Health and Infrastructure in Models of Endogenous Growth

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Abstract

This paper studies the optimal allocation of government spending between infrastructure and health (which affects labor productivity as well as household utility) in an endogenous growth framework. A key feature of the model is that infrastructure affects not only the production of goods but also the supply of health services. The first part considers the case where health enters as a flow in production and utility, whereas the second focuses on a "stock" approach. Growth- and utility-maximizing rules for output taxation and the allocation of public spending are derived. It is shown, in particular, that the welfare-maximizing share of spending on health exceeds the growth-maximizing share.

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> The annual loss of life from filth and bad ventilation are greater than the loss from death or wounds in which the country has been engaged in modern times...
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> The primary and most important measures, and at the same time the most practicable, and within the recognized province of public administration, are drainage, the removal of all refuse of habitations, streets, and roads, and the improvements of the supplies of water...
>
> That by the combinations of all these arrangements it is probable that... an increase of 13 years at least, may be extended to the whole of the labouring classes.
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> E. Chadwick, *The Sanitary Conditions of the Labouring Population* (1842).

1 Introduction

The effect of health on economic growth has been the subject of much recent empirical and analytical research. A key premise of the literature is that good health enhances worker productivity and stimulates growth. Bloom, Canning, and Sevilla (2004), in a sample consisting of both developing and industrial countries, found that good health (proxied by life expectancy) has a sizable, positive effect on economic growth. A one-year improvement in the population's life expectancy contributes to an increase in the long-run growth rate of up to 4 percentage points. Sala-i-Martin, Doppelhofer, and Miller (2004) also found that initial life expectancy has a positive effect on growth, whereas the prevalence of malaria, as well as the fraction of tropical area (which may act as a proxy for exposure to tropical diseases) are both negatively correlated with growth. Jamison, Lau and Wang (2004), using a sample of 53 countries, found that improvements in health (as measured by the survival rate of males aged between 15 and 60) accounted for about 11 percent of growth during the period 1965-90. In countries like Bolivia, Honduras and Thailand, health improvements added about half of a percentage point to the annual rate of growth in income per capita. According to the estimation results of Gyimah-Brempong and Wilson (2004), between 22 and 30 percent of the transition growth rate of per capita income in Sub-Saharan Africa can be attributed to health factors. Along the same lines, Weil (2005), using microeconomic data (such as height and adult survival rates) to build a measure of average health, found that as much as 22.6 percent of the cross-country variation in income per capita is due to health factors—roughly the same as the share accounted for by human capital from education, and larger than the share accounted for by physical capital. Conversely, estimates by the United Nations (2005) suggest that malaria (which claims each year the lives of 1 million people in poor countries and infects 300 million more) has slowed economic growth in Sub-Saharan Africa by 1.3 percentage point a year. According to a recent report on HIV-AIDS by the same institution, in Sub-Saharan Africa—a region where on average 7 out of 100 adults, and up to a quarter of the population in the southern part of the continent, are HIV-positive—the epidemic has reduced annual growth rates by anywhere between 0.5 to 1.6 percentage points (see UNAIDS (2004)).[1]

Accounting for health factors in models of economic growth is important for studies focusing on developing countries—particularly the low-income ones, where health indicators are the weakest. An important issue in that regard relates to the fact the provision of health services requires the use of public resources. At the macroeconomic level, there is therefore a potential trade-off between health and other services that governments can provide—such as education, security, legal protection, and infrastructure

[1]It should be noted, however, that with respect to industrial countries, some studies have found evidence of reverse causation. By raising real incomes, economic growth may enable individuals to spend more on health services. In addition, as shown by Benos (2004), there is also evidence of nonlinearities in the relationship between health and growth.

services. Understanding the nature of this trade-off is important because complementarity effects may exist at the microeconomic level between these various components. There is much evidence, in particular, regarding the relationship between infrastructure and health (see the summary by Brenneman and Kerf (2002)). It has been shown, for instance, that access to safe water and sanitation helps to improve health, as recognized long ago by Edwin Chadwick.[2] Studies by Behrman and Wolfe (1987), Lavy et al. (1996), Lee, Rosenzweig, and Pitt (1997), Leipziger et al. (2003), and Wagstaff and Claeson (2004, pp. 170-74) found that access to clean water and sanitation infrastructure helps to reduce infant mortality. In addition, recent surveys suggest that in some African cities, the death rate of children under five is about twice as high in slums (where water and sanitation services are poor, if not inexistent), compared to other urban communities.

By reducing the cost of boiling water, access to electricity may also help to improve hygiene and health. Availability of electricity is essential for the functioning of hospitals and the delivery of health services (vaccines require continuous and reliable refrigeration to retain their effectiveness). Getting access to clean energy for cooking in people's homes (as opposed to smoky traditional fuels, such as wood, crop residues, and charcoal) improves health outcomes, by reducing indoor air pollution and the incidence of respiratory illnesses. Last but not least, better transportation networks may also contribute to easier access to health care, particularly in rural areas. Recent data produced by national Demographic and Health Surveys in Sub-Saharan Africa show that a majority of women in rural areas rank distance and inadequate transportation as major obstacles in accessing health care (see African

[2] Chadwick's work led to the passage, in 1848, of the Public Health Act in England, which among other measures gave boroughs responsibility for drainage, water supplies, and paving of roads.

Union (2005)). In Morocco, a program developed in the mid-1990s to expand the network of rural roads led—in addition to reducing production costs and improving access to markets—to a sizable increase in visits to primary health care facilities and clinics (see Levy (2004)). At a more formal level, Wagstaff and Claeson (2004, pp. 170-74) found, using cross-section regressions, that road infrastructure (as measured by the length of the paved road network) had a significant effect on a number of health indicators, such as infant and female mortality rates.

Despite the compelling nature of the microeconomic evidence, the link between health and infrastructure has not received much attention in the existing literature on government spending and endogenous growth. In fact, most of this literature does not account in a satisfactory manner for the macroeconomic effects of health services. In those papers that account for government spending on "utility-enhancing services" (as for instance Barro (1990) and Turnovsky (1996, 2000)), these services are generally described as a government-provided consumption good; examples that are often provided include defense and security. However, This approach is unsatisfactory to account for health services. The reason is that models of this type almost invariably introduce a dichotomy in the composition of public spending—expenditure on utility-enhancing services is generally assumed not to affect the production side, whereas production-related spending (such as infrastructure) is assumed to have no effect on utility-enhancing services—for the very reason that these services are usually directly related only to an exogenous component of government spending.

This paper takes a broader perspective on the relationship between health, infrastructure, and growth. It examines the optimal allocation of government spending between health and infrastructure in an endogenous growth framework where public spending is an input in the production of final goods as well as health services. In addition, and in line with the foregoing discussion, infrastructure services are assumed to affect the production of goods as well as the provision of health services. Put differently, what matters is not only spending on health *per se*, but the combination of public spending on health and infrastructure. As noted earlier, to function properly, hospitals need access to electricity. With inadequate water, sanitation and waste disposal facilities, hospitals cannot provide the services that are expected from them. As far as I know, this paper is the first to examine the implications of complementarity between health and infrastructure services in production, while accounting at the same time for substitutability through the government budget constraint, for the optimal allocation of government expenditure in a growing economy.

The model also assumes, more conventionally, that individuals can provide effective services from human capital only if they are healthy. Thus, by enhancing productivity, health influences growth indirectly, in addition to affecting individual welfare.[3] More precisely, health is treated as labor-augmenting, rather than assumed to enter the production function as a separate factor. It is "effective" labor (educated labor multiplied by the stock of public health services) that is used in production. A lower flow of health services reduces therefore the number of effective working days embodied in each worker. At the same time, health services enter in the household's utility function and therefore affect welfare directly. As a result, there is an optimal allocation of expenditure between health and infrastructure which

[3] See Zagler and Durnecker (2003) for a more detailed discussion of this channel. van Zon and Muysken (2001) proposed an early model along these lines. However, they analyzed only steady-state solutions and did not derive explicitly optimal allocation rules, as I do here.

depends on technology for producing goods and health services, as well as household preferences.

