

# EXCHANGE RESTRICTIONS AND DEVALUATION CRISES

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## ABSTRACT

This paper develops a model of devaluation cycles for an economy in which foreign exchange transactions take place through both an official and a parallel market. The exchange rate differential determines the propensity to under-invoice exports. The authorities are assumed to follow a devaluation rule that relates official parity changes to the level of reserves. The analysis shows that such a rule leads to periodic cycles of erosion and recovery of foreign reserves, which bear some similarities to recent devaluation episodes in developing countries.

## I. INTRODUCTION

The literature on balance-of-payments crises pioneered by Krugman (1979) examines the consequences of incompatible monetary, fiscal, and exchange rate policies for the balance of payments of a small open economy. Krugman showed that, under perfect foresight, domestic credit creation in excess of money demand growth leads to a gradual loss of reserves, and ultimately to a speculative attack against the currency that forces the abandonment of the fixed exchange rate regime. This attack always occurs *before* the central bank would have run out of reserves in the absence of speculative capital flows.

Krugman's analysis has been extended in several directions.<sup>1</sup> Although the focus of the literature has been mainly on the transition from a fixed exchange rate to a post-collapse

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floating exchange rate, several studies have examined the case where the central bank devalues the currency instead of floating it.<sup>2</sup> This scenario appears particularly appropriate for the analysis of balance-of-payments crises in developing countries. Edwards (1989) and Edwards and Montiel (1989), for instance, characterize a typical crisis as follows. Fiscal imbalances lead to rates of domestic credit expansion above money demand growth, which typically translate into an excess demand for goods, traded and nontraded, in addition to increased demand for foreign financial assets. The excess demand for tradables generates a loss of foreign reserves for the central bank, while the excess demand for nontradables translates into higher prices for those goods. As a consequence, the real exchange rate appreciates. If fiscal and monetary policies are not altered, the central bank will eventually "run out" of reserves, and a balance-of-payments crisis will ensue. In such a framework, a nominal devaluation may correct the real exchange overvaluation produced by the expansionary fiscal and credit policies. But as long as the devaluation is not accompanied by a reversal of the unsustainable macroeconomic policy stance, nominal parity changes can only yield temporary improvements in the balance of payments; without fiscal restraint, there will be recurrent balance-of-payments crises that may eventually lead to a devaluation-inflation spiral.<sup>3</sup> A similar "devaluation cycle" has been formalized by Kamin (1993) and—in a different context—by Wyplosz (1986).

The purpose of this paper is to describe an alternative macroeconomic process leading to recurrent balance-of-payments crises and devaluation cycles. The model developed here assumes that the economy operates an informal, dual exchange rate system with a fixed official exchange rate and a freely determined parallel rate—a framework that appears particularly appropriate for developing countries.<sup>4</sup> In this framework—as in standard models of balance-of-payments crises—a devaluation is triggered when foreign reserves hit a lower threshold level. The mechanism through which reserves evolve, however, is related to the exchange rate differential, and results from widespread under-invoicing of export remittances. With a well-defined devaluation rule, agents are able to anticipate future official parity changes, and such expectations are reflected immediately in the behavior of the parallel exchange rate.

The remainder of the paper is organized as follows. Section II sets out the analytical framework. Section III determines the behavior of the parallel market premium and foreign reserves before and after an official parity change. Section IV examines the stability of reserves/devaluation cycles. Section V summarizes the results and briefly discusses the empirical regularities predicted by the analysis.

## II. THE ANALYTICAL FRAMEWORK

Consider a small open economy that operates a dual exchange rate arrangement in which an official, pegged exchange rate coexists with a freely determined parallel rate. The official rate applies to most current account transactions, while the parallel rate is used for capital account transactions and current account transactions that are not allowed to take place through the official market for foreign exchange. Agents are endowed with perfect foresight

and hold only domestic and foreign currencies in their portfolios. Domestic output, which consists of a single exportable good, is sold entirely abroad and is taken as exogenous. In each period, exporters must determine which proportion of their foreign exchange earnings they will surrender at the official exchange rate, and which proportion they will repatriate via the parallel market.

