Artin groups and monoids; normal forms and the group word problem

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Statement of results

Joint work with Holt, and sometimes Ciobanu.

Let G be an Artin group of large type in its standard presentation. Then

- the sets of all geodesics and all shortlex minimal geodesics are regular, and G is shortlex automatic (HR, PLMS 2011);
- $\exists$ straightforward algorithms to reduce words to geodesic or shortlex minimal geodesic form, and hence solve word problem, in at worst quadratic time (HR, PLMS 2011);
- parabolic subgroups embed convexly in G (HR, PLMS 2011);
- if extra large type, G satisfies the rapid decay condition (CHR, 2012).

All but (possibly) the last result extend to Artin groups with some commuting relations (HR, 2012).

The word and conjugacy problems

Let $G = \langle X \mid R \rangle$ be a group, $|X| < \infty$.

G has **soluble word problem** if $\exists$ algorithm (equiv. a halting Turing machine) to decide, given any input word w over X, the question:

$$w =_G 1?$$

G has **soluble conjugacy problem** if $\exists$ algorithm to decide, given any pair of input words w, v over X, the question:

$$\exists ? g \in G : gw =_G vg.$$

It's well known that both problems are theoretically insoluble even for finitely presented groups, and that there are groups with soluble word problem but insoluble conjugacy problem.

But that are many groups for which both problems are soluble.

Solving the word problem

- Dehn solved the word problem for surface groups in linear time, using length reducing rewrite rules; Dehn's solution extends to all word hyperbolic groups.

  In fact both decision problems are soluble in linear time in any word hyperbolic group.

- An explicit solution to the word problem is often found by reduction to a normal form, e.g. $\{x_1^{r_1} x_2^{r_2} \cdots x_k^{r_k}\}$ in abelian groups, nilpotent groups, for appropriate generating sets $x_1, \ldots, x_k$.

- For automatic groups, which have regular normal forms satisying a fellow traveller property, the word problem can be solved in quadratic time. For biautomatic groups, the conjugacy problem is also soluble.

Coxeter groups

A Coxeter group W is defined by a presentation

$$\langle x_1, x_2, \cdots, x_n \mid x_i^2 = 1, \overbrace{x_i x_j x_i \cdots}^{\mathsf{m_{ij}}} = \overbrace{x_j x_i x_j \cdots}^{\mathsf{m_{ij}}}, \quad i \neq j \in \{1, 2, \ldots, n\} \rangle$$

$$\mathsf{m_{ij}} \in \mathbb{N} \cup \infty, \mathsf{m_{ij}} \geq 2.$$

The relations split into **involutionary** and **braid** relations. The groups are well known. Both word and conjugacy problem are solved.

Tits described an exponential time combinatorial solution to the word problem:

A positive word is non-geodesic $\iff$ application of some sequence of braid relations transforms it to a word with some x_i^2 as a subword.
So any word w can be transformed to a geodesic representative, of length 0 $\iff$ $w =_W 1$.

. . . Coxeter groups

There's also a quadratic time algorithm due to the embedding of W as a group of reflections in $GL_n(\mathbb{R})$.

Moreover, Coxeter groups are shortlex automatic (Brink, Howlett). So the set of shortlex minimal geodesics is regular (so is the set of all geodesics). Reduction to this takes quadratic time and solves the word problem.

Artin groups are closely related to Coxeter groups, which arise from them as quotients.

But in general it's not known if the word or conjugacy problems are soluble in Artin groups.

Introducing Artin groups

An **Artin group** G is defined by a presentation

$$\langle x_1, x_2, \cdots, x_n \mid \overbrace{x_i x_j x_i \cdots}^{\mathsf{m_{ij}}} = \overbrace{x_j x_i x_j \cdots}^{\mathsf{m_{ij}}}, \quad i \neq j \in \{1, 2, \ldots, n\}\rangle$$
$$\mathsf{m_{ij}} \in \mathbb{N} \cup \infty, \mathsf{m_{ij}} \geq 2,$$

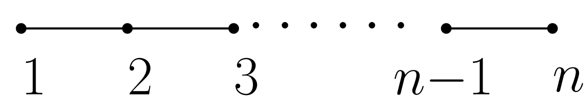
and naturally has a Coxeter group W as a quotient.

It's associated with a Coxeter matrix $(\mathsf{m_{ij}})$, and Coxeter graph Γ, with vertex set $X = \{x_i : i = 1, 2, \ldots, n\}$. We write $G = G(\Gamma)$, $W = W(\Gamma)$.