The remainder of the paper is organized as follows. Section II presents the framework, which assumes that all public services are provided free of charge and financed by a distortionary tax on output. Section III derives the balanced-growth path and discusses the dynamic properties of the model. Section IV examines the short- and long-run effects of an increase in spending shares on infrastructure, health, and education. The issue that we address is whether (given that the production of educated labor and health services depend on infrastructure services) an increase in public spending on infrastructure the most efficient method to stimulate long-run growth. As noted earlier, the provision of each category of services requires resources and this (given the overall constraint on tax revenues) creates trade-offs. The role of technology and preferences in determining the growth- and welfare-maximizing allocations of public expenditure are explored in Section V. The last section of the paper offers some concluding remarks and discusses some future research perspectives.

2 A Basic Framework

Consider an economy with a constant population and an infinitely-lived representative household who produces and consumes a single traded good. The good can be used for consumption or investment. The government spends on infrastructure and produces health services, free of charge. It levies a flat tax on output to finance its expenditure.

2.1 Production

Output, Y, is produced with private physical capital, K_P, public infrastructure services, G_I, and "effective" labor, defined as the product of the quantity of labor and productivity, A. With zero population growth, and the population size normalized to unity, assuming that the technology is Cobb-Douglas yields[4]

$$Y = G_I^{\alpha} A^{\beta} K_P^{1-\alpha-\beta}, \tag{1}$$

where $\alpha, \beta \in (0, 1)$. Health is thus labor-augmenting, as often assumed in micro-level studies of nutrition and labor productivity.

Productivity depends solely on the availability of health services, H:

$$A = H^{\varepsilon}, \tag{2}$$

where $\varepsilon > 0$ is a constant elasticity. For simplicity, I will assume in what follows a linear relationship, so that $\varepsilon = 1$. This assumption is consistent with the results of Knowles and Owen (1997, Table 3). Using population per physician and population per hospital bed as proxies for health services, they found an estimate of ε that varies between 0.81 and 1.04 for their sub-sample of low-income countries.

Combining (1) and (2) yields

$$Y = (\frac{G_I}{K_P})^{\alpha}(\frac{H}{K_P})^{\beta} K_P, \tag{3}$$

which implies that in the steady-state, with constant ratios of G_I/K_P and H/K_P, the output-capital ratio is also constant.

[4]Throughout the paper, the time subscript t is omitted whenever doing so does not result in confusion. A dot over a variable is used to denote its time derivative.

2.2 Household Preferences

With C denoting consumption, the household's instantaneous utility function is given by

$$U = \frac{(C^{\kappa} H^{1-\kappa})^{1-1/\sigma}}{1-1/\sigma}, \quad \kappa \in (0,1), \, \sigma \neq 1, \tag{4}$$

where $1-\kappa$ measures the relative contribution of health to utility and σ is the intertemporal elasticity of intertemporal substitution. Utility is thus nonseparable in consumption and health services. This specification is similar to the one used by Barro (1990), Lee (1992), and van Zon and Muysken (2001), among others.[5] The critical difference, however, is that in those papers, it is utility-enhancing public *spending* that enters directly in the utility function, whereas in the present case what matters is health services, which are produced (as discussed below) through a combination of public spending on infrastructure and health. To ensure that the instantaneous utility function has the appropriate concavity properties in C and H, the restriction $\kappa(1-1/\sigma) < 1$ is imposed on σ and κ.

The household maximizes the discounted present value of utility

$$\max_{C} V = \int_{0}^{\infty} U \exp(-\rho t) dt, \tag{5}$$

subject to the resource constraint

$$C + \dot{K}_P = (1-\tau)Y, \tag{6}$$

where $\tau \in (0,1)$ is the tax rate on income. For simplicity, the depreciation rate of private capital is assumed to be zero.

[5] A closely related specification, used for instance by Corsetti and Roubini (1996) and Turnovsky (1996), is $(1-1/\sigma)^{-1}(CH^{\kappa})^{1-1/\sigma}$, where now $\kappa > 0$.

Note that health considerations could also be introduced by assuming that poor health is reflected in a low value of the intertemporal elasticity of substitution, σ, a greater preference for the present (that is, a high value for ρ), or reduced longevity (as in von Zon and Muysken (2001)). However, such complications may generate multiple equilibria (as in Chakrabarty (2002) for instance) and are not pursued here.

2.3 Production of Health Services

Production of health services requires combining labor, and government spending on both infrastructure and health (G_I and G_H, respectively). Assuming also a Cobb-Douglas technology yields, and given that population is constant

$$H = G_I^{\mu} G_H^{1-\mu}, \tag{7}$$

where $\mu \in (0, 1)$. The provision of health services takes place therefore under constant returns to scale.

2.4 Government

The government spends on infrastructure and health services, and levies a flat tax on output at the rate τ. It cannot issue debt claims and therefore must keep a balanced budget at each moment in time. The government budget constraint is thus given by

$$G_H + G_I = \tau Y. \tag{8}$$

Both categories of spending are taken to be a constant fraction of tax revenue:

$$G_h = \upsilon_h \tau Y, \quad \text{for } h = H, I. \tag{9}$$

The government budget constraint can thus be rewritten as

$$\upsilon_H + \upsilon_I = 1. \tag{10}$$

3 The Decentralized Equilibrium

In the present setting, a decentralized equilibrium is a set of infinite sequences for the quantities $\{C, K_P\}_{t=0}^{\infty}$, such that $\{C, K_P\}_{t=0}^{\infty}$ maximizes equation (5) subject to (6), and the path $\{K_P\}_{t=0}^{\infty}$ satisfies equation (6), for given values of the tax rate, τ, and the spending shares υ_h, with $h = H, I$, which must also satisfy constraint (10).

This equilibrium can be characterized as follows. The household solves problem (5) subject to (4) and (6), taking the tax rate, τ, and the supply of health services, H, as given.[6] Using (2) and (1), the current-value Hamiltonian for this problem can be written as

$$L = \frac{(C^{\kappa}H^{1-\kappa})^{1-1/\sigma}}{1-1/\sigma} + \lambda \left\{ (1-\tau)(\frac{G_I}{K_P})^{\alpha}(\frac{H}{K_P})^{\beta}K_P - C \right\},$$

where λ is the co-state variable associated with constraint (6). From the first-order condition $dL/dC = 0$ and the co-state condition $\dot{\lambda} = -dL/dK_P$, optimality conditions for this problem can be written as, with $s \equiv (1-\tau)(1-\alpha-\beta)$,

$$\kappa(\frac{H}{C})^{1-\kappa}(C^{\kappa}H^{1-\kappa})^{-1/\sigma} = \lambda, \tag{11}$$

$$s(\frac{G_I}{K_P})^{\alpha}(\frac{H}{K_P})^{\beta} = s(\frac{Y}{K_P}) = \rho - \dot{\lambda}/\lambda, \tag{12}$$

together with the budget constraint (6) and the transversality condition

$$\lim_{t\to\infty} \lambda K_P \exp(-\rho t) = 0. \tag{13}$$

[6] By taking H as given, it is assumed that the household does not internalize the fact that, by increasing output through its consumption and capital accumulation decisions, it may contribute to generating higher tax revenue and public expenditure on health services.