Formally, the model is described by the following log-linear equations, where all parameters are defined as positive:

$$m_t - p_t = \alpha i_t, \quad (1)$$

$$i_t = i^* + \dot{s}_t - \gamma(s_t - e_t), \quad (2)$$

$$m_t = \nu(e_t + R_t) + (1 - \nu)D_t, \quad 0 < \nu < 1 \quad (3)$$

$$p_t = \Theta s_t + (1 - \Theta)e_t, \quad 0 < \Theta < 1 \quad (4)$$

$$\dot{R}_t = -\Phi(s_t - e_t), \quad (5)$$

$$\dot{D}_t = \mu / (1 - \nu), \quad (6)$$

$$e_t \begin{cases} e_0 & \text{for } R_t > 0, \\ e_0 + \delta & \text{for } R_t = 0. \end{cases} \quad (7)$$

where  $m_t$  denotes the nominal money stock,  $p_t$  the domestic price level,  $D_t$  domestic credit,  $R_t$  the foreign currency value of the stock of net foreign assets held by the central bank,  $e_t$  the official exchange rate,  $s_t$  the parallel exchange rate,  $i_t$  the domestic nominal interest rate and  $i^*$  the (constant) foreign interest rate. All variables are measured in logarithms except interest rates.

Equation (1) describes money market equilibrium. Equation (2) depicts the interest parity condition, modified for a dual exchange-rate system.<sup>5</sup> Equation (3) is a log-linear approximation that defines the domestic money stock as a weighted average of domestic credit and foreign reserves. Equation (4) indicates that the price level depends on the official and parallel exchange rates. This results from the assumption that some commercial transactions are settled in the parallel market, with the domestic price of such imports reflecting the marginal cost of foreign exchange—that is, the parallel rate. The purchasing power parity assumption is assumed to hold therefore at a composite exchange rate, and the foreign price level is set to unity (so that its logarithm is zero) for simplicity.<sup>6</sup> Equation (5) describes the behavior of net foreign assets. The negative effect of the premium—defined as the difference between the official and the parallel exchange rates—on the behavior of reserves results from its impact on the propensity to under-invoice exports. The higher the parallel exchange rate is relative to the official rate, the greater the incentive to falsify export invoices and to divert export proceeds to the unofficial market.<sup>7</sup> Equation (6) indicates that the stock of credit is assumed to grow at a constant rate,  $\mu / (1 - \nu)$ . Finally, equation (7) describes the devaluation

rule followed by the central bank. The official exchange rate is maintained (at the level  $e_0$ , assuming no devaluation has occurred between periods 0 and  $t$ ) as long as reserves remain positive. A devaluation (of size  $\delta$ ) is implemented when reserves reach a lower value of zero. As in Rodriguez (1978) and Kamin (1993), the size of the official parity change is assumed to be high enough to ensure a (temporary, at least) increase in reserves, and is taken as exogenous for simplicity.<sup>8</sup>

### III. SOLUTION AND DYNAMICS

To solve the model described above, it is convenient to define the following variables:

$$z_t = (1 - v)(D_t - e_t), \quad (8)$$

$$\rho_t = s_t - e_t. \quad (9)$$

Therefore  $z_t$  is the "forcing variable" of the system, while  $\rho_t$  denotes the parallel market premium. Since both variables are a function of the official exchange rate, they are both subject to discrete jumps.

Using equations (8) and (9) and setting  $i^* = 0$ , the system described by equations (1)–(6) can be written as

$$\begin{bmatrix} \dot{\rho}_t \\ \dot{R}_t \end{bmatrix} = \begin{bmatrix} (\Theta + \alpha\gamma) / \alpha & -v / \alpha \\ -\Phi & 0 \end{bmatrix} \begin{bmatrix} \rho_t \\ R_t \end{bmatrix} + \begin{bmatrix} -z_t / \alpha \\ 0 \end{bmatrix}. \quad (10)$$

Equations (10) form a non-homogenous system of first-order differential equations in  $\rho_t$  and  $R_t$ , which is saddle-point stable.<sup>9</sup> Denoting by  $\lambda_1$  the negative root and by  $\lambda_2$  the positive root, the general solution can be written as

$$\rho_t = \rho_t^* + C_1 k_1 e^{\lambda_1 t} + C_2 k_2 e^{\lambda_2 t}, \quad (11a)$$

$$R_t = R_t^* + C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}, \quad (11b)$$

where  $k_h = v / [\alpha\lambda_h - (1 - \Theta - \alpha\gamma)]$ ,  $h = 1, 2$ ;  $k_1 < 0$ ,  $k_2 > 0$ .