The name comes from Emil Artin, who introduced **braid groups**

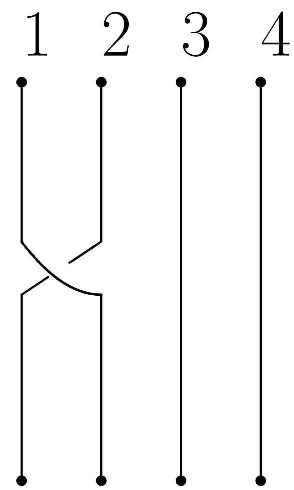
$$\langle x_1, x_2, \cdots, x_n \mid x_i x_{i+1} x_i = x_{i+1} x_i x_{i+1}, \quad x_i x_j = x_j x_i, \forall i + 1 < j\rangle$$

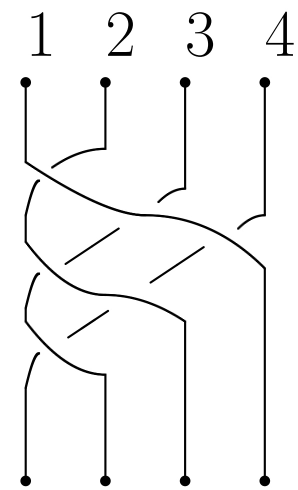
in his 1926 paper **Theorie der Zöpfe**, and a 1947 Annals paper; these have type A_n.



The Artin group of type A_n is faithfully represented as the group of braids on $n+1$ strings,

Generator x_1





Half twist

$x_1 x_2 x_3 x_1 x_2 x_1$
$= (x_2 x_1 x_3)^2$

and maps onto the symmetric group $\mathcal{S}_{n+1}$, generated as a Coxeter group by the transpositions $(1,2), \cdots, (n, n+1)$. The braid representation of this Artin group provides a natural solution to the word problem. Artin solved the word problem, Chow computed the centre in 1948.

Garside

Higman's student Garside (1969) introduced a normal form for the braid group $G(A_n)$, solved word and conjugacy problems, computed centre.

• The normal form represents each braid as a product $\Delta^k w$, $k \in \mathbb{Z}$. Δ is the (positive) half twist braid, Δ^2 is central (generates centre when $n > 1$), and w is a positive word of minimal length. Garside chose w to be least in a reverse lexicographic order.

• Two positive words equal in the group are equal in the braid monoid.

• Two positive words representing elements conjugate in the group are conjugate by a positive word, hence conjugate in the braid monoid.

Garside saw similar behaviour in some other groups, including Artin groups of types C_3 and H_3.

Artin groups of finite type

An Artin group $G(\Gamma)$ has **finite** (or spherical) **type** if the associated Coxeter group $W(\Gamma)$ is finite.

In Inventiones articles published simultaneously in 1972, Brieskorn-Saito and Deligne solved the word and conjugacy problems and computed centres for these groups, generalising Garside's work.

Brieskorn and Saito introduced the term **Artin group**, and defined a normal form analogous to Garside's for each Artin group G of finite type.

Every element of G has normal form $\Delta^k w_1 w_2 \cdots w_m$. $\Delta = \Pi^h$, Π a product of distinct generators, Δ^2 is central. each w_i is positive, a left divisor of Δ.

The monoid of positive words is isomorphic to the Artin monoid.

Deligne constructed a $K(\pi;1)$ for each Artin group $G = G(\Gamma)$ of finite type, that is, a complex with fundamental group G, all higher homotopy groups trivials. He built on work of Brieskorn.

Let Y_W be the complement in $\mathbb{C}^n$ of the complexification of the reflecting hyperplanes in $\mathbb{R}^n$ of the Coxeter group $W(\Gamma)$. W acts on Y_W, and $X_W := Y_W/W$ is contractible with fundamental group G, a $K(\pi;1)$ for G.

The universal cover $\tilde{X_W}$ of X_W is homotopy equivalent to the complex of cosets in G of the subgroups G_J generated by subsets $\{x_i : i \in J\}$ of X, on which G acts by left multiplication.

Using the action of G on this **building**, Deligne could solve the word and conjugacy problems, and compute the centre.

Some general results about Artin groups

Let $G = G(\Gamma)$ be an Artin group, $W = W(\Gamma)$.

• Each **parabolic** subgroup G_J is an Artin group over the (induced) subdiagram of Γ with vertex set J, and

$$G_J \cap G_K = G_{J \cap K},$$

i.e. **parabolic subgroups embed**.

This was proved by van der Lek (1983) in his thesis (seems not to be well known), then reproved by Paris (1997).

