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Foreign currency deposits and the demand for money in developing countries

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Abstract

This paper examines the relative demands for domestic and foreign currency deposits by residents of developing countries. A dynamic currency substitution model that incorporates forward-looking rational expectations is specified and estimated for a group of ten developing countries. The results indicate that the foreign rate of interest and the expected rate of depreciation of the parallel market exchange rate are important factors in the choice between holding domestic money or foreign currency deposits abroad. Furthermore, the model appears to fit the data better than a conventional partial-adjustment model.

JEL classification: E41; F32; C52

Keywords: Currency substitution; Money demand; Forward-looking expectations

1. Introduction

Currency substitution, the process whereby foreign money holdings substitute for domestic money balances as a store of value, unit of account and medium of exchange, has become a pervasive phenomenon in many developing countries. Such substitution, or "dollarization" as it has come to be called, has been observed in countries that differ widely in levels of financial development, in the degree of integration with the rest of the world, and in types of exchange rate regimes and practices. In some cases, particularly where high and variable inflation rates and uncertainty about domestic policies have prevailed for a substantial period of time, the use of foreign currency has become so prevalent

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that a large proportion of domestic sales and contracts are transacted in foreign currency.

A number of empirical studies on currency substitution in developing countries have been conducted over the past few years in an effort to isolate the factors that give rise to this phenomenon (see Calvo and Végh, 1992). Most of the studies, however, are based on two somewhat restrictive assumptions. The first is that existing empirical models implicitly impute "myopic" behavior to the agents whose decisions they describe, by assuming that these agents determine the optimal composition of their money holdings without regard for the future. When dynamic features are introduced, they almost invariably take the form of the addition of the (one-period) lagged dependent variable, with partial adjustment frequently being invoked as a rationalization [1]. The restrictive nature of this approach to dynamic modeling in econometrics has been clearly demonstrated (see, for instance, Hendry et al., 1984). This procedure unduly constrains the lag structure at the outset of the empirical investigation. Furthermore, the estimated coefficients of the lagged dependent variable are usually large, implying implausibly long adjustment lags. Similarly, when expectations are introduced, it is generally through a backward-looking process, such as adaptive expectations. However, the adaptive expectations hypothesis has been shown to be optimal only under restrictive circumstances (see Pesaran, 1988).

A second limitation of existing studies results from the fact that the degree of currency substitution is usually estimated without explicitly accounting for the existence of foreign currency holdings. This occurs in part because of the notorious difficulties involved in obtaining information on the existing stocks of foreign currency assets. A number of countries have, however, allowed foreign currency deposits in their banking system [2]. However, except for a few cases (see Ramirez-Rojas, 1985, El-Erian, 1988, Melvin, 1988), such data have not been systematically exploited in the existing literature. Moreover, there is the data collected and published by the International Monetary Fund on foreign currency deposits held abroad by country of origin of residents [3]. These data show, for instance, that foreign currency deposits held abroad by residents of developing countries rose from about $80 billion in 1981 to $450 billion by the end of 1990, an annual average rate of increase of nearly 50 percent. For developing countries

[1] This tendency seems to result from the influence of the literature on money demand. Generalizations of the partial adjustment framework are discussed by Goldfeld and Sichel (1990).

[2] See Dodsworth et al. (1987) for a discussion of the evolution and implications of domestic holdings of foreign currency bank deposits in developing countries. In Egypt, for instance, a large percentage of the foreign currency deposits in the banking system are held in the form of interest-bearing time deposits. In Bolivia, prior to November 1982, dollar-denominated bank deposits were legal but were suspended at that time until September 1985.

[3] These data are derived from reports on the geographic distribution of the foreign assets and liabilities of deposit banks made by the authorities of 33 international banking centers.

as a whole, such deposits represented on average over 120 percent of the official foreign exchange reserves and about 43 percent of either imports or exports over the period 1981–1990 (see Agénor and Khan, 1992). Although such data are not comprehensive, in that they do not include foreign currency deposits held at home or foreign currency notes in the hands of the public, they provide nevertheless information that can be quite useful in studying the process of currency substitution.

The purpose of this paper is to formulate and estimate a dynamic, forward-looking model of currency substitution, using data on deposits held abroad for a group of ten developing countries: Bangladesh, Brazil, Ecuador, Indonesia, Malaysia, Mexico, Morocco, Nigeria, Pakistan and the Philippines. The model, which incorporates the assumption of rational expectations, is developed in two steps. The desired composition of currency holdings is first derived from an optimizing model of household behavior. Actual currency holdings are then determined in a multi-period costs-of-adjustment framework, following an approach that has been developed in the recent literature on "buffer stock money", notably by Cuthbertson and Taylor (1987, 1990)). The multi-period costs-of-adjustment framework results in an empirical specification that incorporates both backward- and forward-looking components. One of the more appealing features of the empirical implementation of the model is that it does not require information on the domestic interest rate. In previous studies, the lack of a suitable domestic interest rate series has inhibited the estimation of currency substitution models relying on relative rates of return between domestic and foreign currencies that are based on interest rates differentials [4]. The approach taken here circumvents this particular problem, utilizing instead data on the foreign interest rate and the premium on the domestic currency in the parallel exchange market. The resulting model is estimated using quarterly data by an errors-in-variables procedure for the sample of ten developing countries.

The remainder of the paper is divided into three sections. The next section (Section 2) presents the theoretical framework. Section 3 outlines the estimation procedure and presents the empirical results. Finally, Section 4 provides an overall assessment of the theoretical and empirical analysis.

