

ALGEBRAIC GEOMETRY

Problem Sheet 6

- (1) Suppose that k does not have characteristic 2 or 3.

For $a \in k$, let V_a denote the surface in \mathbb{A}^3 defined by the equation

$$x^3 + y^3 + z^3 - 3a(x^2 + y^2 + z^2) - a^2 = 0.$$

You may assume that this polynomial generates the ideal $\mathbb{I}(V_a)$.

For which values of a does V_a have singular points? For each a , find all the singular points of V_a .

- (2) Let V, W be affine varieties. Let $v \in V$ and $w \in W$.

Prove that $V \times W$ is non-singular at (v, w) if and only if V is non-singular at v and W is non-singular at w .

- (3) Let $V \subseteq \mathbb{A}^n$ be a reducible affine algebraic set, with irreducible components V_1 and V_2 . Let $x \in V_1 \cap V_2$. Prove that

$$T_x V_1 + T_x V_2 \subseteq T_x V.$$

Is $T_x V_1 + T_x V_2$ always equal to $T_x V$?

(Here, $T_x V_1 + T_x V_2$ means the vector space spanned by $T_x V_1$ and $T_x V_2$.)