• The complex $X_W = Y_W/W$ has fundamental group G (Lek's thesis). It's homotopically equivalent to the **Deligne complex** of cosets of parabolic subgroups of finite type.

• The Artin monoid embeds (in natural embedding) in the group (Paris,2001)

Some questions

Let $G = G(\Gamma)$ be an Artin group.

- Does G have soluble word problem, soluble conjugacy problem, a good normal form?
- What is the centre of G?
- Is G torsion-free?
- Is the subgroup $\langle x_i^2 \rangle$ free, modulo any obvious commutation relations? This is **Tits' conjecture**, which followed results of Appel and Schupp for large type groups.
- Is $X_W = Y_W/W$ a $K(\pi;1)$ for G? - the $K(\pi;1)$ **conjecture** for G.

The answers are known for various types of Artin groups G, for which either a good normal form or a good action of G is known.

Large and extra-large type

In 1983,1984, Appel and Schupp defined and studied Artin groups G first of **extra-large**, and then of **large type**, that is

$$\mathsf{m}_{\mathsf{ij}} \geq 4, \quad \forall \mathsf{i}, \mathsf{j}, \quad \text{and} \quad \mathsf{m}_{\mathsf{ij}} \geq 3, \quad \forall \mathsf{i}, \mathsf{j}.$$

They used small cancellation arguments to prove

- the word, generalised word and conjugacy problems are soluble in G,
- parabolic subgroups embed,
- G is torsion-free,
- subgroups $\langle x_i^2, 1 \leq i \leq n \rangle$ are free (this led to Tits' conjecture).

Appel and Schupp's technique

Standard presentations of each G_{ij} satisfy $C(4)-T(4)$ small cancellation conditions, implying soluble word and conjugacy problems for G_{ij}.

Where $\mathcal{R}_{ij}$ is the set of non-empty cyclically reduced representatives of the identity in G_{ij}, and $\mathcal{R} := \cup_{ij}\mathcal{R}_{ij}$, the presentation $\langle X \mid \mathcal{R}\rangle$ for G satisfies $C(2m)$, with $m := \min(\mathsf{m_{ij}})$.

Triangle-free and locally spherical Artin groups

An Artin group is

triangle-free if $\nexists$ distinct i, j, k, $\mathsf{m_{ij}}, \mathsf{m_{ik}}, \mathsf{m_{jk}} < \infty$,

locally non-spherical if 3-generator parabolic subgroup has finite type
(generalises both triangle-free and large type)

Pride (Inventiones 1986) used small cancellation to prove triangle-free groups have soluble word and conjugacy problems and satisfy Tits' conjecture, and proved parabolic subgroups embed.

Chermak (J. Alg 1998) solved word problem in locally non-spherical groups in exp. time, described algorithm to reduce words to geodesics, rewriting within 2-generator subwords to either reduce length or maintain length but not increase syllable length; he also proved parabolic subgroups embed.

An Artin group $G(\Gamma)$ has

FC-type if any full subgraph of Γ in which $m_{ij} < \infty, \quad \forall i, j$ has finite type (generalises finite type).

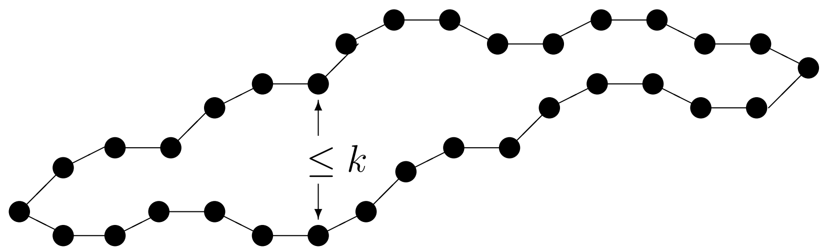
right-angled type (a graph group) if $m_{ij} \in \{2, \infty\}, \forall i, j$.

For G of FC-type, Altobelli (1996, thesis) described a normal form and solved word problem in quadratic time. G is asynchronously automatic wrt Altobelli's form. Altobelli and Charney (2000) constructed another normal form using Niblo and Reevs' 'cube path' techniques on the Deligne complex, a CAT(0) cuve complex.

The family of RAAGs, contains $F_2 \times F_2$, which has insoluble subgroup membership problem (Mihailova), and a group which is FP_2 but not finitely presented Bestvina and Brady (Inventiones, 1997). RAAGs were proved biautomatic by Hermiller and Meier (1995) by direct construction.

Normal forms and automatic structures

A group G is automatic if it has a normal form L that is regular (can be recognised by a finite state automaton) and satisfies a particular fellow traveller property: $v, w \in L$ must fellow travel if $v =_G wx$. G is biautomatic if v, xw also fellow travel when $v =_G xw$.