2. An optimizing model with foreign currency balances

This section describes a dynamic model of currency holdings with forward-looking rational expectations in a developing-country context. First, the optimal

[4] The absence of adequate data on market interest rates is not solely a reflection of insufficiently developed financial markets. As discussed in Agénor and Montiel (1996), sizable and relatively sophisticated informal financial markets exist in a large number of developing countries. However, few, if any, countries collect and report systematically information on the behavior of interest rates in these markets, thus preventing the use of such data in time-series econometric studies.

composition of currency holdings is derived, and then actual currency holdings are determined in a multi-period costs-of-adjustment framework.

2.1. Optimal composition of currency holdings

The hypothetical world of this model consists of a small open developing economy with no commercial banks and a dual exchange rate regime. The dual exchange market consists of an official market for foreign exchange, in which the official rate is pegged, and a legal (or quasi-legal) parallel market for foreign exchange. Commercial transactions are settled partly in the official market at the exchange rate $\bar{e}_t$. The rest of commercial transactions and all capital transactions are settled in the parallel market at the exchange rate s_t, which is determined by market forces [5].

Total output, which consists of a single exportable good only, is assumed to be fixed at level q. The price of the good is set on world markets. In each period, producers surrender a given proportion of their foreign exchange earnings at the official exchange rate and repatriate the remaining proceeds via the parallel market. There is no domestic expenditure on the domestically produced good; thus, total output is exported and total consumption is imported. Domestic agents consume two goods, both of them produced abroad. The first good is imported through legal channels, without restriction, at the official exchange rate. The second good is assumed to be prohibited by law, and thus can only be smuggled in and traded illegally in the economy [6]. The domestic price of the legally imported good is therefore influenced by the official exchange rate, and is given by $\bar{e}_t p_w$ under purchasing power parity, where p_w denotes the (given) world price. Similarly, the domestic price of the smuggled good reflects the marginal cost of foreign exchange (that is, the parallel market rate) and is given by $s_t \tilde{p}_w$. In what follows, the price of the legally imported good is used as the numéraire. Setting $p_w = \tilde{p}_w = 1$, the unit of account is therefore the official exchange rate, while the relative price of the legally imported good in terms of the illegally traded good is given by the inverse of (one plus) the parallel market premium, defined as $\rho_t = s_t / \bar{e}_t$.

Residents are assumed to hold four types of assets: domestic money, foreign money, bonds denominated in domestic currency and bonds denominated in foreign currency. The domestic-currency bond yields i units of domestic currency after one period of time and has a fixed face value equal to one unit of domestic money. The foreign-currency bond pays i^* units of foreign currency after one

[5] Both $\bar{e}_t$ and s_t are defined as the number of units of foreign currency per unit of domestic currency.

[6] Alternatively, the second good can be a "luxury good" that cannot be imported at the official exchange rate, but can be imported legally through the parallel market.

period and has a face value of one foreign-currency unit. Domestic and foreign money are imperfectly substitutable, non-interest-bearing assets. Although both consumption goods are imported, it is assumed, as in Calvo and Végh (1992), that legal restrictions require private agents to use the domestic currency in some domestic transactions – such as those involving transactions with the government and purchases of large-ticket items. Following Calvo and Végh (1990), domestic agents also hold foreign currency in addition to their own currency because of a "liquidity-in-advance" constraint on purchases of both goods. Goods are perishable so that domestic residents must allocate their wealth, in each period, between the available financial assets.

There is a single representative agent who maximizes a discounted sum of future instantaneous utilities. The individual's objective function takes the form [7]:

$$\sum_{t=0}^{\infty} \gamma^t V(c_t, \tilde{c}_t), \tag{1}$$

where $\gamma = 1/(1+\alpha)$, and $\alpha > 0$ is the rate of time preference [8].

At the beginning of every period t, the agent holds M_{t-1} units of domestic money, F_{t-1} units of foreign money, B_{t-1} units of domestic – currency bonds and b^*_{t-1} units of foreign-currency bonds. He then receives his output endowment, nominal interest payments $i_{t-1}B_{t-1}$ and $i^*_{t-1}b^*_{t-1}$, and decides on consumption of imported goods. He allocates his implied end-of-period nominal assets A_t among M_t, F_t, B_t and b^*_t.

Formally, the individual's accumulation program must satisfy the constraints:

$$A_t = M_t + B_t + s_t(b^*_t + F_t), \tag{2}$$

$$\begin{aligned} \Delta A_t \equiv A_t - A_{t-1} = {} & (1-\sigma)\bar{e}_t q + \sigma s_t q + i_{t-1}B_{t-1} + \bar{e}_t i^*_{t-1} b^*_{t-1} \\ & - (\bar{e}_t c_t + s_t \tilde{c}_t) + \Delta s_t (b^*_{t-1} + F_{t-1}). \end{aligned} \tag{3}$$

Eq. (2) defines nominal wealth as the sum of holdings of domestic and foreign currency, and holdings of domestic and foreign bonds [9]. Eq. (3) represents the flow budget constraint of the representative consumer. The parameter σ denotes the fraction of total export earnings repatriated via the parallel market, and for simplicity is assumed to be constant [10]. The domestic-currency value of exports channelled through the official market for foreign exchange is, therefore,

[7] For simplicity, the presentation of the model assumes away uncertainty. The possibility of random shocks is explicitly accounted for in the determination of the actual currency ratio.

[8] $V(\cdot)$ is defined for all $(c_t, \tilde{c}_t) > (0, 0)$, and is assumed to be strictly concave and twice continuously differentiable.