Equation (11) can be rewritten as

$$C = (\kappa/\lambda)^{1/[1-\kappa(1-1/\sigma)]} H^{(1-\kappa)(1-1/\sigma)/[1-\kappa(1-1/\sigma)]}.$$

Taking logs of this expression and differentiating with respect to time yields

$$\frac{\dot{C}}{C} = -\nu_1(\frac{\dot{\lambda}}{\lambda}) + \nu_2(\frac{\dot{H}}{H}), \tag{14}$$

where $\nu_1 \equiv 1/[1-\kappa(1-1/\sigma)] > 0$, and $\nu_2 \equiv (1-\kappa)(1-1/\sigma)\nu_1$. Thus, if $\kappa = 1$, this equation yields the familiar result $\dot{C}/C = -\sigma\dot{\lambda}/\lambda$. Note also that $\upsilon_2 < 1 \ \forall \sigma \neq 1$, and that $\nu_1 < 1$, $\nu_2 < 0$ if $\sigma < 1$.

From (1),

$$\frac{\dot{Y}}{Y} = \alpha(\frac{\dot{G}_I}{G_I}) + \beta(\frac{\dot{H}}{H}) + (1-\alpha-\beta)(\frac{\dot{K}_P}{K_P}).$$

Using (7), which implies that $\dot{H}/H = \dot{Y}/Y$ (as a result of constant returns to scale) and (9), which also implies that $\dot{G}_I/G_I = \dot{Y}/Y$, yields $\dot{Y}/Y = \dot{K}_P/K_P$. Substituting this result in (14), together with (12), yields

$$\frac{\dot{C}}{C} = \nu_1 \left\{ s(\frac{Y}{K_P}) - \rho \right\} + \nu_2(\frac{\dot{K}_P}{K_P}), \tag{15}$$

which can be rewritten as, with $c = C/K_P$:

$$\frac{\dot{c}}{c} = \nu_1 \left\{ s(\frac{Y}{K_P}) - \rho \right\} - (1-\nu_2)(\frac{\dot{K}_P}{K_P}). \tag{16}$$

Now, from (3),

$$\frac{Y}{K_P} = (\frac{G_I}{Y})^{\alpha/(1-\alpha-\beta)}(\frac{H}{Y})^{\beta/(1-\alpha-\beta)},$$

which can be combined with the budget constraint (6) to give

$$\frac{\dot{K}_P}{K_P} = \frac{(1-\tau)Y}{K_P} - c = (1-\tau)(\frac{G_I}{Y})^{\alpha/\eta}(\frac{H}{Y})^{\beta/\eta} - c, \tag{17}$$

where $\eta \equiv 1 - \alpha - \beta \in (0, 1)$. From equations (7) and (9),

$$H = (\upsilon_I^{\mu} \upsilon_H^{1-\mu}) \tau Y, \tag{18}$$

which can be substituted in (17), together with (9), to give

$$\frac{\dot{K}_P}{K_P} = (1 - \tau)(\upsilon_I \tau)^{\alpha/\eta} [(\upsilon_I^{\mu} \upsilon_H^{1-\mu}) \tau]^{\beta/\eta} - c = \Lambda - c. \tag{19}$$

Substituting this result in (16) yields the following nonlinear differential equation in c:

$$\frac{\dot{c}}{c} = (1 - \nu_2)c + [\frac{s}{1 - \tau}\nu_1 - (1 - \nu_2)]\Lambda - \nu_1 \rho. \tag{20}$$

This equation, together with the transversality condition (13), determines the dynamics of the decentralized economy.

On the balanced-growth path (BGP), consumption and the stock of private capital grow at the same constant rate $\gamma = \dot{C}/C = \dot{K}_P/K_P$, so $\dot{c} = 0$. But, given that $\nu_2 < 1$, the equilibrium is (globally) unstable. Thus, to be on the BGP, the economy must start there.

Setting $\dot{c} = 0$ in (20) yields the economy's steady-state level of the consumption-capital ratio:

$$\tilde{c} = \Lambda + \frac{\nu_1(\rho - \eta\Lambda)}{1 - \nu_2}.$$

Substituting this result in (19) yields the steady-state growth rate as

$$\gamma = \frac{\nu_1}{1 - \nu_2}(\eta\Lambda - \rho), \tag{21}$$

which is positive as long as $\rho < \eta\Lambda$. Thus, the model has no transitional dynamics; following a shock, the consumption-capital ratio must jump immediately to its new equilibrium value. It then follows from (21) that the economy is always on its steady-state growth path. Because H/C is constant and $\dot{H}/H = \dot{K}_P/K_P$ along that path, equation (11) implies that $\dot{\lambda}/\lambda = -\gamma/\sigma$. Thus, the transversality condition (13) is satisfied along the BGP if $\gamma(1-1/\sigma) - \rho < 0$, that is,[7]

$$\rho > \left\{1 + \frac{\nu_1(1-1/\sigma)}{1-\nu_2}\right\}^{-1} \frac{\nu_1\eta\Lambda(1-1/\sigma)}{1-\nu_2}.$$

Noting that $\nu_1(1-1/\sigma)/(1-\nu_2) = \sigma - 1$, this expression can be rewritten as

$$\rho > \sigma^{-1}(\sigma-1)\eta\Lambda. \tag{22}$$

Condition (22) is automatically satisfied if $\sigma \in (0,1)$. If $\sigma > 1$, it imposes an upper bound on the admissible value of the tax rate or one of the spending shares. For simplicity, it will be assumed in what follows that $\sigma < 1$. The transversality condition (13) therefore holds irrespective of the particular values obtained from the analysis of optimal public decisions.

4 Optimal Policies

I now consider the growth and welfare effects of an increase in the tax rate, taking the composition of spending as constant (that is, $d\tau > 0$ and $d\upsilon_I = d\upsilon_H = 0$), as well as a revenue-neutral shift in government spending from health to infrastructure (that is, $d\tau = 0$ and $d\upsilon_I = -d\upsilon_H$), assuming that the allocation of spending is set arbitrarily.

Consider first the growth effects. From (21),

$$\text{sg}\left\{\left.\frac{d\gamma}{d\tau}\right|_{d\upsilon_h=0}\right\} = \text{sg}\left\{-1 + (1-\tau)(\frac{\alpha+\beta}{\tau\eta})\right\}, \quad h = I, H \tag{23}$$

[7] The condition $\rho > \gamma(1-1/\sigma)$ is also necessary to guarantee that the integral in (5) remains bounded.

$$\text{sg}\left\{ \left.\frac{d\gamma}{d\upsilon_I}\right|_{d\tau=0} \right\} = \text{sg}\left\{ (\frac{\alpha+\mu\beta}{\upsilon_I}) - \frac{\beta(1-\mu)}{\upsilon_H} \right\}. \tag{24}$$

Both of these expressions are in general ambiguous. The reason, in the case of an increase in the tax rate, is the familiar trade-off examined by Barro (1990), which implies a hump-shape relationship between τ and γ. Equation (23) implies that the growth-maximizing tax rate is given by

$$\tau^* = \alpha + \beta. \tag{25}$$

Thus, formula (25) generalizes Barro's tax-and-spending rule to the case where spending on health has a positive effect on the marginal product of capital (by increasing labor productivity), in addition to infrastructure services. It accounts therefore for both *direct* and *indirect* effects of government spending on production. Of course, had it been assumed that the elasticity ε differs from unity in (2), the optimal tax rate would also depend on how responsive productivity is with respect to health. More specifically, the impact of β on the optimal tax rate would be weighted by ε.