In any automatic group the word problem is soluble in quadratic time; the conjugacy problem is soluble in biautomatic groups.

- Brieskorn-Saito normal form was proved to give automatic structure first for braid groups (Thurston,1992), then for Artin groups of finite type (Charney, 1992); a related form makes these groups biautomatic.
- 1999, Dehornoy and Paris defined Garside monoids, as monoids with Garside elements Δ, the same properties of division, Garside elements Δ. Garside groups are defined as groups of quotients. The word and conjugacy problems are solved in Garside groups as in Artin groups of finite type, and biautomaticity is proved.
- RAAGs were proved biautomatic by Hermiller and Meier (1995).
- Automaticity of extra-large type (Peifer, 1996) and many large type (Brady, McCammond, 2000) proved using small cancellation arguments.
- FC type Artin groups were proved asynchronously automatic (Altobelli, 1996).

Statement of recent results (again)

Joint work with Holt, and sometimes Ciobanu.

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Rapid decay

A finitely generated group G satisfies **rapid decay** if the operator norm $||.||_*$ for the group algebra $\mathbb{C}G$ is bounded by a constant multiple of the Sobolev norm $||.||_{s,r,\ell}$, for some length function ℓ on G.

Rapid decay is relevant to the Novikov and Baum-Connes conjectures. e.g. it was used in Connes and Moscovic's proof of the Novikov conjecture for word hyperbolic groups.

Explaining the notation for rapid decay

We define, for $\phi \in \mathbb{C}G$,

$$||\phi||_* = \sup_{\psi \in \mathbb{C}G} \frac{||\phi * \psi||_2}{||\psi||_2}, \quad \phi * \psi(g) = \sum_{h \in G} \phi(h)\psi(h^{-1}g),$$

$$||\psi||_2 = \sqrt{\sum_{g \in G} |\psi(g)|^2}, \quad ||\phi||_{2,r,\ell} = \sqrt{\sum_{g \in G} |\phi(g)|^2 (1 + \ell(g))^{2r}}.$$

A function $\ell : G \to \mathbb{R}$ is a length function if

$$\ell(1_G) = 0, \quad \ell(g^{-1}) = \ell(g), \quad \ell(gh) \leq \ell(g) + \ell(h), \quad \forall g, h \in G.$$

Reducing to shortlex minimal form; our approach

We build on the work of Mairesse and Mathéus, who characterised geodesics in dihedral Artin groups.

We shall see that in an Artin group of large type a word that is not shortlex minimal can be reduced using a sequence of length preserving moves within 2-generator subwords, combined with free cancellation.

In some sense this is analogous to the reduction process that solves the word problem in Coxeter groups (Tits' algorithm), as was Chermak's exponential time algorithm. But it's more directed than both, and so faster (at worst quadratic).

Notation

Recall we have

$$G = \langle a_1, \ldots, a_n \mid \overbrace{a_i a_j a_i \cdots}^{\mathsf{m_{ij}}} = \overbrace{a_j a_i \cdots}^{\mathsf{m_{ij}}}, \quad i < j, \mathsf{m_{ij}} \in \mathbb{N} \cup \infty \rangle.$$

We write ${}_m(a,b)$ for the alternating product $aba\cdots$ of length m, and $(a,b)_m$ for $\cdots bab$.

Large type means that $\mathsf{m_{ij}} \geq 3$. We may assume that some $\mathsf{m_{ij}}$ are finite, since otherwise the group is free.

A word over X is a string over $X^{\pm} := X \cup X^{-1}$, i.e. an element of $X^{\pm *}$. We use $=$ for equality of strings, $=_G$ for equality of group elements, $|w|$ for the string length of w, $|w|_G$ for $\min\{|u| : u =_G w\}$.

The shortlex word order puts u before v ($u <_{\mathrm{slex}} v$) if either $|u| < |v|$ or $|u| = |v|$ but u precedes v lexicographically.

Dihedral Artin groups

The geodesics of all dihedral Artin groups have been studied by Mairesse and Mathéus. Let DA_m be the dihedral Artin group

$$DA_m = \langle a, b \mid {}_m(a, b) = {}_m(b, a) \rangle.$$

For any word w over a, b, we define $p(w)$, $n(w)$ as follows:

$p(w)$ is the minimum of m and the length of the longest positive alternating subword in w,

$n(w)$ is the minimum of m and the length of the longest negative alternating subword in w.