[9] Foreign currency holdings and holdings of foreign bonds are valued at the unofficial exchange rate, since capital transactions are settled in the parallel market.

[10] In a more general formulation σ would be endogenous and related to the size of the parallel market premium as, for instance, in Bhandari and Végh (1990).

$(1-\sigma)\bar{e}_t q$. Interest payments on foreign bonds, $i^*_{t-1} b^*_{t-1}$, are assumed to be repatriated at the official exchange rate. The quantities of the legally and illegally imported goods consumed by the representative consumer are given by c_t and $\tilde{c}_t$, respectively. The last term in Eq. (3) represents valuation effects on the stock of foreign currency and foreign bonds.

Dividing Eq. (2) by $\bar{e}_t$ yields real wealth (measured in terms of the price of the legally imported good) as:

$$a_t = m_t + b_t + \rho_t(b^*_t + F_t), \tag{2'}$$

where $a_t \equiv A_t/\bar{e}_t$, $m_t \equiv M_t/\bar{e}_t$, and $b_t \equiv B_t/\bar{e}_t$. Similarly, dividing Eq. (3) by $\bar{e}_t$ and re-arranging yields:

$$\begin{aligned} a_t - a_{t-1} = {} & [1+\sigma(\rho_t - 1)]q + [i_{t-1} - \epsilon_t(1+i_{t-1})]b_{t-1} - (c_t + \rho_t\tilde{c}_t) \\ & + [i^*_{t-1} + \Delta\rho_t(1+\epsilon_t)]b^*_{t-1} + \Delta\rho_t(1+\epsilon_t)F_{t-1} - \epsilon_t m_{t-1}, \end{aligned} \tag{3'}$$

where $\epsilon_t \equiv \Delta\bar{e}_t/\bar{e}_t$ and $\Delta\rho_t/\rho_t = \Delta s_t/s_t - \epsilon_t$.

In addition to constraints (2) and (3), the consumer is subject to a liquidity-in-advance constraint, which requires him to hold both domestic and foreign money in order to carry out transactions [11]:

$$c_t + \rho_t\tilde{c}_t \leq L(m_t, \rho_t F_t). \tag{4}$$

Eq. (4) indicates that total real expenditure on both goods cannot exceed the flow of liquidity services produced by the use of domestic and foreign currencies [12]. The "liquidity services" production function $L(.)$ is assumed concave, homogeneous of degree one, twice-continuously differentiable, with partial derivatives given by [13]:

$$\partial L/\partial m > 0, \partial L/\partial(\rho F) > 0,$$

$$\partial L^2/\partial m^2 < 0, \partial L^2/\partial(\rho F)^2 < 0, \partial L^2/\partial m\partial(\rho F) > 0.$$

[11] See Boyer and Kingston (1986) for the use of the cash-in-advance constraint as a microeconomic foundation for currency substitution. Similarly, Greenwood and Kimbrough (1987) motivate the existence of a parallel currency market with a cash-in-advance requirement that forces individuals to accumulate foreign currency (either officially or illegally) before they can consume. The specification of (4) assumes that domestic and foreign currency are equally important in meeting expenditures in the formal and informal markets. A more general approach would be to specify separate constraints to allow for different currency intensities in the two markets. Such a model, however, is difficult to estimate as the size of the informal market is typically unknown.

[12] Strictly speaking, in discrete time the approximation to the Calvo and Végh (1990) liquidity-in-advance constraint would require using beginning-of-period stocks in Eq. (4). The formulation used here is, however, more convenient analytically.

[13] The assumption on the cross derivative of L rules out perfect substitutability between domestic and foreign currencies, and therefore eliminates corner solutions. If there is no limit on substitutability, the Kareken–Wallace indeterminacy result would hold (see Sargent, 1987b, pp. 188–92).

To simplify the algebra somewhat, it will be assumed that the consumer's instantaneous utility is a separable, logarithmic function, and that the liquidity services function is of the Cobb–Douglas form, therefore constraining to unity the elasticity of substitution between domestic and foreign currency:

$$V(c_t,\tilde{c}_t) = log c_t + \log \tilde{c}_t, \tag{5}$$

$$L(m_t,\rho_t F_t) = (m_t)^{\delta}(\rho_t F_t)^{1-\delta}. 0 \leq \delta \leq 1 \tag{6}$$

The consumer chooses an optimal sequence $\{c_t, \tilde{c}_t, m_t, b_t, b_t^*, F_t\}_{t=0}^{\infty}$ to maximize (1) subject to (2)–(6). Assuming that Eq. (4) holds with equality, it is shown in the Appendix that the optimality conditions for this control problem yield the following solution for the composition of money holdings:

$$\frac{m_t}{\rho_t F_t} = \left(\frac{\delta}{1-\delta}\right)\left[\frac{i_t^*}{(1-\epsilon_{t+1})i_t}\right], \tag{7a}$$

and,

$$(1-\epsilon_{t-1})i_t = (i_t^* + \Delta\rho_{t+1}(1+\epsilon_{t+1}))/\rho_t + \epsilon_{t+1}. \tag{7b}$$