Consider now a revenue-neutral increase in the share of spending on infrastructure. The ambiguous impact on growth results from two conflicting effects. A rise in the share of spending on infrastructure tends to raise the marginal product of capital, which raises investment and growth, *both* directly and indirectly, through its effect on the production of health services. At the same time, the reduction in public spending on health lowers growth by reducing labor productivity. The net effect depends on the parameters characterizing the technology for producing goods and health services. With $\mu = 0$ for instance, an increase in spending in infrastructure would raise growth if the initial composition of spending υ_I/υ_H exceeds the ratio of elasticities in the production of goods, α/β.

From the budget constraint (10) and (24), the growth-maximizing share of spending on infrastructure can be shown to be

$$\upsilon_I^* = \frac{\alpha + \mu\beta}{\alpha + \beta}, \tag{26}$$

which is in general greater than α. The "strict" Barro rule (which would relate the share of spending only to the elasticity of output with respect to infrastructure services) is thus sub-optimal. In the particular case where $\mu = 0$, that is, the "standard" case where health services are produced only with government spending on health, $\upsilon_I^* = \alpha/(\alpha + \beta)$, which is also greater than α; and with $\mu = 1$, all spending should be allocated to infrastructure ($\upsilon_I^* = 1$).[8] Naturally enough, the higher is the elasticity of output of health services with respect to spending on infrastructure (the higher μ is), the lower should be the share of spending on health.

Consider now the welfare-maximizing allocation. From (6) and (8), the economy's consolidated budget constraint can be written as

$$Y = C + \dot{K}_P + (G_H + G_I),$$

that is, using (8),

$$\dot{K}_P = (1 - \tau)Y - C, \tag{27}$$

From (1) and (7), $Y = G_I^{\alpha+\mu\beta} G_H^{\beta(1-\mu)} K_P^{\eta}$. Using again (9), as well as (18), yields

$$Y = \tau^{(\alpha+\beta)/\eta} \upsilon_I^{(\alpha+\mu\beta)/\eta} \upsilon_H^{\beta(1-\mu)/\eta} K_P. \tag{28}$$

Using this result, together with (5) and (18), taking into account the fact that, from the government budget constraint, $\upsilon_H = 1 - \upsilon_I$, and denoting

[8] See Agénor (2005*a*, 2005*b*, 2005*c*) for a more detailed discussion of these growth-maximizing rules in a related model with human capital accumulation.

by ζ_P the co-state variable associated with (27) the government's problem is therefore to maximize

$$L = \frac{\{C^{\kappa}[\{\upsilon_I^{\mu}(1-\upsilon_I)^{1-\mu}\}\tau Y]^{1-\kappa}\}^{1-1/\sigma}}{1-1/\sigma} + \zeta_P[(1-\tau)Y - C],$$

with respect to C, υ_I, τ, and K_P, subject to (28). The first-order optimality conditions with respect to C, υ_I, and τ are given by

$$\kappa(\frac{H}{C})^{1-\kappa}[C^{\kappa}H^{1-\kappa}]^{-1/\sigma} = \zeta_P, \tag{29}$$

$$(1-\kappa)(\frac{C}{H})^{\kappa}[C^{\kappa}H^{1-\kappa}]^{-1/\sigma}\left\{\frac{\alpha(1-\mu)+\mu}{\eta\upsilon_I} - \frac{(1-\mu)(1-\alpha)}{\eta(1-\upsilon_I)}\right\}H \tag{30}$$

$$= -\zeta_P(1-\tau)Y\left\{\frac{\alpha+\mu\beta}{\eta\upsilon_I} - \frac{\beta(1-\mu)}{\eta(1-\upsilon_I)}\right\},$$

$$(1-\kappa)(\frac{C}{H})^{\kappa}[C^{\kappa}H^{1-\kappa}]^{-1/\sigma}(\frac{H}{\eta\tau}) = \zeta_P Y\left\{1-(1-\tau)\frac{(\alpha+\beta)}{\eta\tau}\right\}. \tag{31}$$

Dividing equation (29) by (30), and (31) by (30), yields, after manipulations,

$$\tau^{**} = (\alpha+\beta) + \frac{1-\kappa}{\kappa}(\frac{C}{Y}), \tag{32}$$

$$\upsilon_I^{**} = \frac{1}{1+\Omega}\left\{\frac{\alpha+\mu\beta}{\alpha+\beta} + [\alpha(1-\mu)+\mu]\Omega\right\} \in (0,1), \tag{33}$$

where

$$\Omega \equiv \frac{1-\kappa}{\kappa(1-\tau)(\alpha+\beta)}(\frac{C}{Y}) > 0,$$

and C/Y is constant in the steady state.[9]

In the particular case where $\kappa = 1$, so that utility does not depend on the (flow) supply of health services, $\Omega = 0$ and formulas (32) and (33) are

[9] The solution for τ is admissible only if the steady-state value of the consumption-output ratio is not too high, where υ_I^{**} is always less than unity (see equation (34) below, where both υ_I^{*} and $\alpha(1-\mu)+\mu$ are less than unity). Note also that the complete dynamics of the model under a centralized planner is not fully characterized here; this can be done also the lines discussed in the previous section and the Appendix.

identical to (25) and (26). In general, however, this is not the case. The utility-maximizing tax rate exceeds the growth-maximizing rate. The magnitude of the wedge depends on κ; because $d\tau^{**}/d\kappa < 0$, the greater the role of health services in utility, the larger the difference between the two rates. Note also that the welfare-maximizing tax rate does *not* depend on the technology for producing health services.

Using (26), formula (33) can be rewritten as

$$\upsilon_I^{**} = \frac{\upsilon_I^* + [\alpha(1-\mu) + \mu]\Omega}{1+\Omega} \in (0,1) \tag{34}$$

from which it can readily be verified that $\upsilon_I^{**} < \upsilon_I^*$. Thus, the welfare-maximizing share of spending on infrastructure is lower than the growth-maximizing share.

Intuitively, spending on health services is now more "valuable" to the central planner, given its complementarity with consumption. Choosing an income tax rate that exceeds the growth-maximizing rate entails a fall in the balanced growth rate, which tends, on the one hand, to lower welfare. On the other, however, an increase in the tax rate induces the household to shift resources from investment to consumption, as well as a higher output of health services (see (18)). This tends to increase welfare. With $\kappa < 1$, the positive effect dominates if the optimal tax rate is higher than the growth-maximizing value.

Similarly, choosing a share of spending on infrastructure that is lower than the growth-maximizing rate reduces the growth rate but also leads to a reallocation of government outlays toward health services. If μ is not too high, this reallocation leads to a higher output of health services, and thus higher productivity, which tends to mitigate the drop in public outlays in infrastructure. In turn, with $\kappa < 1$, the increase in output of health services translates into a higher level of consumption (and thus lower investment) and an increase in welfare. This positive welfare effect dominates the negative effect of a lower growth rate. The higher μ is, the smaller the difference between the two solutions. In the limit case where $\mu = 1$, formula (26) yields $\upsilon_I^* = 1$, so that, from (34),

$$\upsilon_I^{**} = \frac{\upsilon_I^* + \Omega}{1 + \Omega} = 1,$$

which shows that both the growth- and welfare-maximizing solutions imply that all tax resources should be allocated to infrastructure.

5 A Stock Approach

I now extend the analysis to consider the case where the flow of health services is proportional to the *stock* of capital in health, K_H, which is itself augmented by combining government spending on infrastructure with spending on health. Specifically, equation (2) is replaced by

$$H = K_H, \tag{35}$$

whereas the production function becomes

$$Y = G_I^{\alpha} K_H^{\beta} K_P^{1-\alpha-\beta} = (\frac{G_I}{K_P})^{\alpha} (\frac{K_H}{K_P})^{\beta} K_P. \tag{36}$$

The production of public capital in health is given by, using (9),

$$\dot{K}_H = G_I^{\mu} G_H^{1-\mu} = (\upsilon_I^{\mu} \upsilon_H^{1-\mu}) \tau Y, \tag{37}$$

where, for simplicity, a zero depreciation rate is assumed. Thus, to accumulate health capital requires spending not only on health *per se*, but also on infrastructure. Health capital can therefore be thought of as a composite asset. It comprises, for instance, not only a hospital building in a particular location, but also the road (or portion of road) that gives access to it. The "conventional" treatment corresponds, again, to $\mu = 0$.