Example:

For $w = aba^{-1}baba^{-1}bababa^{-1}b^{-1}$, in DA_3, we have $p(w) = 3$, $n(w) = 2$. The following theorem tells us that w is non-geodesic.

Theorem (Mairesse,Mathéus, 2006)

In a dihedral Artin group DA_m, a word w is geodesic iff $p(w)+n(w) \leq m$, and is the unique geodesic representative of the element it represents if $p(w) + n(w) < m$.

The **Garside element** Δ in a dihedral Artin group is used in rewriting. Δ is represented by ${}_m(a, b)$ and either Δ or Δ^2 is central (depending on whether m is even or odd. If m is odd, Δ conjugates a to b.

We write δ for the permutation of $\{a, b, a^{-1}, b^{-1}\}^*$ induced by conjugation by Δ.

Examples:

In DA_3, $\Delta = aba =_G bab$. $a^\Delta = b$, and $\delta(ab^3a^{-1}) = ba^3b^{-1}$

In DA_4, $\Delta = abab =_G baba$. $a^\Delta = a$, and $\delta(ab^3a^{-1}) = ab^3a^{-1}$

Critical words in dihedral Artin groups.

We call a geodesic word v critical if it has the form

$$_p(x,y)\xi(z^{-1},t^{-1})_n \quad \text{or} \quad _n(y^{-1},x^{-1})\delta(\xi)(t,z)_p,$$

where $\{x,y\} = \{z,t\} = \{a,b\}$, $p = p(v)$, $n = n(v)$, $p+n = m$. (We add an extra condition when p or n is zero.)

We define an involution τ on the set of critical words that swaps the words in each pair (as above). That $w =_G \tau(w)$ follows from the equations

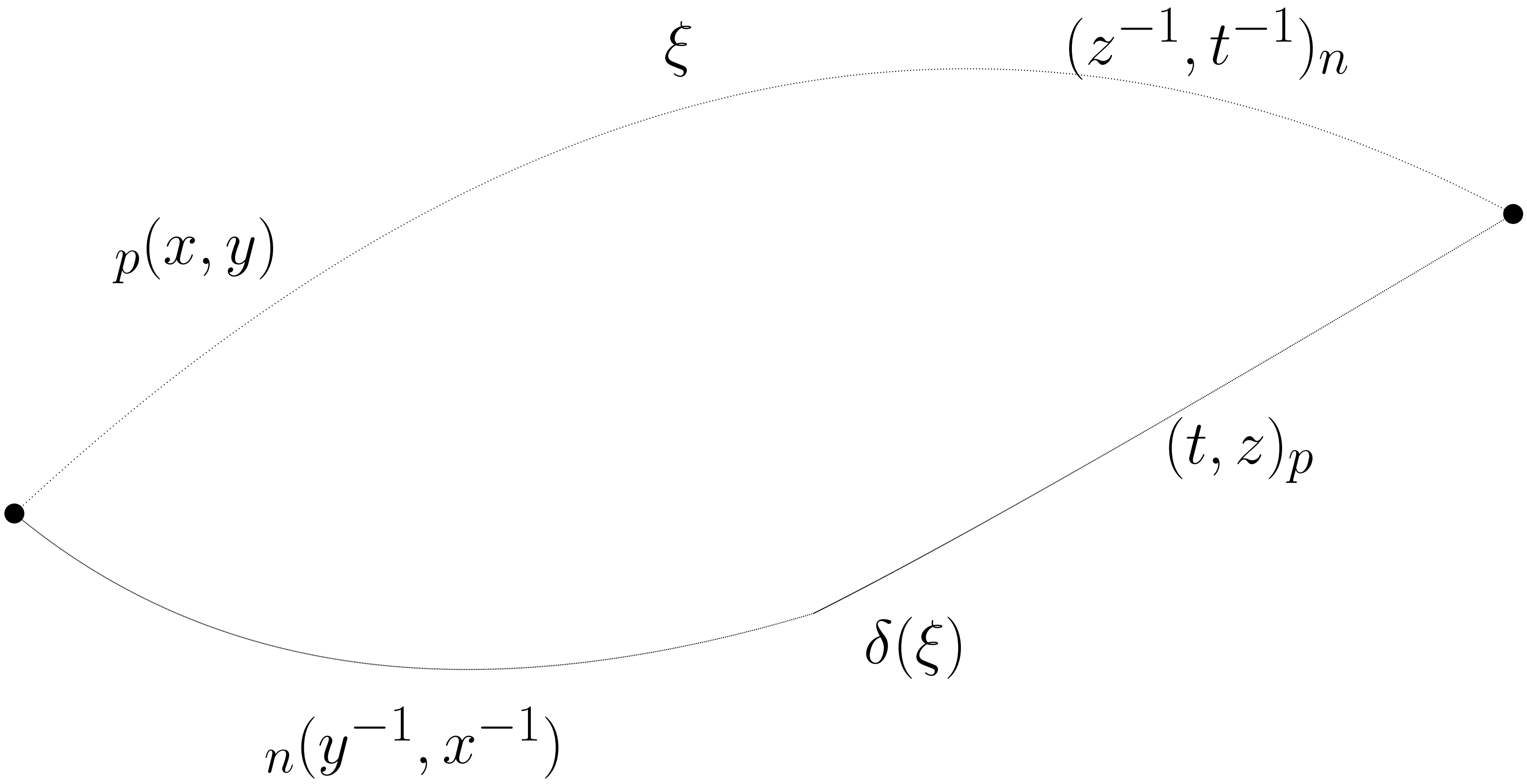
$$_p(x,y) =_G {}_n(y^{-1},x^{-1})\Delta, \qquad \Delta\xi =_G \delta(\xi)\Delta, \quad \Delta(z^{-1},t^{-1})_n =_G (t,z)_p$$