When $\epsilon_{t+1} \equiv 0$ (that is, under fixed exchange rates), Eqs. (7a) and (7b) become:

$$m_t/\rho_t F_t = [(1-\delta)/\delta](i_t^*/i_t), \tag{7a$'$}$$

$$i_t = (i_t^* + \Delta s_{t+1})/\rho_t. \tag{7b$'$}$$

Consider first Eqs. (7a′) and (7b′). Essentially, Eq. (7a′) defines a demand function for the currency ratio $m_t/\rho_t F_t$, which relates the marginal rate of substitution between domestic and foreign currencies inversely to the ratio of their opportunity costs. Eq. (7b′) represents an interest parity condition that has been used in several models of dual exchange markets, notably the continuous-time models of Obstfeld (1986) and Bhandari and Végh (1990). It implies that an expected future depreciation of the domestic currency in the parallel market for foreign exchange would cause domestic residents to shift out of domestic money into foreign money and vice versa. Eqs. (7a) and (7b) generalize Eqs. (7a′) and (7b′) so as to account for fluctuations in the official exchange rate. By substituting the interest parity condition (7b) in Eq. (7a), the domestic interest rate can be eliminated, a procedure that will prove particularly useful at the estimation stage, because of the lack of published time-series data on market-determined rates in many developing countries.

2.2. Adjustment costs and actual currency holdings

The desired or "long-run" composition of currency holdings given by Eqs. (7a) and (7b), by using the definition of ρ_t, can be written as:

$$q_t^d \equiv \log(M_t/s_t F_t) = \kappa_0 + \kappa_1 z_t, \kappa_1 > 0 \tag{7$''$}$$

where $z_t \equiv \log(i_t^* / [(i_t^* + \Delta\rho_{t+1}(1+\epsilon_{t+1}))/\rho_t + \epsilon_{t+1}])$, using the interest parity condition (7b) [14]. This formulation abstracts from institutional and structural factors, such as the transactions costs incurred in the exchange of currencies (including the probability of being caught in fraudulent activities), or the degree of diversification of the domestic financial system, that may influence the currency ratio. These factors would be reflected in the size of the coefficient κ_1.

Following the "buffer stock" approach of Cuthbertson and Taylor (1987, Cuthbertson and Taylor, 1990), we assume that agents pursue their currency ratio target subject to the costs of being out of long-run equilibrium and the costs of adjustment in currency holdings [15]. These costs of adjustment may be thought of as costs incurred during reallocation of the individual's monetary portfolio and may result, for instance, from the existence of regulations that restrict possibilities to acquire foreign exchange through official channels, or from restrictions on transfers of funds abroad. Formally, therefore, the representative agent undertakes a two-stage decision process. First, he determines the desired path of the currency ratio, $\{q_{t+k}^d\}_{k=0}^{\infty}$, and then makes an independent decision concerning the speed of adjustment towards this path.

Agents choose the short-run currency ratio so as to minimize the expected discounted present value of a quadratic loss function, L, conditional on information available at time $t-1$:

$$L = E_{t-1} \sum_{k=0}^{\infty} \gamma^k \left[\alpha_0 \left(q_{t+k} - q_{t+k}^d \right)^2 + \alpha_1 \left(q_{t-k} - q_{t+k-1} \right)^2 \right], \tag{8}$$

where α_i are positive weights and E_{t-1} denotes the conditional expectation operator for information up to time $t-1$. The actual currency ratio, q_t, is chosen so as to minimize (8), for $k = 0, 1, 2,...$ The solution to this optimization problem is provided in Appendix A. It is given by:

$$E_{t-1} q_t = \lambda_1 q_{t-1} + (1-\lambda_1)(1-\gamma\lambda_1) \sum_{j=0}^{\infty} (\gamma\lambda_1)^j E_{t-1} q_{t+j}^d, \tag{9}$$

where $0 < \lambda_1 < 1$ is the stable root of the Euler equation obtained from the first-order conditions. Using (7″) and (9), the equation describing the behavior of the actual currency ratio is therefore given by:

$$q_t = (1-\lambda_1)\kappa_0 + \lambda_1 q_{t-1} + (1-\lambda_1)(1-\gamma\lambda_1)\kappa_1 \sum_{j=0}^{\infty} (\gamma\lambda_1)^j E_{t-1} z_{t+j} + \nu_t \tag{10}$$

[14] It can be shown that by using a *CES* liquidity function (instead of the Cobb–Douglas formulation given in Eq. (6)), the more general restriction $\kappa_1 > 0$ (instead of $\kappa_1 = 1$) would hold.

[15] For an evaluation and extensions of the Cuthbertson–Taylor forward-looking model of buffer stock money, see Muscatelli (1988), Swamy and Tavlas (1989) and Pesaran (1991).

where ν_t denotes a random disturbance, which results from the imposition of rational expectations.

Eq. (10) shows that the currency ratio depends on a backward-looking component, q_{t-1}, and forward-looking variables, a geometrically-declining weighted sum of the opportunity-cost variable. For instance, a 1 percent increase in z_t in the current period and all future periods leads to an immediate rise in the currency ratio of $(1-\lambda_1)(1-\gamma\lambda_1)\kappa_1$ percent, which thereafter continues to rise gradually, due to the existence of adjustment costs, to its new desired path. If the opportunity cost variable z_t is generated by a random walk process, then the forward-looking model described by Eq. (10) reduces to a form that is of the partial adjustment (or adaptive expectations) variety (see Nickell, 1985).

