Study group on abelian varieties 2023

Weeks are numbered starting from the beginning of the semester, not from the beginning of the study group.

Week	Date	Topic	Reference	Speaker
2	4 Oct	Introduction to study group		Martin
3	11 Oct	Complex tori and Hermitian forms	HS A.5.0, A.5.1	Bijay
4	18 Oct	Cartier divisors	HS A.2.2	William
5	25 Oct	Linear systems	HS A.3.1, A.3.2	Pedro
6	1 Nov	Riemann–Roch for complex tori	HS A.5.2, A.5.3	Andrew
7	8 Nov	Appell–Humbert theorem and dual abelian variety analytically	HS Exercise A.5.5 (cf. BL 2.2, 2.4)	Martin
*8	15 Nov	Abelian varieties algebraically	HS A.7.1	Vahagn
9	22 Nov	Theorem of the cube and divisors	HS A.7.2 up to A.7.2.9	Raymond
*10	29 Nov	Dual abelian variety algebraically	HS A.7.2.10, A.7.3	Pedro
11	6 Dec	No meeting		
12	13 Dec	Tate module and homomorphisms	Milne I.7, I.10, Mumford 19	Bijay

* Week 8 (15 Nov): Engineering Building, 1A.029
* Week 10 (29 Nov): Frank Adams 2

References:

Hindry and Silverman, *Diophantine Geometry: An Introduction*
Milne, `https://www.jmilne.org/math/CourseNotes/AV.pdf`
Mumford, *Abelian varieties*
Birkenhake and Lange, *Complex abelian varieties*

Week 3. Complex tori and Hermitian forms

Definition of complex torus
Definition of Riemann forms, link with alternating forms (A.5.0.2)
Homomorphisms of complex tori
Torsion subgroup
If time: Poincaré irreducibility theorem

Week 4. Cartier divisors

Definition of Cartier divisors, linear equivalence, Picard group
Example of hypersurface divisor in $\mathbb{P}^n$
Pull back of divisors
Definition of $L(D)$
If time (very ambitious!): Invertible sheaves (A.3.3)

Week 5. Linear systems

Definition of linear systems
Linear systems and maps to projective space
Ample divisors (Thm. A.3.2.1)
Finiteness of $\ell(D)$ (Thm A.3.2.7)
(Examples are more valuable than proofs)

Week 6. Riemann–Roch for complex tori

Definition of theta functions
The map $\{$theta functions$\} \to \{$Riemann forms$\}$
Pfaffian of alternating form
Riemann–Roch for complex tori (statement)
Lefschetz embedding theorem (sketch proof)

Week 7. Appell–Humbert theorem and dual abelian variety analytically

Semi-characters
Exact sequence $0 \to \mathrm{Hom}(V, S^1) \to \mathrm{Pic}(V/\Lambda) \to \mathrm{NS}(V, \Lambda) \to 0$
Dual complex torus is an abelian variety
If time: homomorphism $\Phi_D \colon A \to \hat{A}$

Week 8. Abelian varieties algebraically

Definition of abelian variety
Abelian varieties are smooth
Morphisms between abelian varieties (A.7.1.2)
Abelian varieties are commutative (A.7.1.3)
Isogenies (Milne Prop 7.1(a)–(c))
If time: $[n_A]$ is an isogeny when $\mathrm{char}(k) \nmid n$ (part of A.7.2.7)

Week 9. Divisors on abelian varieties

Statement of theorem A.7.2.1, analytic proof
Mumford's formula (A.7.2.5)
Torsion subgroup (A.7.2.7)
Theorem of the square (A.7.2.9)

Week 10. Dual abelian variety algebraically

$K(D)$ and ampleness (A.7.2.10)
Homomorphism Φ_c (A.7.3.1)
Definition of dual abelian variety, Poincaré divisor class
Existence of dual abelian variety (A.7.3.4)

Week 12. Tate module and homomorphisms

Definition of Tate module (Milne I.7.3, I.10.5)
Poincaré reducibility theorem (Milne I.10.1)
Injectivity of $\mathrm{Hom}(A, B) \to \mathrm{Hom}(T_\ell A, T_\ell B)$ (Milne I.10.6, compare with statement of I.10.15)
Degree is a homogeneous polynomial function on $\mathrm{End}(A)$ (Milne I.10.13, may not have time for proof)
Characteristic polynomial of endomorphisms (Milne I.10.9)
If time: consequence for rank of $\mathrm{End}(A)$ (Mumford, p. 169, Corollary), characteristic polynomial of endomorphisms acting on $T_\ell A$ (Milne I.10.20 / Mumford, p. 167, Thm 4)