The two words begin and end with different generators.

We call application of τ to a critical subword of w a **τ-move** on w.

We'll use τ-moves in much the same way that Tits' algorithm for Coxeter groups uses the braid relations.

Two critical words related by a τ-move.



Example:

In $G = DA_3$, aba^{-1} is critical, and $\tau(aba^{-1}) = b^{-1}ab$.

Applying that τ-move 2 (but not 3) times to the non-geodesic word w we met before,, we see that

$$w = (aba^{-1})b(aba^{-1})bab(aba^{-1})b^{-1} \rightarrow (aba^{-1})b(b^{-1}ab)bab(b^{-1}ab)b^{-1},$$

and the final word freely reduces to $abbbaa$, which is geodesic.

It is straightforward to derive the following from Mathéus and Mairesse' criterion for geodesics.

Theorem (Holt, Rees, PLMS 2011)

If w is freely reduced over $\{a, b\}$ then w is shortlex minimal in DA_m unless it can be written as $w_1w_2w_3$ where w_2 is critical, and $w' = w_1\tau(w_2)w_3$ is either less than w lexicographically or not freely reduced.

Applying τ moves in Artin groups of large type.

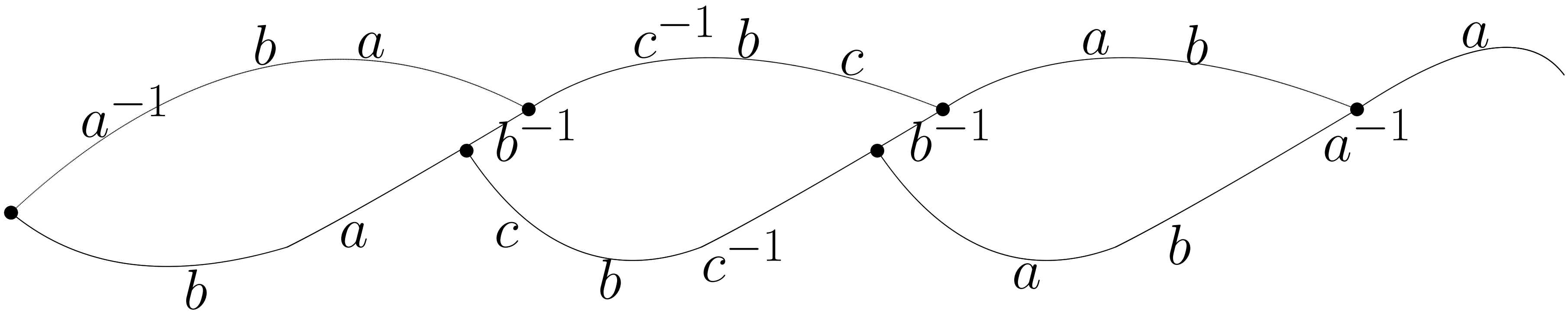
When we have more than 2 generators, we reduce to shortlex minimal form using sequences of τ-moves, each on a 2-generator subword.

Example:

$$G = \langle a, b, c \mid aba = bab,\ aca = cac,\ bcbc = cbcb \rangle$$

First consider $w = a^{-1}bac^{-1}bcaba$. The 2 generator subwords $a^{-1}ba$, $c^{-1}bc$, aba are all geodesic in the dihedral Artin subgroups (in fact also in G). The two maximal a, b subwords are critical in DA_3. Applying a τ-move to the leftmost critical subword creates a new critical subword, to which we can then apply a τ-move.