3. Econometric methodology and estimation results

This section first describes the estimation procedure and then discusses the results obtained from estimation of Eq. (10). As noted in the Introduction, one needs to recognize several shortcomings and limitations in the data. The definition of ''foreign money balances'' in a currency substitution framework may cover several components, such as foreign currency deposits held in the domestic financial system, foreign money held abroad by domestic residents and foreign currency notes circulating domestically. Data on all three components are extremely difficult to obtain for any country. Moreover, even when it is possible to isolate, say, foreign currency deposits in the domestic financial system, it is not always possible to identify whether the deposits are held by foreigners or by domestic residents. A particularly important source of error in the measurement of foreign money balances relates to the amount of foreign currency notes held by the public (in this case, domestic residents) in the country. The quantitative importance of this component is extremely difficult to evaluate, especially in a country where parallel currency markets are important. Therefore, we focus here on residents' holdings of foreign currency deposits held in foreign banks abroad. Such a procedure does not take into account foreign currency deposits held in domestic banks and domestic nonbank foreign money balances. Accordingly, this measure will underestimate the magnitude of currency substitution if residents hold significant foreign currency deposits in domestic banks and if a significant portion of financial transactions in foreign currency takes place outside the formal banking system [16]. Finally, we assume that money and bonds can be subsumed into money

[16] Note also that the measures of domestic and foreign currency holdings used below for estimation include holdings of firms as well as households, while the theoretical model developed earlier pertains only to consumer money demand. The lack of data on business currency holdings prevented us from appropriately adjusting for this component.

holdings, and therefore the empirical analysis deals only with domestic and foreign currency assets.

3.1. Estimation method

To estimate Eq. (10), multi-step ahead predictions for the determinants of the currency ratio must be derived. Several alternative procedures can be used for modeling the expectational terms appearing in the model. These procedures can be conveniently grouped into two classes: "substitution" methods, and "errors-in-variables" techniques. In the first case, the unknown future expectations entering the equation to be estimated are replaced by generated or predicted values derived from a forecasting equation for the driving process. This gives rise to the "generated regressors" situation, which has been extensively analyzed by Pagan (1984). In this paper, we use the errors-in-variables technique suggested by Wickens (1986) [17].

Another problem that needs to be handled when estimating Eq. (10) is the existence of nonlinearities introduced by the presence of the discount factor γ in several terms. In principle, estimation of the model in a restricted form would require the use of nonlinear least squares. Here, instead, the model is estimated in an unrestricted form and the parameter constraints imposed by the rational expectations assumption are tested for three different values of γ (0.1, 0.5 and 1.0), avoiding thereby the complications that would result from the application of a nonlinear estimation procedure. The restrictions result from the geometric term $(\gamma\lambda_1)^j$, which appears in the coefficients attached to the successive future expected values of the opportunity-cost variable z. Given the estimated value of λ_1 obtained from the coefficient on q_{t-1} in Eq. (10), these parameter restrictions (also known as "backward-forward" restrictions) are tested using a Wald test [18].

3.2. Empirical results

Using the econometric procedure described above, the model described by Eq. (10) was estimated for ten developing countries: Bangladesh, Brazil, Ecuador, Indonesia, Malaysia, Mexico, Morocco, Nigeria, Pakistan and the Philippines.

[17] In the application of this technique below, we do not account for the moving average property of the error process, which results from the errors-in-variables transformation (see Wickens, 1986, Cuthbertson, 1990). Our estimates, although not fully efficient, are nevertheless consistent. Other limited-information methods (such as the Generalized *IV* technique or the Generalized *2SLS* procedure discussed by Cuthbertson, 1990) are only asymptotically efficient and are of limited use in the small sample size used here.

[18] Let $\mu_j \equiv (1-\lambda_1)(1-\gamma\lambda_1)\kappa_1(\gamma\lambda_1)^j$ denote the estimated coefficient on z_{t+j}, where $j \geq 0,... n$. The backward-forward restrictions are given by the n relationships:

$$\mu_0 / \mu_j = 1/(\gamma\lambda_1)^j, j = 1,...n$$

While these countries were selected primarily because of data availability, the sample is fairly representative of developing countries in general. Data are quarterly and seasonally unadjusted for the period October 1981 to June 1991 [19]. Seasonal dummies were included in all regressions but are omitted in the following tables for convenience.

Table 1 presents unrestricted estimates for the parameters of Eq. (10) [20]. We initially experimented with up to four leads on the expectational variable z_t for each country, and then "tested down" until an adequate specification was found [21]. The results indicate that the model performs fairly well in most cases. The coefficient of determination is reasonably high for most countries, except perhaps Malaysia, indicating that the regression accounts for a sizable fraction of the variance of the currency ratio for these countries. The Lagrange multiplier test statistic indicates the absence of autocorrelation of the residuals in all cases. With the exception of Mexico, there is no evidence of departure from normality of the residuals, as shown by the Jarque–Bera test statistic, and evidence of (autoregressive conditional) heteroscedasticity seems to appear only for Bangladesh. The predictive Chow test, which examines the ex ante forecasting capability of the model over the period April 1988–January 1990 indicates parameter instability only for Morocco, and only at a 5 percent significance level [22]. Finally, for three countries, Brazil, Malaysia and Mexico, there is no evidence of expectational effects beyond a quarter ahead, suggesting for these economies a fairly short forward horizon and/or a very high discount rate. Interestingly enough, Brazil and

[19] Data on foreign currency deposits held abroad were obtained from *International Financial Statistics* (*IFS*, line 7*xrd*), published by the International Monetary Fund. The domestic money stock is measured by narrow money (*IFS* line 34). The "world" interest rate is measured by the three-month Eurodollar rate (*IFS* line 60*d* for the UK). The official exchange rate (*IFS* line *ae*) is the end-of-period exchange rate between the domestic currency and the US dollar; the parallel exchange rate (obtained from the *World Currency Yearbook*) is similarly defined.

