossi (1988) and accords well with the evidence on saving elasticities reported by Fry (1988, p. 142). The particular form chosen for the under-invoicing share, although arbitrary, provided reasonable results in the simulation experiments for low values of ν.[21] The required reserve ratio on bank deposits, μ, is set to 0.4, an assumption which is consistent with reserve ratios actually observed

in many highly inflationary, financially-repressed economies. The estimates of the (semi-) elasticities of the demand for bank deposits relative to the curb interest rate ($\alpha_1 = -0.6$) and the administered deposit rate ($\alpha_2 = 1.1$) reported in table 4.2 are slightly lower than those reported by van Wijnbergen (1982, p. 156; 1985) for Korea ($\alpha_1 = -0.9, \alpha_2 = 1.6$), but experiments with values closer to these last estimates did not change the results significantly.

The results were subjected to limited sensitivity analysis, using an alternative parameter set with plausible values. However, sensitivity analysis cannot "prove" in any sense that the chosen parameter values are "true" values, nor that the conclusions derived are completely general. With this is mind, we now turn to a quantitative analysis of macroeconomic policies.

Appendix: Summary Equations of the Model

The equations of the model, in summary form, are as follows:[22]

Supply of home goods

$$\log Q_h = 1/(1-\alpha)\,[\alpha \log P_h - \alpha_N \log P_n - \alpha_L \log \omega]. \qquad \alpha, \alpha_L, \alpha_N < 1 \qquad (A1)$$

Supply of export goods

$$\log Q_x = 1/(1-\alpha)\,[\alpha \log P_x - \alpha_N \log P_n - \alpha_L \log \omega]. \qquad (A2)$$

Supply price of exports

$$P_x = [b\Phi + s(1-\tau_x)(1-\Phi)]P_x^*. \qquad 0 < \tau_x < 1 \qquad (A3)$$

Nominal wage contracts

$$\Delta \log \omega = \Psi(\log \bar{\omega} - \log \omega_{-1}). \qquad 0 \leqslant \Psi \leqslant 1 \qquad (A4)$$

Market-clearing wage

$$\bar{\omega} = \alpha_L (P_h Q_h + P_x Q_x)/\bar{L}. \qquad (A5)$$

Official output at factor cost

$$PZ = P_h Q_h + (1-\tau_x)(1-\Phi) s P_x^* Q_x - P_n N. \qquad (A6)$$

Demand for imported intermediate inputs

$$N = \alpha_N (P_h Q_h + P_x Q_x)/P_n. \qquad (A7)$$

Domestic price level

$$P = P_h^{\delta} P_I^{1-\delta}. \qquad 0 < \delta < 1 \qquad (A8)$$

Domestic price of final imports

$$P_I = (s^{\sigma} b^{1-\sigma}) P_I^*. \qquad 0 \leq \sigma \leq 1 \qquad (A9)$$

Private sector expenditure

$$\log E^p = h_1 \log Z^d + h_2 \log (A_{-1}/P) - h_3 (i_L - \mathrm{E}\pi_{+1}). \qquad 0 < h_1 < 1 \qquad (A10)$$

Real disposable income

$$PZ^d = (1-\tau) PZ + \Phi b P_x^* Q_x + \rho i_L [L^p_{-1} - (1-\mu) D^p_{-1}]. \qquad 0 < \tau < 1 \qquad (A11)$$

Demand for the home good

$$Q_h = G_h + \delta P E^p / P_h. \qquad (A12)$$

Financial repression tax

$$\rho = (i_L - i_c)/i_L. \qquad 0 \leq \rho \leq 1 \qquad (A13)$$

Financial wealth

$$A = M + bF^p - L^p. \qquad (A14)$$

Budget constraint of the private sector

$$\Delta A = P(Z^d - E^p) + i_d D^p_{-1} + i_f^* b F^p_{-1} - i_c L^p_{-1} + \Delta b F^p_{-1}. \qquad (A15)$$

Demand for foreign assets

$$\log [bF^p/(A - CC)] = \lambda_0 - \lambda_1 i_L - \lambda_2 i_d + \lambda_3 i_f. \qquad (A16)$$

Rate of return on foreign assets

$$i_f \cong i_f^* + E\tilde{b}_{+1}. \qquad (A17)$$

Demand for domestic currency notes

$$\log(CC/P) = \varphi_1 \log(Z) + \varphi_2 \log(\Phi q P_x Q_x/P). \qquad \varphi_2 > \varphi_1 \tag{A18}$$

Demand for bank deposits

$$\log(D^p/(A - CC)] = \alpha_0 - \alpha_1 i_L + \alpha_2 i_d - \alpha_3 i_f. \tag{A19}$$

Private demand for final imports

$$I^p = (1 - \delta) PE^p/P_I. \tag{A20}$$

Official sales of foreign exchange

$$F_S = P_n^* N + P_I^* G_I + \sigma P_I^* I^p. \qquad 0 \le \sigma \le 1 \tag{A21}$$

Official current account

$$C = (1 - \Phi) P_x^* Q_x + i_f^* (F_{-1}^g + R_{-1}) - F_S. \tag{A22}$$

Under-invoicing share

$$\Phi = 1 - [(1 - \tau_x)/q]^\nu, \quad q \equiv b/s, \quad \nu > 1, \quad 0 \le \Phi \le 1. \tag{A23}$$