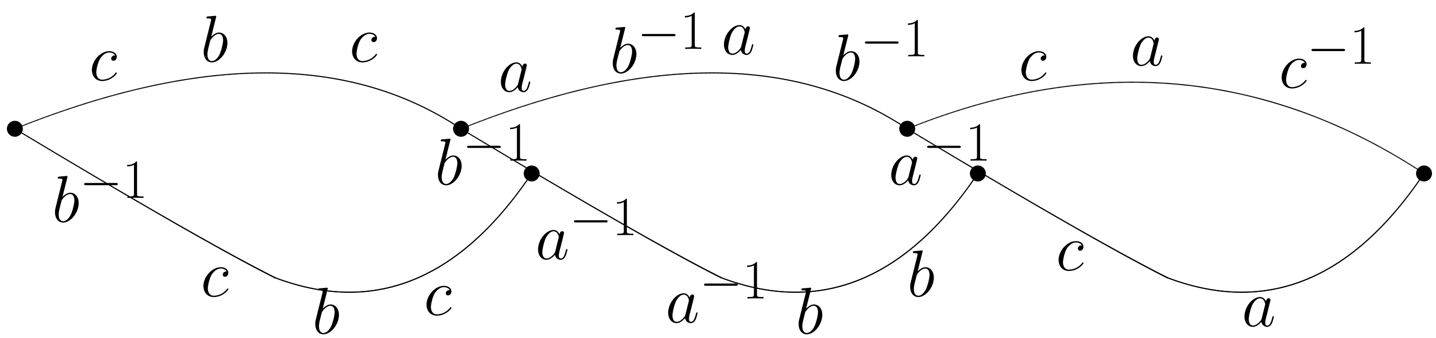
In fact, a sequence of 3 τ-moves transforms w to a word that is not freely reduced. The free reduction is then $bacbc^{-1}ab$,



Reducing $a^{-1}bac^{-1}bcaba$.

We call a sequence of τ-moves like this a **rightward length reducing sequence.**

Now consider $w = cbcab^{-1}ab^{-1}cac^{-1}$, in which cac^{-1} is critical. Applying a τ-move to this critical subword creates a new critical subword, $ab^{-1}ab^{-1}a^{-1}$, to which we can then apply a further τ-move. After one more τ-move, w is transformed to the word $w' = b^{-1}cbca^{-1}a^{-1}bbca$, of the same length as w but preceding w lexicographically.



We call a sequence like this a **leftward lex reducing sequence.**

A shortlex automatic structure

Let L be the regular set of words that excludes w iff it admits **either** a rightward length reducing sequence of τ-moves **or** a leftward lex areducing sequence of τ-moves. Certainly L contains all shortlex minimal reps. To prove shortlex automaticity we need the following.

Proposition (Holt, Rees)

If $w \in L$ but $wg \notin L$ then

either (1) a single rightward sequence of τ-moves on w transforms w to a word $w'g^{-1}$, and $w' \in L$

or (2) a single leftward sequence of τ-moves on wg transforms wg to a word w'' less than wg, and $w'' \in L$.

NB: w and w', w'' k=fellow travel, for $k = 2\max\{\mathsf{m_{ij}} : \mathsf{m_{ij}} < \infty\}$.

Crucial to the proof of the proposition:

Application of a single sequence of τ-moves to a word preserves the sequence of pairs of generators that appear in successive, overlapping, maximal 2-generator subwords.

We can check that this is valid for the reductions

$$a^{-1}bac^{-1}bcaba \to bacbc^{-1}ab \quad \text{and}$$
$$cbcab^{-1}ab^{-1}cac^{-1} \to b^{-1}cbca^{-1}a^{-1}bbca$$