[20] Unit root tests and cointegration tests between the currency ratio and the opportunity cost ratio were also conducted. The results of these tests, together with a discussion of the relationship between the forward-looking model estimated here and an error-correction representation, are presented in Agénor and Khan (1992). In general, the currency ratio and the opportunity cost ratio are stationary in levels. For the three countries for which both variables appeared to be integrated of order one, we found evidence of cointegration in only one case. However, given the small size of our sample and the low power of the cointegration test used (the Dickey–Fuller test), this result may not be highly significant.

[21] The appropriate length of the expectation horizon was based on a joint F-test of all z variables in the equation. In some cases examined here, adding expectational terms for periods further ahead produced problems of multicollinearity. In other cases, longer-term expectations were statistically insignificant.

[22] The ability of the estimated currency ratio equation to generate good out-of-sample predictions represents an important diagnostic test for the hypothesis of parameter stability. Persistent out-of-sample overprediction of the currency ratio for instance, would tend to indicate the occurrence of a shift in the underlying relationship.

Table 1
Estimation results: forward-looking model (errors-in-variables procedure)

Variables	Country									
	Bangladesh	Brazil	Ecuador	Indonesia	Malaysia	Mexico	Morocco	Nigeria	Pakistan	Philippines
Constant	1.478	0.396	0.321	0.186	0.793	−0.042	0.389	−0.113	0.183	0.151
	(2.131)	(1.729)	(1.527)	(1.159)	(2.221)	(−0.247)	(1.387)	(−0.662)	(1.227)	(1.736)
q_{t-1}	0.431	0.915	1.132	0.891	0.587	0.869	0.804	0.364	0.877	0.759
	(1.496)	(10.563)	(8.745)	(4.051)	(2.408)	(8.157)	(4.809)	(1.906)	(9.665)	(5.213)
z_t	−0.145	−0.348	−0.018	0.439	−4.103	−0.273	−1.151	−0.405	0.112	0.103
	(−1.193)	(−1.504)	(−0.082)	(0.331)	(−0.699)	(−0.347)	(−1.165)	(−1.224)	(0.316)	(0.764)
z_{t+1}	0.026	0.544	−0.623	0.821	4.906	1.081	−0.239	0.158	−0.016	−0.610
	(0.017)	(1.813)	(−1.506)	(2.934)	(1.733)	(1.813)	(−0.782)	(0.263)	(−0.035)	(−0.816)
z_{t+2}	0.381	–	0.661	−1.523	–	–	1.766	0.448	0.287	0.834
	(2.190)		(1.557)	(−0.547)			(2.481)	(1.991)	(1.833)	(2.870)
z_{t+3}	−0.498	–	−0.801	−0.279	–	–	–	−0.245	–	–
	(−0.779)		(−2.117)	(−0.667)				(−0.319)		
z_{t+4}	1.186	–	–	0.996	–	–	–	0.624	–	–
	(1.659)			(2.507)				(2.362)		
$\bar{R}^2$	0.892	0.797	0.871	0.927	0.384	0.748	0.756	0.796	0.839	0.689
$\hat{\sigma}$	0.219	0.153	0.156	0.078	0.196	0.157	0.087	0.153	0.079	0.141
lm(8)	0.138	1.326	0.725	0.207	1.146	0.034	0.041	0.113	0.064	0.453
arch(8)	2.493	1.012	0.546	0.223	1.075	0.088	0.493	1.117	0.481	0.974
JQ	0.739	0.119	1.094	1.753	0.701	7.159	0.809	0.359	1.641	1.503
Chow	0.241	1.197	0.375	1.954	0.947	0.531	2.197	1.473	0.263	0.065
κ_1	1.669	2.341	–	4.165	1.944	6.167	1.918	0.911	3.114	1.357
	(1.907)	(6.003)		(1.930)	(1.297)	(6.213)	(1.797)	(1.768)	(8.416)	(2.074)

Estimation period is October 1982–March 1990. Instruments are 4 lags of q and z, seasonal dummies and the constant term. $\bar{R}^2$ denotes the adjusted coefficient of determination, $\hat{\sigma}$ the estimated standard error of the regression, *JQ* the Jarque–Bera normality test statistic and *lm(n)* is the Lagrange multiplier test statistic for serial correlation in the residuals of order up to n. *Arch(n)* is the Engle test statistic for autoregressive conditional heteroscedasticity of order up to n. *Chow* denotes the Chow predictive failure test over October 1988–March 1990. Numbers in parentheses denote t-statistics. A – indicates that the parameter is set to zero or is not defined. See Harvey (1990) for a description of the diagnostic tests reported in this table.

Table 2
Test statistics for backward-forward restrictions

Country	Restrictions	Discount factor		
		$\gamma = 0.1$	$\gamma = 0.5$	$\gamma = 1.0$
Bangladesh	4	0.382	0.298	0.367
Brazil	1	1.657	1.922	2.068
Ecuador	3	10.82	8.087	4.846
Indonesia	4	1.879	1.895	1.701
Malaysia	1	1.486	2.326	2.788
Mexico	1	1.117	0.932	0.751
Morocco	2	1.756	1.739	2.003
Nigeria	4	0.751	0.859	1.004
Pakistan	2	0.681	0.503	0.291
Philippines	2	0.765	0.726	0.741

The backward-forward restrictions are tested using a Wald test statistic, which is distributed asymptotically as $\chi^2(m)$, where m is the number of restrictions. The critical values for rejection of the null hypothesis are, for 1, 2, 3 and 4 restrictions, 3.84, 5.99, 7.82 and 9.49, respectively, at a 5 percent significance level, and 2.71, 4.61, 6.25 and 7.78, respectively, at a 10 percent significance level.