The  $\rho_t^*$  and  $R_t^*$  in equations (11) denote the steady-state values of the premium and the stock of reserves, respectively, and  $C_1$  and  $C_2$  are as yet undetermined coefficients.<sup>10</sup> If private agents have no information about what policies the authorities will implement when reserves hit their lower level of zero, stability would require setting  $C_2 = 0$  in equations (10). Using an initial condition on reserves would then allow the determination of  $C_1$ . However, since the authorities follow a well-defined rule that relates official parity changes to the behavior of reserves and since agents are well aware of the trigger mechanism it involves,  $C_2$  will not be zero. As shown below, both coefficients are determined by a set of constraints imposed by the dynamic properties of the model.

Before proving this result, we need to calculate the steady-state values of the premium and foreign reserves. Using the method of undetermined coefficients, the solution for  $\rho_t^*$  can be conjectured to be

$$\rho_t^* = \kappa_0 + \kappa_1 R_t + \kappa_2 z_t. \quad (12)$$

Substituting equation (12) in equations (10) and solving yields

$$\kappa_0 = \alpha \kappa_2 \mu / (\Theta + \alpha \gamma + \alpha \Phi \kappa_1),$$

$$\kappa_1 = \frac{1}{2\alpha\Phi} \{ -(\Theta + \alpha\gamma) \pm [(\Theta + \alpha\gamma)^2 + 4\nu\alpha\Phi]^{1/2} \},$$

$$\kappa_2 = 1/(\Theta + \alpha\Phi\kappa_1).$$

There are two values of  $\kappa_1$ , one positive, the other negative. To choose among these two values, observe that substituting equation (12) in the second equation of the system (10) yields a differential equation in  $R_t$  which, to be stable, requires selection of the positive value of  $\kappa_1$  — a condition which implies that  $\kappa_1 = -\lambda_1 / \Phi$  and ensures that  $\kappa_0, \kappa_2 > 0$ . To obtain the long-run solution for  $R_t^*$ , substitute equation (12) in the second equation of (10), so that

$$\dot{R}_t = -\Phi\kappa_1 R_t - \Phi(\kappa_0 + \kappa_2 z_t),$$

which is a differential equation in  $R_t$ . Using the relationship given above between  $\kappa_1$  and  $\lambda_1$ , the solution to this equation can be shown to be given by

$$R_t^* = R_0 e^{\lambda_1 t} - \frac{\Phi\kappa_2 z_0}{\lambda_1} \{ e^{\lambda_1 t} - 1 \} \quad (13)$$

$$- (\Phi\kappa_2 \mu) \frac{e^{-\lambda_1 t}}{\lambda_1^2} (\lambda_1 t - 1) - (\Phi\kappa_2 \mu / \lambda_1^2) - \frac{\Phi\kappa_0}{\lambda_1} \{ e^{\lambda_1 t} - 1 \},$$

where  $R_0$ , the initial value of reserves at  $t = 0$ , is taken as given.

To determine the complete solution requires now the imposition of three conditions.

1. We have an initial condition on the (predetermined) level of foreign reserves, which we assume to be  $R_0 = 0$ .
2. We know that reserves reach a lower level at time (say)  $\tau$ , which we also take to be  $R_\tau = 0$ .
3. At  $\tau$ , when reserves are depleted, the official exchange rate is devalued, implying an instantaneous jump in the premium by  $-\delta$ .

