

MATH10202 Linear Algebra A 2019-20

Exercise Sheet 0

These exercises are to do **in the Week 1 Feedback Supervision**. They revise material from **MATH10101 Foundations of Pure Mathematics** which will be needed again this term, so **you may wish to take your MATH10101 notes to the supervision**.

0.1. Let $f : A \rightarrow B$ be a function. What does it mean for f to be (a) injective, (b) surjective and (c) bijective?

0.2. Decide whether each of the following functions is (a) injective, (b) surjective or (c) bijective:

(i) $g : \mathbb{C} \rightarrow \mathbb{C}$, $g(x) = x^2$;

(ii) $k : \{1, 2\} \rightarrow \{1, 2\}$, $k(x) = 3 - x$.

0.3. Consider the function:

$$h : \mathbb{R}^2 \rightarrow \mathbb{R}^2, \quad h(x, y) = \left(\frac{1}{\sqrt{2}}x - \frac{1}{\sqrt{2}}y, \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y \right)$$

Try applying h to some pairs (x, y) . If we think of each pair (x, y) as the coordinates of a point in the plane, can you describe geometrically the action of the function h ? Do you think h is injective, surjective and/or bijective?

0.4. If A is a set, what does $|A|$ denote?

0.5. Let $f : A \rightarrow B$ be a function. Mark each of the following statements true or false:

(i) If f is bijective then $|A| = |B|$;

(ii) If $|A| = |B|$ then f is bijective;

(iii) If f is surjective then $|A| < |B|$;

(iv) If $|A| > |B|$ then f is not injective;

(v) If $|A| = 1$ then f is injective.

(vi) f is surjective if and only if $\text{Im} f = B$.

(vii) All flying pigs carry umbrellas.

(viii) Every bijective function has an inverse.

For those which are false, find a counterexample.

0.6. Suppose we have two straight lines in the plane. Let $S \subseteq \mathbb{R}^2$ be the set of points on one line, and $T \subseteq \mathbb{R}^2$ the set of points on the other. What are the possible values of $|S \cap T|$?

0.7. What are the invertible elements of \mathbb{Z}_6 (with modulo 6 arithmetic)?