Mexico are countries that have experienced considerable macroeconomic instability over the estimation period, a phenomenon which has often been viewed as conducive to a high degree of substitution between domestic and foreign currencies.

Wald test statistics for the validity of the parameter restrictions imposed by rational expectations are shown in Table 2, for $\gamma = 0.1$, $\gamma = 0.5$ and $\gamma = 1.0$. The results indicate, first, that the effect of varying the value of γ on the reported test statistics is negligible. Second, they also indicate that the restrictions are rejected for only one country at a 5 percent significance level (Ecuador, and only for low values of the discount factor) and for Malaysia as well (for high values of γ) at a 10 percent significance level.

Since the restrictions imposed by rational expectations cannot be rejected in most cases, the unrestricted parameters can be used to estimate the long-run elasticity of the currency ratio with respect to the opportunity cost variable, κ_1. The estimated value of κ_1, together with its approximate t-value, are shown at the bottom of Table 1. This elasticity is calculated by setting $z_t = z_{t+1} = \ldots$ in the regression equation and dividing the parameter sum by 1 minus the coefficient on the lagged currency ratio, as implied by Eq. (10) [23]. The results indicate that the long-run elasticity coefficients are significant and in the expected range (except perhaps for Nigeria, which seems to be on the low side) and are statistically significant at conventional levels, except for Malaysia.

[23] The estimated value of κ_1 may lack efficiency because unrestricted parameter estimates are used and because the infinite sum appearing in Eq. (10) is truncated.

For comparison purposes, a "conventional" partial adjustment model, relating the currency ratio to its lagged value and only the current rate of depreciation of the parallel market exchange rate, was also estimated. Comparison with the forward-looking model was made on the basis of standard regression diagnostic tests and of a non-nested procedure due to MacKinnon et al. (1983). The results, which are reported in detail in Agénor and Khan (1992), suggest that the forward-looking model performs better than the partial adjustment model for almost all countries in the sample. Given that the forward-looking model is rigorously derived from an optimizing framework, and that it provides an adequate representation of the data, it can be viewed as a better alternative to more conventional partial adjustment models.

4. Conclusions

The purpose of this paper has been to derive and test a currency substitution model in which portfolio decisions depend upon forward-looking variables. The long-run, or equilibrium, currency ratio is derived from an optimizing model in which agents can hold domestic and foreign currencies, and the actual currency ratio is derived in a framework that is based on a multi-period cost of adjustment scheme, involving rational forward-looking expectations. Empirical implementation of the model for ten developing countries has proven encouraging. The model had statistically significant coefficients as implied by the theory, and its forecasting ability appeared satisfactory. Essentially, a substantial amount of the variation in the ratio of domestic to foreign money balances is accounted for by changes in foreign interest rates and in the premium in the parallel exchange market. A formal comparison of the forward-looking model with a conventional, partial-adjustment specification using a non-nested testing procedure shows that the former model is preferred in almost all cases.

Aside from the use of a rational-expectations framework, two other important methodological and empirical features of the paper are worth mentioning. First, use is made of data on foreign currency deposits held abroad. Such data have not been employed before in the literature dealing with currency substitution in developing countries. Second, the model does not require information on domestic interest rates, a formulation that appears to be an important advantage in the case of developing countries where appropriate time-series data on market-based interest rates are generally not readily available. In this sense the paper goes beyond other empirical studies of currency substitution and the approach outlined can be a useful framework upon which to build. One possible extension of this model would be to introduce separate liquidity constraints, to capture the fact that purchases in illegal markets may be more foreign-currency intensive than transactions conducted through official markets. To the extent that adequate information on the relative sizes of the formal and informal markets can be obtained, the impact of the difference in the intensity of currency use on the degree of currency substitution could be estimated.

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Appendix A

In this appendix we first derive the necessary conditions for the optimal individual optimization problem described in Eqs. (1)–(6). Let λ_t^1, λ_t^2 and λ_t^3 denote the Lagrange multipliers associated with, respectively, the portfolio constraint (2′), the flow constraint (3′), and the cash-in-advance constraint (4). Necessary conditions for optimality are found by maximizing the Lagrangian:

$$\begin{aligned}\mathscr{L} = \sum_{t=0}^{\infty} \gamma^t \Big\{ & V(c_t, \tilde{c}_t) + \lambda_t^1 [a_t - m_t - b_t - \rho_t(b_t^* + F_t)] - \lambda_t^2 [a_t - a_{t-1} \\ & - [1 + \sigma(\rho_t - 1)] q - [i_{t-1} - \epsilon_t(1 + i_{t-1})] b_{t-1} + (c_t + \rho_t \tilde{c}_t) \\ & - [i_{t-1}^* + \Delta\rho_t(1 + \epsilon_t)] b_{t-1}^* - \Delta\rho_t(1 + \epsilon_t) F_{t-1} + \epsilon_t m_{t-1}] \\ & - \lambda_t^3 [c_t + \rho_t \tilde{c}_t - (m_t)^\delta (\rho_t F_t)^{1-\delta}] \Big\}, \end{aligned} \tag{A1}$$

with respect to c_t, $\tilde{c}_t$, m_t, b_t, b_t^*, F_t and a_t, $\forall t$. Using Eq. (5) yields [24]:

$$\partial V/\partial c_t = 1/c_t = \lambda_t^2 + \lambda_t^3, \tag{A2a}$$

$$\partial V/\partial \tilde{c}_t = 1/\tilde{c}_t = \rho_t(\lambda_t^2 + \lambda_t^3), \tag{A2b}$$

$$\lambda_t^3 \delta(\rho_t F_t/m_t)^{1-\delta} = \lambda_t^1 + \gamma\lambda_{t+1}^2 \epsilon_{t+1}, \tag{A2c}$$

$$\gamma\lambda_{t+1}^2 [i_t - \epsilon_{t+1}(1 + i_t)] = \lambda_t^1, \tag{A2d}$$

$$\gamma\lambda_{t+1}^2 [i_t^* + \Delta\rho_{t+1}(1 + \epsilon_{t+1})] = \lambda_t^1 \rho_t, \tag{A2e}$$

$$\gamma\lambda_{t+1}^2 \Delta\rho_{t+1}(1 + \epsilon_{t+1}) + \lambda_t^3(1 - \delta)(m_t/\rho_t F_t)^\delta = \lambda_t^1 \rho_t, \tag{A2f}$$

$$\gamma\lambda_{t+1}^2 + \lambda_t^1 = \lambda_t^2. \tag{A2g}$$

[24] In addition to Eqs. (A2), the optimal solution must satisfy the lifetime resource constraint $\lim_{t\to\infty} (1 + \gamma)^{-t} a_t \geq 0$. Using Eq. (3′), this inequality can be written in a form that restricts the present value of spending on consumption and on the services of domestic and foreign real balances to the value of real wealth at the beginning of period 0.

First, note that from (A2a) and (A2b), $c_t = \rho_t \tilde{c}_t$, which yields, using (4), holding with equality, $2c_t = L(m_t, \rho_t Ft)$. These equations determine the path of consumption once the composition of currency holdings is known. Second, note that from (A2d) and (A2e),

$$i_t - \epsilon_{t+1}(1 + i_t) = (1/\rho_t)\left[i_t^* + \Delta\rho_{t+1}(1 + \epsilon_{t-1})\right], \tag{A3}$$

which yields Eq. (7b) in the text.

From (A2e), we can substitute out for λ_{t+1}^2 in (A2c) and (A2f). Dividing the resulting expressions for (A2c) by (A2f) and using (A3) yields Eq. (7a) in the text.

Using Eq. (8), we now examine how short-run currency holdings are determined. The solution of this optimization problem follows Sargent (1987a, pp. 393–396). Necessary first-order conditions for minimization of Eq. (8), with respect to q_t $(k = 0, 1, 2,...)$, are a set of stochastic Euler equations and a transversality condition, which are given by the following equations:

$$\gamma E_{t+k-1} q_{t+k+1} + \phi E_{t+k-1} q_{t+k} + q_{t+k-1} = -\alpha E_{t+k-1} q_{t+k}^d, \tag{A4a}$$

$$\lim_{T\to\infty} E_{t-1}\gamma^T\left[(\alpha_0 + \alpha_1)q_{t+T} - \alpha_1 q_{t+T-1} - \alpha_0 q_{t+T}^d\right] = 0, \tag{A4b}$$

where $\alpha = \alpha_0/\alpha_1$ and $\phi = -(1 + \alpha + \gamma)$. Eq. A4a can be rewritten as

$$(1 + \phi\gamma^{-1}B + \gamma^{-1}B^2)E_{t+k-1}q_{t+k-1} = -(\alpha/\gamma)E_{t+k-1}q_{t+k}^d, \tag{A5}$$

where B is the information-neutral backshift operator, defined such that $B^{-1}\mathrm{E}_{t-1}z_{t+k} = \mathrm{E}_{t-1}z_{t+k+1}$. Factoring the polynomial on the left-hand side of (A5) yields:

$$(1 - \lambda_1 B)(1 - \lambda_2 B) = 1 - (\lambda_1 + \lambda_2)B + \lambda_1\lambda_2 B^2 = 1 + \phi\gamma^{-1}B + \gamma^{-1}B^2. \tag{A6}$$

Equating powers of B gives $-\phi = \lambda_1\gamma + (1/\lambda_1)$ and $\lambda_2 = 1/\lambda_1\gamma$. Without loss of generality, assume that $\lambda_1 < \lambda_2$. As shown by Sargent (1987a, pp. 201–202), it can be established that $0 < \lambda_1 < 1/\gamma < \lambda_2$. The solution to the Euler equations which satisfies the transversality condition can be shown to be [25]:

$$E_{t+k-1}q_{t+k} = \lambda_1 q_{t+k-1} + \alpha\gamma^{-1}\lambda_2^{-1}\sum_{j=0}^{\infty}(\lambda_2^{-1})^j E_{t+k-1}q_{t+k+j}^d. \tag{A7}$$

Using the above relationship between λ_1 and λ_2, and setting $\tau = t + k$, (A7) can be written as:

$$E_{\tau-1}q_\tau = \lambda_1 q_{\tau-1} + (1 - \lambda_1)(1 - \gamma\lambda_1)\sum_{j=0}^{\infty}(\gamma\lambda_1)^j E_{\tau-1}q_{\tau+j}^d. \tag{A8}$$

[25] To satisfy the transversality condition, we need, in fact, to impose some restrictions on the exponentional orders of the sequences (q_t) and (q_t^d); see Sargent (1987a, p. 201).

Assuming that the actual currency ratio differs from its rationally expected value only by a zero-mean, serially uncorrelated disturbance term yields Eq. (9) in the text.

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