Imposing the first and second conditions on equation (11b) and using equation (13) yields

$$R_0 = C_1 + C_2 = 0, \quad (14)$$

$$R_\tau = R_\tau^* + C_1 e^{\lambda_1 \tau} + C_2 e^{\lambda_2 \tau} = 0, \quad (15)$$

where

$$R_{\tau}^* = -\frac{\Phi\kappa_2\mu}{\lambda_1^2} \{1 + e^{\lambda_1\tau} (\lambda_1\tau - 1)\} - \frac{\Phi}{\lambda_1} (\kappa_0 + \kappa_2 z_0) \{e^{\lambda_1\tau} - 1\}.$$

Finally, from the definition of the premium—and assuming that no official devaluation has occurred during the interval  $(0, \tau)$ , so that  $\Delta e_{\tau} = e_{\tau} - e_0$ —the third condition indicates that

$$\rho_{\tau} - \rho_0 = -\Delta e_{\tau} = -\delta.$$

Together with equation (11a), this yields

$$-\delta = \rho_{\tau}^* - \rho_0^* + C_1 k_1 (1 - e^{\lambda_1\tau}) + C_2 k_2 (1 - e^{\lambda_2\tau}). \quad (16)$$

From equation (12),

$$\rho_{\tau}^* - \rho_0^* = \kappa_2 (z_{\tau} - z_0) = -\kappa_2 \delta. \quad (17)$$

Substituting equation (17) in equation (16) yields, together with equations (14) and (15) a nonlinear, simultaneous equation system in  $C_1$ ,  $C_2$  and  $\tau$ . In general, no explicit analytical solution is possible, but a numerical simulation method can be used to solve for these values. By use of the implicit function theorem, it can nevertheless be shown that the “collapse time”  $\tau$  is inversely related to the elasticity of money demand,  $\alpha$ , and positively related to the under-invoicing coefficient,  $\Phi$ .

To obtain the solution for the parallel exchange rate from the above setup is straightforward. Assume that  $n \geq 1$  jumps have occurred, as of period  $t$ . The official exchange rate is therefore given by

$$e_t = e_0 + n\delta, \quad (18)$$

and from equation (9) and equation (11a),  $s_t$  is given by

$$s_t = e_0 + n\delta + \rho_t^* + C_1 \kappa_1 e^{\lambda_1 t} + C_2 \kappa_2 e^{\lambda_2 t}. \quad (19)$$

The behavior of the system during a devaluation cycle is described in Figure 1. The first panel of the figure shows the cyclical (clockwise) behavior of foreign reserves. The expansionary credit policy raises the parallel market premium and reduces the demand for domestic currency assets. But since agents anticipate the future devaluation of the official exchange rate, the parallel market premium is initially negative. Accordingly, reserves tend to increase in the first phase of the cycle and reach a maximum at point A. In the process, however, the rate of depreciation of the parallel exchange rate increases, thus leading to an accelerated reduction in domestic currency holdings. Beyond point A, reserves start falling at an increasing rate—while the rate of change of the premium slows down—and are eventually depleted, triggering an official parity change (point B), which starts a new cycle. The second panel of the figure shows the behavior of the official exchange rate as well as the parallel exchange rate, derived in equation (19). A positive premium is associated with falling reserves, while a negative premium is associated with an increasing stock of net