The instantaneous utility function in (4) also has K_H replacing H. The budget constraints, (6) and (10), remain the same.

As shown in the Appendix, the model can be manipulated to give a system of two nonlinear differential equations in $c = C/K_P$ and $k_H = K_H/K_P$ (see equations (A14) and (A15)). These equations, together with the initial condition k_H^0, and the transversality condition (13), determine now the dynamics of the decentralized economy. The BGP is now a set of sequences $\{c, k_H\}_{t=0}^{\infty}$, such that for the initial condition k_H^0, and for given spending shares and tax rate, equations (A14) and (A15) in the Appendix and the transversality condition (13) are satisfied, with consumption and the stocks of private capital and public capital in health all growing at the same constant rate $\gamma = \dot{C}/C = \dot{K}_H/K_H = \dot{K}_P/K_P$.

From equations (A12) and (A13) in the Appendix, the economy's growth rate can be written in the equivalent forms

$$\gamma = \tau^{1/(1-\alpha)} \upsilon_H^{1-\mu} \upsilon_I^{\omega} \tilde{k}_H^{-\eta/(1-\alpha)}, \tag{38}$$

$$\gamma = \frac{\nu_1 s}{1-\nu_2} (\upsilon_I \tau)^{\alpha/(1-\alpha)} \tilde{k}_H^{\beta/(1-\alpha)} - \frac{\nu_1}{1-\nu_2} \rho, \tag{39}$$

where $\tilde{k}_H$ denotes the steady-state value of k_H and $\omega \equiv \mu + \alpha/(1-\alpha)$. As also shown in the Appendix, the equilibrium is saddlepoint stable and the BGP is unique. The model is thus locally determinate.

Transitional dynamics can be analyzed using the phase diagram depicted in Figure 1. The upward-sloping curve HH corresponds to combinations of (c, k_H) for which $\dot{k}_H = 0$, whereas the downward-sloping curve CC corresponds to combinations of (c, k_H) for which $\dot{c} = 0$. The saddlepath, SS, has a negative slope. As before, a budget-neutral shift in spending toward infrastructure has an ambiguous effect on the growth rate and the consumption-private capital ratio, c. In the "standard" case where $\mu = 0$, it also lowers unambiguously the ratio of health capital to private capital, k_H. But in general, if μ is sufficiently high, the steady-state value of k_H may actually increase. The positive effect of an increase in infrastructure spending may thus outweigh the negative effect of lower spending on health services on the stock of health capita. Graphically, CC always shifts to the left, whereas HH can shift in either direction, depending on the parameters of the model. If μ and α/β are relatively high, curve HH shifts to the right (as illustrated in the upper panel of the figure), and the new equilibrium (point E') is characterized by a higher capital ratio and a lower consumption-capital ratio. By contrast, if μ and α/β are relatively low, curve HH shifts to the left (as depicted in the lower panel). At the new equilibrium, the public-private capital ratio is lower the consumption-capital ratio is lower. In both cases the adjustment path corresponds to the sequence EAE'.

From equations (38) and (39), it can readily be established that the growth-maximizing tax rate and share of spending on infrastructure are again given by (25) and (26). Thus, the growth-maximizing allocation of government expenditure does not depend on whether it is the flow of spending on health, or the stock of health capital, that matters in determining household utility and productivity.

Given that the model has now transitional dynamics, calculating the tax rate and share of spending on infrastructure that maximize lifetime utility is more involved. In general, the rates of growth of consumption and health capital are not constant during the transition to the BGP, and neither is their ratio. Formally, from (9) and (35),

$$Y = (\upsilon_I \tau)^{\alpha/(1-\alpha)} K_H^{\beta/(1-\alpha)} K_P^{\eta/(1-\alpha)}. \tag{40}$$

Using this result, together with (5) and (37), the government's problem is now to maximize

$$L = \frac{(C^\kappa K_H^{1-\kappa})^{1-1/\sigma}}{1-1/\sigma} + \zeta_P[(1-\tau)Y - C] + \zeta_H[\upsilon_I^\mu (1-\upsilon_I)^{1-\mu}\tau Y],$$

with respect to C, υ_I, τ, K_H, and K_P, with Y defined in (40), and with ζ_H denoting the co-state variable associated with equation (37). The solution of this problem yields the transversality conditions

$$\lim_{t\to\infty} \zeta_h K_h \exp(-\rho t) = 0, \quad h = P, H,$$

together with the following optimality conditions:

$$\kappa(\frac{K_H}{C})^{1-\kappa}[C^\kappa K_H^{1-\kappa}]^{-1/\sigma} = \zeta_P, \tag{41}$$

$$\zeta_P \frac{\alpha(1-\tau)}{(1-\alpha)\upsilon_I} + \zeta_H \left\{ \frac{\omega}{\upsilon_I} - \frac{1-\mu}{1-\upsilon_I} \right\} \upsilon_I^\mu (1-\upsilon_I)^{1-\mu}\tau = 0, \tag{42}$$

$$\zeta_P[\frac{\alpha(1-\tau)}{(1-\alpha)\tau} - 1] + \zeta_H \upsilon_I^\mu (1-\upsilon_I)^{1-\mu}[1 + \frac{\alpha}{1-\alpha}] = 0, \tag{43}$$

$$(\frac{C}{K_H})^\kappa \frac{[C^\kappa K_H^{1-\kappa}]^{-1/\sigma}}{(1-\kappa)^{-1}} + \frac{\beta Y}{(1-\alpha)K_H} \left\{ (1-\tau)\zeta_P + \frac{\zeta_H \upsilon_I^\mu \tau}{(1-\upsilon_I)^{\mu-1}} \right\} = \rho\zeta_H - \dot{\zeta}_H, \tag{44}$$

$$\frac{\eta Y}{(1-\alpha)K_P} \left\{ (1-\tau)\zeta_P + \zeta_H \upsilon_I^\mu (1-\upsilon_I)^{1-\mu}\tau \right\} = \rho\zeta_P - \dot{\zeta}_P. \tag{45}$$

Conditions (42) and (43) can be combined to give

$$\frac{\alpha(1-\tau)}{(\alpha-\tau)(1-\alpha)} = \omega - \frac{(1-\mu)\upsilon_I}{1-\upsilon_I},$$

which can be written in implicit form

$$\Phi(\upsilon_I, \tau; \mu) = 0. \tag{46}$$

To solve explicitly for the optimal tax rate and share of spending on infrastructure requires solving jointly equations (41), (44), (45), (46), together with (6) and (37). Together they can be combined into a dynamic system in c, k_H, $z = \zeta_P/\zeta_H$, and either τ or υ_I (with the other variable derived from (46)). In general, therefore, τ and υ_I will be constant only when the economy settles on its balanced growth path; and given the complexity of the model, the determination of the welfare-maximizing, time-varying policies can only be done numerically. Because the transitional dynamics (and thus cumulative welfare calculations) are sensitive to the choice of parameters and initial values, general results are difficult to establish. Restricting the analysis only to the balanced growth path, as in Barro (1990) for instance, does not allow clear-cut results either.