Net foreign assets of the central bank

$$\Delta R = C - \Delta F^g. \tag{A24}$$

Unreported current account

$$C_U \equiv \Delta F^p = \Phi P_x^* Q_x + i_f^* F_{-1}^p - (1 - \sigma) P_I^* I^p. \tag{A25}$$

Balance sheet of commercial banks

$$RR + L^p = D^p + L^b. \tag{A26}$$

Reserve holdings by commercial banks

$$RR = \mu D^p. \qquad 0 \le \mu \le 1 \tag{A27}$$

Interest rate on bank deposits

$$i_c = i_d/(1 - \mu). \tag{A28}$$

Balance sheet of the central bank

$$CC + RR = sR + (L - L^p) - \Omega. \tag{A29}$$

Total domestic credit

$$L = L^p + (L^b + L^g). \tag{A30}$$

Central bank's net worth

$$\Omega = \Omega_{-1} + i_f^* s R_{-1} + i_c (L_{-1}^b + L_{-1}^g) + \Delta s R_{-1} - t^g. \tag{A31}$$

Government deficit

$$D = \tau PZ + i_f^* s F_{-1}^g + \tau_x (1 - \Phi) s P_x^* Q_x + t^g - (P_h G_h + s P_I^* G_I) - i_c L_{-1}^g \tag{A32}$$

Deficit financing

$$L^g = L_{-1}^g - D + s \Delta F^g. \tag{A33}$$

Money stock

$$M = CC + D^p. \tag{A34}$$

Parallel market premium

$$p = q - 1. \tag{A35}$$

Notes

1 In principle, unlimited access to the official market may generate the incentive to over-invoice imports of intermediate goods, and this should depend on the same factors affecting the propensity to under-invoice exports (see below). For simplicity, we abstract from this possibility.

2 Recall that domestic currency notes were excluded in chapter 3.

3 Foreign borrowing of private agents that is repatriated (as it would be) through the parallel exchange market does not affect the model, since only *net* private holdings of foreign-currency denominated assets matter.

4 For the derivation of equation (4.1′), see Islam (1984).

5 The price index (2) and the consumption and import behavioral equations that follow reflect the assumption that the instantaneous utility function for consumption of the private sector is of the Cobb-Douglas type.

6 Market prices of imported goods depend on the marginal cost of foreign exchange. Thus, an equation like (3) would hold when certain categories of goods can be imported freely at the official exchange

rate, while others cannot. If all imported goods are rationed, σ would be zero.

7 If there is a production lag between the time when labor is paid and that when output is sold, total labor costs would also depend on the informal interest rate. See, for instance, Buffie (1984).

8 Conventional interest income is not included in (4.13), because the contribution of such income to household resources is already captured by the wealth term A_{-1}/P in (4.12).

9 It can be noted that $(1-\mu)i_L$ represents the interest rate that banks would pay on deposits, in the absence of interest rate ceilings.

10 In the previous chapter, the financial repression tax was assumed to affect wealth (stocks) rather than income (flows).

11 Note that foreign currency holdings are valued at the parallel market exchange rate, b. Also note that since loans in the informal credit market take place between households and firms, they cancel out in the definition of private sector wealth.

12 See chapter 1.

13 Note that the familiar adding-up constraints on the interest-bearing components of wealth bear on the parameters of equations (4.17) and (4.19) only, and not on (4.20). Consequently, in the former equations, the quantity $A - CC$ (which measures interest-bearing financial wealth) appears as the scale variable.

14 Note that this is consistent with the foreign exchange regime assumed above, since imports of consumer goods are not subject to quantity constraints, they are simply channeled through two different markets. Under this specification, the quantity of consumer goods imported through the official market is responsive to relative price changes, but the share of such goods in total private consumption is not.

15 This formulation allows us to link the rationing scheme with the determination of the domestic price of imports.

16 This is, admittedly, a simplyfing assumption. In practice, one is likely to observe repatriation through both legal and illegal channels. The decision regarding which market agents choose to surrender foreign exchange receipts derived from interest payments should also depend on the level of the premium. The above assumption simplifies the workings of the model considerably.

17 The importance of a proper accounting of central bank net worth is emphasized by Anand and van Wijnbergen (1989).

18 Whalley (1985) provides a perceptive discussion of the pros and cons of calibration versus estimation procedures, in the context of Computable General Equilibrium models.

19 In macroeconomics, simulation is most often associated with specific,

estimated econometric models. By contrast, Camilleri *et al.* (1984), Nguyen and Turnovsky (1983) and Levin (1989) use simulation as it is used here, as basically a theoretical tool to analyze abstract and fairly general models. They described this alternative approach to simulation as "theoretical simulation", in order to highlight its complementarity with the solution of small analytical models.

20 Note that not all parameter estimates are chosen from empirical studies which explicitly incorporate rational expectations, or any other particular assumption in our model. However, the suitability of parameter values for our purposes depends not only on the presence or absence of particular assumptions in the models from which they are derived but also on the overall structure of those models. Therefore, values which come from models incorporating rational expectations, or any other particular assumption, but whose structure differs from our model will not be necessarily superior to values obtained from other types of models.

21 Empirical evidence is too sparse to provide any guidance as to whether or not the under-invoicing function is appropriate.

22 Note that using (4.14) and (4.31), equation (4.13) has been rewritten as:

$$PZ^d = (1-\tau)PZ + \Phi bP_x^* Q_x + \rho i_L [L^p_{-1} - (1-\mu) D^p_{-1}].$$