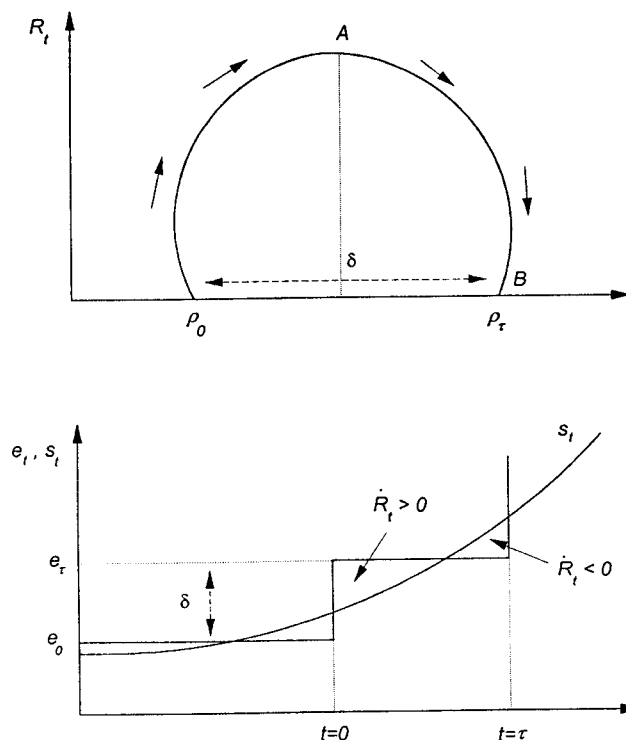


Figure 1. Reserves and the Premium in a Devaluation Cycle

foreign assets. At the moment the official parity change is triggered (that is, at period  $\tau$ ), the premium falls to its initial value.<sup>11</sup>

#### IV. CONVERGENCE OF CYCLES

An important question in the present model is to determine whether a steady cycle will eventually emerge. From equation (7), we have

$$z_\tau - z_0 = (1 - \nu) (\mu\tau - \Delta e_\tau), \tag{20}$$

To prevent a jump in the forcing variable if a devaluation occurs at  $\tau$  requires that  $z_\tau - z_0 = 0$ . Assuming that no devaluation has ever occurred before period  $\tau$  implies that  $\Delta e_\tau = \delta$ . Substituting these results in equation (20) yields

$$\tau = (1 - \nu) \delta / \mu, \tag{21}$$

which determines the length of the devaluation cycle compatible with a steady cycle.<sup>12</sup> If, for instance,  $\tau < \delta / (1 - \nu)\mu$ , devaluations occur “too frequently” implying that  $z_0$  will be *falling* with each successive cycle. On the contrary, for  $\tau > \delta / (1 - \nu)\mu$ ,  $z_0$  will be rising at

the beginning of each cycle. For a given  $z_0$  and a given configuration of parameters, there is therefore no guarantee that the system will approach a stable cycle over time.

## V. SUMMARY AND CONCLUSIONS

This paper has extended the literature on balance-of-payments crises so as to consider an economy in which the monetary authorities operate a dual exchange rate arrangement characterized by a fixed official exchange rate and a floating parallel rate. A crucial assumption of the model is that the exchange rate differential affects the exporters' decision regarding the market through which export proceeds should be repatriated. The analysis showed that when the central bank follows a well-defined devaluation rule triggered by changes in net foreign assets, an exchange rate-reserves cycle will emerge. The cycle emerges because ultimately, the new exchange rate is inconsistent with the underlying macroeconomic policy stance (that is, an overly expansionary credit policy), generating therefore periodic crises. The expectation that falling reserves will eventually prompt a devaluation of the official exchange rate induces speculative rises in the parallel rate (as agents switch away from domestic currency assets) that accelerate the rate of depletion of reserves and brings forward in time the parity change. If fiscal and monetary policies are maintained on an expansionary course, however, the premium will eventually begin rising again, and reserves will start depleting, until a new cycle is started by an official parity change. As long as a positive rate of growth of domestic credit is maintained, the economy will undergo a succession of reserve crises and devaluations.

Despite its simplicity—the analysis does not, for instance, explicitly consider the demand for foreign currency or domestic financial wealth, which represents a budget constraint as agents swap domestic against foreign money<sup>13</sup>—the model predicts several types of empirical regularities that should be observed in periods surrounding devaluation episodes in developing countries. First, in the years preceding a devaluation, credit policy should indicate an expansionary stance, foreign reserves should be falling, and the parallel market premium should display a tendency to increase. Second, following a devaluation, the parallel market premium should jump downwards, and foreign reserves should recover fairly rapidly.

These empirical predictions have, in fact, been fairly well documented by Edwards (1989), Edwards and Montiel (1989), and Kamin (1993). Edwards, in particular, has shown that devaluation episodes in developing countries have usually been preceded by rising parallel market premia and sharp drops in international reserves. In addition, Kamin (1993) has provided evidence that following a devaluation, the parallel market premium drops substantially while official reserves, as well as recorded exports—as a result of reduced underinvoicing—tend to increase. The evidence, however, does not suggest that the reserves/devaluation cycle has a fairly regular shape, as described in the model developed here. The assumption of a varying threshold that triggers the official parity change and/or uncertainty about the devaluation rule itself can easily generate a pattern of reserves movements that is less regular and may provide a very close approximation to the type of behavior that has been observed in the actual experience of developing countries.