6 Concluding Remarks

This paper studied the optimal allocation of government spending between health and infrastructure in an endogenous growth framework. The amount of effective labor services that a worker can provide was assumed to be proportional to his average health. In turn, average health is proportional to the total amount of health services produced in the economy. Thus, by enhancing the productivity of individuals (through higher intakes of calories or micro-nutrients, for instance), health influences growth directly, in addition to affecting individual welfare. Infrastructure services are assumed to affect the production of goods as well as the provision of health services.

The first part of the paper focused on the case where it is the flow of health services that affects production and utility. It was shown that the model then has no transitional dynamics. The analysis also showed that there is a trade-off in increasing public spending on infrastructure: on the one hand, it leads to an increase in the provision of infrastructure services to production of both goods and health services, which increases growth, but on the other, it lowers resources allocated to health and lowers productivity, which in turn lowers growth. Thus, the long-run effect on steady-state growth is ambiguous; depending on the various parameters of the economy, a revenue-neutral increase in spending on infrastructure can actually lower the growth rate. The growth-maximizing tax rate was shown to be equal to the sum of the elasticities of output with respect to infrastructure services and "effective" labor, whereas the optimal allocation of expenditure between health and infrastructure was shown to depend on the parameters characterizing the technology for producing goods as well as health services.

Moreover, the welfare-maximizing tax rate was found to be higher than the growth-maximizing value, whereas the welfare-maximizing share of spending on infrastructure was shown to be lower than the growth-maximizing solution. Intuitively, choosing for instance the share of spending on infrastructure so as to maximize the growth rate raises, on the one hand, consumption and welfare. On the other, however, it reduces outlays on health, which tends to lower welfare if the supply of health services falls. As long as the elasticity of output of health services with respect to spending on infrastructure is not too high, the government is better off reducing the share of spending that it allocates to infrastructure; in doing so, it ensures that the supply of health services increases. Because health and consumption are complementary in preferences, this ensures that the representative household will increase consumption. Despite the consumption loss associated with a lower rate of capital accumulation (induced by the shift from investment to consumption), the net effect on utility will be positive. Thus, restricting the share of resources allocated to infrastructure to a value below the growth-maximizing rate is welfare improving—as long as direct government spending on health has a positive effect on output of health services. If the production of health services involves only public infrastructure, the growth- and welfare-maximizing shares of spending on infrastructure is the same. Similarly, choosing an income tax rate that exceeds the growth-maximizing rate entails a fall in the balanced growth rate, which tends, on the one hand, to lower welfare. On the other, however, an increase in the tax rate induces the household to shift resources from investment to consumption, as well as higher production of health services. This tends to increase welfare—sufficiently so to ensure that the net effect is positive.

The second part of the paper extended the analysis to consider the case where production and utility depend on the flow of services produced by the *stock* of public capital in health. The production of health capital, in turn, was assumed to result from a combination of government spending on health and infrastructure. The growth-maximizing values of the tax rate and share of spending on infrastructure were shown to be identical to those obtained with the flow approach. Due to the complexity of the model, however, no general results could be established.

The analysis presented in this paper can be extended in various directions. First, alternative financing mechanisms could be considered. The trade-off identified earlier between spending on health and infrastructure depends in part on how spending is financed. With money financing, for instance, higher government outlays on health and infrastructure may not always boost growth. To the extent that fiscal deficits lead to higher inflation, the negative impact on macroeconomic stability and growth of such spending could outweigh the beneficial effects of welfare. An important lesson of the model is that a more effective way to increase welfare may be not be to raise public spending on health, but rather to increase spending on infrastructure, which may be more of a "binding" constraint.

Second, it may be useful to introduce quality considerations. As noted in the recent literature on public infrastructure and growth, congestion costs are important in assessing the impact of public investment. But they may also be important in assessing the effect of health on growth, to the extent that access to public health capital is limited. A recent press release by the World Health Organization noted that hospitals in Sub-Saharan Africa are "getting worse in terms of both the scope and quality of health care they provide." For instance, the number of hospital beds per 1,000 people varies only from 0.9 to 2.9 in the region, compared to 4.0 in the United States and 8.7 in France.[10] Pressure on health capital may alter the quality of the services being produced, and therefore mitigate their growth-enhancing effects. More generally, congestion effects may give rise to nonlinearities in the relationship between health, infrastructure, and growth. In turn, these nonlinearities may explain the persistence of poverty traps, characterized by persistent low growth rates in per capita income (see Agénor and Aizenman (2005)).

Third, the model could be extended to account for the fact that health may have an indirect effect on growth through education and the accumulation of human capital. Good health and nutrition are prerequisites for effective learning. Poor nutritional status can adversely affect children's cognitive development, which may translate into poor educational attainment or higher drop-out rates (see Behrman (1996) and Bundy and others (2005)).

[10] Similarly, the number of doctors per 100,000 people is 16 in sub-Saharan Africa, compared to between 33 and 48 in South Asia, and 200 and 300 in developed countries.

Conversely, increases in life expectancy raise the incentive to invest in education, because the returns to schooling are expected to accrue over longer periods. Moreover, intra-family allocations regarding school and work time of children can be adjusted in the face of disease within the family; in turn, these adjustments may influence the accumulation of physical and human capital and thus long-run growth. For instance, as discussed by Corrigan, Glomm, and Mendez (2005), when parents become ill, children may be pulled out of school to care for them, take on other responsibilities in the household, or work to support their siblings. Agénor and Neanidis (2005) extend the present model to consider the impact of health on education.

Finally, despite the wealth of micro evidence on the effect of infrastructure on health (as discussed in the introduction), there has been few attempts to date to estimate directly, through cross-country regressions, the magnitude of this effect. The foregoing analysis showed, that choosing between a "flow" or "stock" treatment of health services may be largely a matter of analytical convenience, given that (at least if the focus is strictly on growth) optimal solutions are independent of that choice. However, from a practical and policy standpoint, it is important to test empirically for the relevant specification, building perhaps on the results of Gyimah-Brempong and Wilson (2004, Appendix A). Although data limitations may prove severe, these empirical tests may lead to better understanding of the role of infrastructure in the growth process. Indeed, an implication of the model is that standard growth-accounting exercises, may provide misleading estimates of the role of capital (public and private) and labor. If, public capital in infrastructure affects productivity, and thus the effective supply of labor, standard decompositions may under-estimate the contribution of that component. A more appropriate approach to identify the overall effect of public capital would then be to estimate a simultaneous equations model explaining the growth rate per capita and the effective supply of labor.

Appendix
Dynamic Structure, Stability and Uniqueness

The model consists now of equations (6), (8), (9), (12), (14), with $\dot{K}_H/K_H$ replacing $\dot{H}/H$, (36), and (37). These equations are repeated here for convenience:

$$\dot{K}_P = (1-\tau)Y - C, \tag{A1}$$

$$\dot{\lambda}/\lambda = \rho - s(\frac{Y}{K_P}), \tag{A2}$$

$$\frac{\dot{C}}{C} = -\nu_1(\frac{\dot{\lambda}}{\lambda}) + \nu_2(\frac{\dot{K}_H}{K_H}), \tag{A3}$$

$$Y = (\frac{G_I}{K_P})^\alpha(\frac{K_H}{K_P})^\beta K_P, \tag{A4}$$

$$G_H + G_I = \tau Y, \tag{A5}$$

$$G_I = \upsilon_I \tau Y, \quad G_H = (1-\upsilon_I)\tau Y, \tag{A6}$$

$$\dot{K}_H = G_I^\mu G_H^{1-\mu}, \tag{A7}$$

where ν_1 and ν_2 are as defined in the text.