in

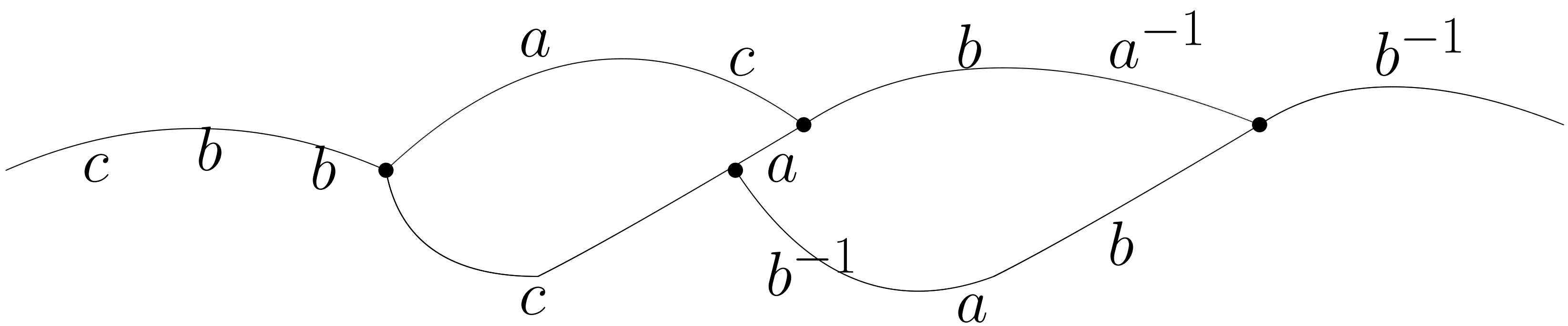
$$G = \langle a, b, c \mid aba = bab,\ aca = cac,\ bcbc = cbcb \rangle.$$

But this is a consequence of large type.

The proposition fails without large type

$$\text{Let} \quad G = \langle a, b, c \mid aba = bab,\ ac = ca,\ bcb = cbc \rangle.$$

Then $w = cbbacba^{-1} \in W$, but wb^{-1} admits a rightward length reducing sequence to $w' = cbbcb^{-1}a$, which then reduces lexicographically, to $w'' = b^{-1}cbbca$. And $w''b <_{\text{slex}} w$.



In the rightward reduction of wb^{-1} a sequence of 5 overlapping max 2-gen subwords collapses to a sequence of 2.

Regularity of the set of all geodesics

Let G be an Artin group of large type over standard generators X.

Theorem (Holt, Rees, PLMS 2011)

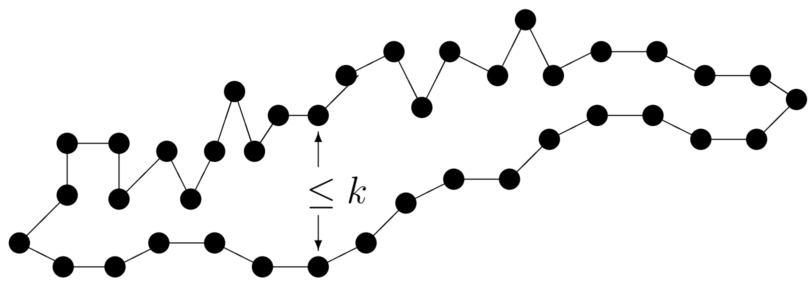
The set of all geodesic words over X is regular.

By a result of Neumann and Shapiro, this follows from the following.

Theorem (Holt, Rees, PLMS 2011)

G satisfies the Falsification by Fellow Traveller Property (FFTP), that is, $\exists k$ such that, for any non-geodesic word v over X,

$$\exists u, \quad |u| < |v|, \quad u =_G v, \quad u \sim_k v$$



Beyond large type

We can move beyond large type, to accommodate some edges in Γ with $\mathsf{m}_{\mathsf{ij}} = 2$, provided that whenever i, j, k are distinct and $\mathsf{m}_{\mathsf{ij}} = 2$,

either $\mathsf{m}_{\mathsf{ik}} = \mathsf{m}_{\mathsf{jk}} = 2$
or at least one of m_{ik} and m_{ij} is infinite.

In this case we need to expand our definition of τ-moves to allow moves that commute a generator past a block of others.

The sets of shortlex minimal geodesics and geodesics are recognised in essentially the same way, and the reduction procedure is just as in the large type case.

Verifying rapid decay

We verify rapid decay by verifying the property

$$\begin{aligned}&\forall \phi, \psi \in \mathbb{C}G, k, l, m \in \mathbb{N},\\ &\qquad |k-l| \leq m \leq k+l, \Rightarrow ||(\phi_k * \psi_l)_m||_2 \leq P(k)||\phi_k||_2||\psi_l||_2.\end{aligned}$$

where ϕ_k means the restriction of ϕ to elements of word length k, and we similarly interpret the subscripted l and m.

Verification of the property requires understanding of the possible ways in which elements of length m can be factorised as products of elements of length k, l.

Further questions

- Can we solve the word problem in linear time Artin groups of large or at least extra-large type?
- Is there a good solution to the conjugacy problem?
- Do these groups have a good biautomatic structure?
- Can we extend the proof of rapid decay to cover more Artin groups? (It's only known already for RAAGs.)

T H A N K

Y O U !