## APPENDIX

Equation (2) is derived as follows (Flood & Marion, 1983). Assume that private agents hold foreign bonds, the principal on which is acquired and repatriated at the parallel exchange rate,  $S_t$ , but that interest income—a current account item—is repatriated at the commercial exchange rate  $E_t$ . Let  $i_t^*$  denote the foreign interest rate.

Consider first the opportunity cost of holding money for a time period of length  $k$ . At the beginning of a period of length  $k$ , 1 unit of domestic currency will buy  $1/S_t$  units of foreign bonds, which may be repatriated at the end of the period at the rate  $E_{t+k}$ . During the period, the  $1/S_t$  units of foreign bonds earn  $ki_t^*/S_t$  in interest income, which, when repatriated and converted in domestic currency, yields an amount  $E_t ki_t^*/S_t$ . The overall return on foreign bonds is therefore equal to

$$\frac{S_{t+k}}{S_t} \left[ 1 + \frac{E_{t+k} ki_t^*}{S_{t+k}} \right] \quad (\text{A1})$$

Hence, the opportunity cost of holding domestic money from time  $t$  to time  $t+k$  is  $ki_t^*$  in the expression

$$1 + ki_t = \frac{S_{t+k}}{S_t} \left[ 1 + \frac{E_{t+k} ki_t^*}{S_{t+k}} \right] \quad (\text{A2})$$

A logarithmic approximation to equation (A2), using  $\log(1+z_t) \cong z_t$ , is given by

$$ki_t = s_{t+k} - s_t + \frac{E_{t+k} ki_t^*}{S_{t+k}} \quad (\text{A3})$$

Dividing each side of equation (A3) by  $k$  and letting  $k \rightarrow 0$  yields

$$i_t = \dot{s}_t + E_t i_t^* / S_t \quad (\text{A4})$$

The last term in this expression can be approximated by

$$E_t i_t^* / S_t \cong i_t^* + i_t^* (e_t - s_t) \cong (1 + \alpha) i_t^* + \gamma (e_t - s_t) + \gamma \alpha, \quad (\text{A5})$$

where  $\gamma$  and  $\alpha$  are, respectively, the mean values of  $(e_t - s_t)$  and  $i_t^*$ . Adopting, for simplicity, the normalization rule  $\alpha = 0$  and substituting equations (A5) in (A4) yields equation (2).

Equation (5) can be derived as follows. Suppose that there is only one exporter in the economy. Since the exporter can repatriate the proceeds of its sales abroad through either the official or the parallel market, the relevant export price—setting, for simplicity, the foreign-currency price of exports equal to unity— $P_x(t)$  is a weighted average of the official exchange rate and the parallel exchange rate:

$$P_x(t) = (1 - \sigma) E_t + \sigma S_t, \quad 0 < \sigma < 1 \quad (\text{A6})$$

where  $\sigma$ , denotes the fraction of total exports earnings repatriated via the parallel market, and  $q_t$  the total production for exports.

Assume now that output and labor costs are constant at  $\bar{q}$  and  $\bar{n}$ , respectively, and that there are rising marginal costs associated with the under-invoicing share,  $\sigma_t$ .<sup>14</sup> The exporter's profits can therefore be written as

$$\Pi = [(1 - \sigma_t) E_t + \sigma_t S_t] \bar{q} - w_t \bar{n} - E_t F(\sigma_t), \quad (\text{A7})$$

where  $w_t$  denotes the wage rate and  $F(\sigma) = F \sigma^\gamma$ , where  $\gamma > 1$ . The particular form chosen for  $F(\cdot)$  satisfies  $F' > 0$  and  $F'' > 0$ , that is, the assumption of rising marginal costs. The exporter therefore maximizes equation (A7) with respect to  $\sigma_t$ . The solution can be shown to be<sup>15</sup>

$$\sigma_t = \left[ \left( \frac{S_t}{E_t} - 1 \right) \bar{q} / \gamma F \right]^{1/(\gamma-1)} \equiv \sigma \left( \frac{S_t}{E_t} \right), \quad \sigma' > 0 \quad (\text{A8})$$

which indicates that a rise in the premium raises the leakage coefficient. Defining changes in reserves as the difference between exports  $(1 - \sigma_t)\bar{q}$  and imports (which are assumed exogenous) yields, using a logarithmic approximation, equation (5) in the text.