Substituting equation (A4) in (A1) yields

$$\dot{K}_P = (1-\tau)(\frac{G_I}{K_P})^\alpha(\frac{K_H}{K_P})^\beta K_P - C. \tag{A8}$$

From (A4) and (A6),

$$G_I = \upsilon_I \tau Y = \upsilon_I \tau (\frac{G_I}{K_P})^\alpha(\frac{K_H}{K_P})^\beta K_P,$$

that is, with $k_H = K_H/K_P$,

$$\frac{G_I}{K_P} = \upsilon_I \tau (\frac{G_I}{K_P})^\alpha k_H^\beta,$$

or equivalently

$$\frac{G_I}{K_P} = (\upsilon_I \tau)^{1/(1-\alpha)} k_H^{\beta/(1-\alpha)}. \tag{A9}$$

Substituting (A9) in (A8) yields

$$\frac{\dot{K}_P}{K_P} = (1-\tau)(\upsilon_I \tau)^{\alpha/(1-\alpha)} k_H^{\beta/(1-\alpha)} - c, \tag{A10}$$

where again $c = C/K_P$.

From (A4) and (A6),

$$G_H = \upsilon_H \tau Y = \upsilon_H \tau (\frac{G_I}{K_P})^{\alpha} (\frac{K_H}{K_P})^{\beta} K_P,$$

so that, using (A9),

$$\frac{G_H}{K_P} = \upsilon_H \tau (\upsilon_I \tau)^{\alpha/(1-\alpha)} k_H^{\beta/(1-\alpha)} = \upsilon_H \tau^{1/(1-\alpha)} \upsilon_I^{\alpha/(1-\alpha)} k_H^{\beta/(1-\alpha)}. \tag{A11}$$

Equation (A7) gives

$$\frac{\dot{K}_H}{K_H} = (\frac{G_I}{G_H})^{\mu} (\frac{G_H}{K_P}) (\frac{K_P}{K_H}),$$

so that, using (A6) and (A11),

$$\frac{\dot{K}_H}{K_H} = (\frac{\upsilon_I}{\upsilon_H})^{\mu} \upsilon_H \tau^{1/(1-\alpha)} \upsilon_I^{\alpha/(1-\alpha)} k_H^{-\eta/(1-\alpha)},$$

where $\eta \equiv 1-\alpha-\beta$, as defined in the text. This expression can be rewritten as

$$\frac{\dot{K}_H}{K_H} = \tau^{1/(1-\alpha)} \upsilon_H^{1-\mu} \upsilon_I^{\omega} k_H^{-\eta/(1-\alpha)}, \tag{A12}$$

where $\omega \equiv \mu + \alpha/(1-\alpha)$. Equation (38) in the text is derived by setting $k_H = \tilde{k}_H$ and $\dot{K}_H/K_H = \gamma$ in (A12).

Using (A2) and (A12), equation (A3) can be rewritten as

$$\frac{\dot{C}}{C} = \nu_1 [s(\frac{Y}{G_I})(\frac{G_I}{K_P}) - \rho] + \nu_2 \tau^{1/(1-\alpha)} \upsilon_H^{1-\mu} \upsilon_I^{\omega} k_H^{-\eta/(1-\alpha)},$$

or, using (A6),

$$\frac{\dot{C}}{C} = \nu_1 s (\upsilon_I \tau)^{-1} (\frac{G_I}{K_P}) + \nu_2 \tau^{1/(1-\alpha)} \upsilon_H^{1-\mu} \upsilon_I^{\omega} k_H^{-\eta/(1-\alpha)} - \nu_1 \rho,$$

that is, using (A9),

$$\frac{\dot{C}}{C} = \nu_1 s(\upsilon_I \tau)^{\alpha/(1-\alpha)} k_H^{\beta/(1-\alpha)} + \nu_2 \tau^{1/(1-\alpha)} \upsilon_H^{1-\mu} \upsilon_I^{\omega} k_H^{-\eta/(1-\alpha)} - \nu_1 \rho. \quad \text{(A13)}$$

Equation (39) in the text is derived by setting $k_H = \tilde{k}_H$ and $\dot{C}/C = \gamma$ in (A13) and using (38) to substitute out for the second term on the right-hand side.

Combining equations (A10), (A13), and (A12) yields, noting that $s \equiv (1-\tau)\eta$:

$$\frac{\dot{c}}{c} = (1-\tau)\nu(\upsilon_I \tau)^{\alpha/(1-\alpha)} k_H^{\beta/(1-\alpha)} + \nu_2 \tau^{1/(1-\alpha)} \upsilon_H^{1-\mu} \upsilon_I^{\omega} k_H^{-\eta/(1-\alpha)} - \nu_1 \rho + c, \quad \text{(A14)}$$

$$\frac{\dot{k}_H}{k_H} = \tau^{1/(1-\alpha)} \upsilon_H^{1-\mu} \upsilon_I^{\omega} k_H^{-\eta/(1-\alpha)} - (1-\tau)(\upsilon_I \tau)^{\alpha/(1-\alpha)} k_H^{\beta/(1-\alpha)} + c, \quad \text{(A15)}$$

with $\nu \equiv \eta\nu_1 - 1 < 0$, given that $\nu_1 < 1$ for $\sigma < 1$, and $\eta < 1$.

To investigate the dynamics in the vicinity of the steady state, equations (A14)-(A15) can be linearized to give

$$\begin{bmatrix} \dot{c} \\ \dot{k}_H \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} c - \tilde{c} \\ k_H - \tilde{k}_H \end{bmatrix}, \quad \text{(A16)}$$

where the a_{ij} are given by

$$a_{11} = \tilde{c}, \qquad a_{21} = \tilde{k}_H,$$

$$a_{12} = \frac{\beta\tilde{c}(1-\tau)\nu}{1-\alpha}(\upsilon_I \tau)^{\alpha/(1-\alpha)} \tilde{k}_H^{-\eta/(1-\alpha)} - \frac{\eta\tilde{c}\nu_2 \tau^{1/(1-\alpha)} \upsilon_H^{1-\mu} \upsilon_I^{\omega}}{1-\alpha} \tilde{k}_H^{-\eta/(1-\alpha)-1},$$

$$a_{22} = -\frac{\eta\tau^{1/(1-\alpha)} \upsilon_H^{1-\mu} \upsilon_I^{\omega} \tilde{k}_H^{-\eta/(1-\alpha)}}{1-\alpha} - \frac{\beta(1-\tau)(\upsilon_I \tau)^{\alpha/(1-\alpha)} \tilde{k}_H^{\beta/(1-\alpha)}}{1-\alpha} < 0,$$

where $\tilde{c}$ and $\tilde{k}_H$ denote the stationary values of c and k_H. Given that $\nu_2 < 0$ for $\sigma < 1$, the second term in the expression for a_{12} is positive; but the first is negative, given that $\nu < 0$. Coefficient a_{12} is thus in general ambiguous. Given that $\partial\nu_1/\partial\sigma$, $\partial\left|\nu_2\right|/\partial\sigma > 0$, it is assumed in what follows that σ is sufficiently small to ensure that $a_{12} > 0$.

From (A16), the slopes of CC and HH in Figure 1 are given by

$$\left.\frac{dc}{dk_H}\right|_{\dot{c}=0} = -\frac{a_{12}}{a_{11}} < 0, \qquad \left.\frac{dc}{dk_H}\right|_{\dot{k}_H=0} = -\frac{a_{22}}{a_{21}} > 0.$$

The consumption-capital ratio c can jump, whereas k_H is predetermined and evolves continuously. Saddlepath stability requires one unstable (positive) root. To ensure that this condition holds, the determinant of the Jacobian matrix of partial derivatives of the dynamic system (A16), Δ, must be negative, that is, $\Delta = a_{11}a_{22} - a_{12}a_{21} < 0$. This condition is always satisfied in the present case. The slope of the saddlepath SS, which is given by $-a_{12}/(\tilde{c} - \varphi)$, where φ is the negative root of the system, is negative.

From (A14), setting $\dot{c} = 0$ yields

$$\tilde{c} = \nu_1\rho - (1-\tau)\nu(\upsilon_I\tau)^{\alpha/(1-\alpha)}k_H^{\beta/(1-\alpha)} - \nu_2\tau^{1/(1-\alpha)}\upsilon_H^{1-\mu}\upsilon_I^{\omega}k_H^{-\eta/(1-\alpha)}. \quad \text{(A17)}$$

Expression (A17) can be substituted in (A15) with $\dot{k}_H = 0$ to yield the implicit form

$$F(\tilde{k}_H) = \frac{\tau^{1/(1-\alpha)}\upsilon_H^{1-\mu}\upsilon_I^{\omega}}{\tilde{k}_H^{\eta/(1-\alpha)}} - \frac{(1-\tau)\eta\nu_1(\upsilon_I\tau)^{\alpha/(1-\alpha)}}{(1-\nu_2)\tilde{k}_H^{-\beta/(1-\alpha)}} + \frac{\nu_1\rho}{1-\nu_2} = 0,$$

from which it can be established that $F_{\tilde{k}_H} < 0$. Thus, $F(\tilde{k}_H)$ cannot cross the horizontal axis from below. Now, we also have $\lim_{k_H \to 0} F(\tilde{k}_H) = +\infty$ and $\lim_{k_H \to +\infty} F(\tilde{k}_H) = -\infty$. Given that $F(\tilde{k}_H)$ is a continuous, monotonically decreasing function of $\tilde{k}_H$, there is a unique positive value of $\tilde{k}_H$ that satisfies $F(\tilde{k}_H) = 0$. From (A17), there is also a unique positive value of $\tilde{c}$. Thus, the BGP is unique.

References

African Union, *Transport and the Millennium Development Goals*, Addis Ababa (February 2005).

Agénor, Pierre-Richard, "Infrastructure, Public Education and Growth with Congestion Costs," Working Paper No. 47, Centre for Growth and Business Cycle Research, University of Manchester (January 2005*a*).

——, "Fiscal Policy and Endogenous Growth with Public Infrastructure," Working Paper No. 59, Centre for Growth and Business Cycle Research, University of Manchester (September 2005*b*).

——, "Schooling and Public Capital in a Model of Endogenous Growth," Working Paper No. 61, Centre for Growth and Business Cycle Research, University of Manchester (September 2005*c*).

Agénor, Pierre-Richard, and Joshua Aizenman, "Public Capital and the Big Push," work in progress, University of Manchester (December 2005).

Agénor, Pierre-Richard, and Kyriakos Nenaidis, "Health, Education and Growth with Public Infrastructure," unpublished, University of Manchester (December 2005).

Barro, Robert J., "Government Spending in a Simple Model of Endogenous Growth," *Journal of Political Economy*, 98 (October 1990), s103-s25.

Behrman, Jere R., "The Impact of Health and Nutrition on Education," *World Bank Research observer*, 11 (February 1996), 23-37.

Behrman, Jere R., and Barbara L. Wolfe, "How Does Mother's Schooling Affect Family Health, Nutrition, Medical Care Usage, and Household Sanitation?," *Journal of Econometrics*, 36 (September 1987), 185-204.

Benos, Nikos, "Fiscal Policy and Economic Growth: Empirical Evidence from OECD Countries," unpublished, University of Cyprus (July 2004).

Bloom, David E., David Canning, and Jaypee Sevilla, "The Effect of Health on Economic Growth: A Production Function Approach," *World Development*, 32 (January 2004), 1-13.

Brenneman, Adam, and Michel Kerf, "Infrastructure and Poverty Linkages: A Literature Review," unpublished, World Bank (December 2002).

Bundy, Donald, and others, "School Health and Nutrition Programs," in *Disease Control Priorities in Developing Countries*, ed. by Dean Jamison et al., 2nd ed., Oxford University Press (New York: 2005).

Chakrabarty, Debajyoti, "Poverty Traps and Growth in a Model of Endogenous Time Preference," unpublished, University of Bonn (May 2002).

Corrigan, Paul, Gerhard Glomm, and Fabio Mendez, "AIDS Crisis and Growth," *Journal of Development Economics*, 77 (June 2005), 107-24.

Corsetti, Giancarlo, and Nouriel Roubini, "Optimal Government Spending and Taxation in Endogenous Growth Models," Working Paper No. 5851, National Bureau of Economic Research (December 1996).

Gyimah-Brempong, Kwabena, and Mark Wilson, "Health, Human Capital, and Economic Growth in Sub-Saharan African and OECD Countries," *Quarterly Review of Economics and Finance*, 44 (May 2004), 296-320.

Jamison, Dean T., lawrence J. Lau, and Jia Wang, "Health's Contribution to Economic Growth in an Environment of Partially Endogenous Technical Progress," in *Health and Economic Growth: Findings and Policy Implications*, ed. by Guillem López-Casasnovas, Berta Rivera and Luis Currais, MIT Press (Boston, Mass.: 2005).

Knowles, Stephen, and P. Dorian owen, "Education and Health in an Effective-Labour Empirical Growth Model," *Economic Record*, 73 (December 1997), 314-28.

Lavy, Victor, John Strauss, Duncan Thomas, and Philippe de Vreyer, "Quality of Health Care, Survival and Health Outcomes in Ghana," *Journal of Health Economics*, 15 (June 1996), 333-57.

Lee, Lung-fei, Mark R. Rosenzweig, and Mark M. Pitt, "The Effects of Improved Nutrition, Sanitation, and Water Quality on Child Health in High-Mortality Populations," *Journal of Econometrics*, 77 (March 1997), 209-35.

Leipziger, Danny, Marianne Fay, Quentin Wodon, and Tito Yepes, "Achieving the Millennium Development Goals: The Role of Infrastructure," Working Paper No. 3163, World Bank (November 2003).

Levy, Hernan, "Rural Roads and Poverty Alleviation in Morocco," unpublished, World Bank (May 2004).

Sala-i-Martin, Xavier, Gernot Doppelhofer, and Ronald I. Miller, "Determinants of Long-Term Growth: A Bayesian Averaging of Classical Estimates (BACE) Approach," *American Economic Review*, 94 (September 2004), 813-35.

Soares, Rodrigo R., "Mortality Reductions, Educational Attainment, and Fertility Choice," *American Economic Review*, 95 (June 2005), 580-601.

Turnovsky, Stephen J., "Optimal Tax, Debt, and Expenditure Policies in a Growing Economy," *Journal of Public Economics*, 60 (April 1996), 21-44.

——, "Fiscal Policy, Elastic Labor Supply, and Endogenous Growth," *Journal of Monetary Economics*, 45 (February 2000), 185-210.

UNAIDS, *2004 Report on the Global AIDS Epidemic*, World Health Organization (Geneva: 2004).

United Nations, *The Millennium Development Goals Report 2005*, United Nations (New York: 2005).

van Zon, Adriaan, and Joan Muysken, "Health and Endogenous Growth," *Journal of Health Economics*, 20 (March 2001), 169-85.

Wagstaff, Adam, and Mariam Claeson, *The Millennium Development Goals for Heath: Rising to the Challenges*, World Bank (Washington DC: 2005).

Weil, David N., "Accounting for the Effect of Health on Economic Growth," Working Paper No. 11455, National Bureau of Economic Research (July 2005).

Zagler, Martin, and Georg Durnecker, "Fiscal Policy and Economic Growth," *Journal of Economic Surveys*, 17 (July 2003), 397-418.

Figure 1

Balanced Growth Path and Revenue-Neutral shift in Spending from Health to Infrastructure