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## NOTES

1. See, in particular, Flood and Garber (1984) who compute, in a linear deterministic framework, the exact time of occurrence of the crisis when the exchange rate is permanently allowed to float. Cumby and van Wijnbergen (1989) consider a crawling peg regime and emphasize the uncertainty regarding the lower limit on reserves that triggers the parity change. For an extensive review of the literature on speculative attacks and balance-of-payments crises, see Agénor, Bhandari, and Flood (1992).
2. Studies in which a reserves crisis yields to a devaluation include Blanco and Garber (1986), Otani (1989), and Willman (1989).
3. Calvo's (1987) optimizing model of collapsing fixed exchange rates also characterizes the process leading to a devaluation crisis by a progressive appreciation of the real exchange rate. Edwards' analysis builds on an earlier paper by Rodriguez (1978), who develops a model of a small open economy with no capital mobility, and where inconsistent fiscal policies lead also to a real overvaluation of the exchange rate, losses in reserves, and eventually to a devaluation crisis. Rodriguez, however, does not explicitly consider the role of devaluation expectations in his analysis.
4. See Agénor (1992) for an overall discussion of parallel currency markets in developing countries.
5. The derivation of equation (2), which follows Flood and Marion (1983) is provided in the Appendix.

6. The coefficient  $\Theta$  can be viewed as an approximation to the share of transactions settled through the parallel market relative to total trade transactions.

7. Models in which a similar mechanism is used include Bhandari and Végh (1990) and Kamin (1993). A simple proof of this result, based on the assumption of exogenous output, is provided in the Appendix. In general, the size of the coefficient  $\Phi$  is related to the degree of risk aversion and the perceived costs of engaging in fake invoicing. We assume that such costs are not prohibitive.

8. A similar devaluation rule is used by Willman (1989). Ideally, the devaluation rule and the size of the parity change should be the outcome of an optimization process, involving a government objective function and well-defined constraints. The alternative procedure followed here consists in specifying what amounts to a "plausible" rule, given the specification of the model.

9. The determinant of the general matrix of equations (10) is equal to  $-v\Phi/\alpha$ , which indicates that the roots of the system are of opposite sign. The roots are  $(1/2\alpha)^{-1}[(\Theta + \alpha\gamma) \pm \{(\Theta + \alpha\gamma)^2 + 4\alpha v\Phi\}^{1/2}]$ .

10. Strictly speaking,  $\rho_t^*$  and  $R_t^*$  cannot be described as "long-run" equilibrium solutions, since negative reserves may not be feasible in the steady-state. We abstract from this complication at this stage.

11. Qualitatively, these results are quite similar to those derived by Kamin (1993). Kamin uses a framework that is in some respects more detailed than the one considered here—the real exchange rate, for instance, is allowed to affect exports—but in others more restrictive—the domestic money stock is taken as exogenous in his model, although reserves are endogenous.

12. As in Wylosz (1986), the length of the period separating two successive devaluations—or the length of the devaluation cycle—is independent of the reserves threshold level,  $R_\tau$ .

13. However, although discrete devaluations do in fact reduce real wealth, nothing of substance would be altered by incorporating changes in wealth into the analysis. The assumption of constant real wealth and instantaneous stock-shift exchanges between domestic and foreign currency holdings was made for simplicity only.

14. In the absence of this feature, there would be nothing in this framework to prevent a corner solution, that is, a situation where exporters would divert 100 percent of their exports receipts to the parallel market. These costs may take the form of penalties assessed by the agencies enforcing exchange controls, or side payments that have to be made to government officials, etc.

15. Note that since we assume that  $0 < \sigma < 1$ , we implicitly imposed the restriction  $\bar{q}(S_t/E_t - 1) < \bar{F}\gamma$  on equation (A8).

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