PROJECTS FOR M.SC. IN PURE MATHEMATICS AND MATHEMATICAL LOGIC

MATH61000 PROJECT

Credits	20
Staff/student contact hours	7
Private study hours	193
Total study hours	200
Assessment	Written report and short oral examination
Level	MSc
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The project is written in the period
week 9 of **semester 1** – first day of the exam period of **semester 2**.

Contact details of project supervisors may be found at https://www.maths.manchester.ac.uk/about/people/academic-and-research-staff/ and at https://research.manchester.ac.uk/en/persons/

Project topics

- **Fusion systems of p-groups** (Cesare G. Ardito)
The project will be about fusion systems, which are new objects being studied in group theory and in particular representation theory. While defining a fusion system in a group is straightforward, the more general definition requires notions from category theory, whose basic concepts will form part of the project. The main goal is to understand how abstract fusion systems are defined, and how this definition is related to fusion systems that come from finite groups.

  The project can then take many different directions in exploring classical or more recent applications of fusion systems in group theory and representation theory, or the existence of exotic fusion systems.

  Fusion systems appeared in many different results, and they form one of the most promising and interesting new research direction in group theory. For instance, Aschbacher's new proof of the classification of finite simple groups (arguably the most important result of the 20th century in group theory) heavily relies on this theory.

  References:
  - M. Linckelmann, An introduction to fusion systems.
  - D. Craven, Fusion Systems: Group theory, representation theory, and topology.
  - M. Aschbacher, R. Kessar, B. Oliver, Fusion Systems in Algebra and Topology.
  - S. MacLane, Categories for the Working Mathematician.

- **Model Theory of Differential Fields** (Vahagn Aslanyan) *This topic may lead to a dissertation in logic.*
  Please contact the project supervisor for more details of this project
- **Topics in Lie theory** (Yuri Bazlov)
  The goal of the project will be to present a proof of a result about Lie algebras not covered in the taught component of the programme. Such a result could be the Poincaré-Birkhoff-Witt theorem which has many different proofs — e.g., via the Diamond Lemma, coalgebras, or deformation theory. You should expect to devote a significant part of the project to investigating ideas, constructions and results from other areas of mathematics which are used in Lie theory. Alternatives to the PBW theorem include Ado's theorem and results on representations of semisimple Lie algebras. A subsequent dissertation could be on an advanced topic in Lie theory or on a topic in algebra where the skills gained in the project will be useful.
  Pre/corequisites: you must have taken a Lie algebras course as part of your undergraduate studies, or be taking it as part of the taught component of the MSc. You need to know linear algebra well (bases, linear maps, quotient spaces). Please contact the supervisor to discuss possible topics and pre-requisite knowledge.
  Reference: M. Lorenz, *A tour of representation theory*, Graduate Studies in Mathematics, vol. 193, American Mathematical Society, Providence, RI, 2018. (Further material will be selected according to the topic chosen.)
- **Knot invariants via quantum groups** (Yuri Bazlov)
  Quantum invariants arose in 1980s at the intersection of topology, algebra and mathematical physics. Three early contributors to the area – Jones, Drinfeld and Witten — were awarded Fields medals in 1990. The project explores an algebraic route to quantum invariants. Having covered the basics of Hopf algebras, you will study the Hopf algebra $U_q(\mathfrak{sl}_2)$, known as Drinfeld's quantum group, to understand its application to calculating the Jones polynomial $J(K)$ of a knot K. A knot is a circle embedded in $\mathbb{R}^3$, but one often works with knots drawn as self-crossing curves on a plane. Knot invariants can show that a given knot cannot be unknotted, or that two given knots are not the same; $J(\cdot)$ is a famous knot invariant which can show that a trefoil knot cannot be deformed into a mirror image of itself. A subsequent dissertation could be on a topic of current interest to algebraists where Hopf algebras play a role.
  Prerequisites: you should be confident when using linear algebra (bases, linear maps; quotient spaces), manipulate power series (e.g., is e^{A+B} the same as $e^A e^B$ if A, B are matrices?) and have seen some group theory and/or ring theory. Tensor products and Lie algebras are helpful, but you could become acquainted with them while doing the project. Knowledge of knot theory, topology or physics beyond basic intuition is not required, yet those who already have it will see new connections between different areas of mathematics.
  References:
  - T. Ohtsuki, Quantum invariants (mainly chapter 3) – core reference.
  - L. Kauffman, Knots and physics – linking the project to knot theory and physics.
  - S. Majid, A quantum groups primer – introduction to Hopf algebras.

  Further references (specialist papers, general-interest essays, video talks) can be found on the project supervisor's web page.

- **Sieve methods and distribution of prime numbers** (Hung Bui)
Sieve methods dated back more than 2000 years ago from Eratosthenes of Cyrene to determine all the primes up to a certain point. The theory has had a remarkable development over the last 100 years, and is now a powerful tool to study the distribution of prime numbers. Most notably is the recent breakthrough of Zhang and Maynard on small gaps between prime numbers, which gives a partial answer to the famous Twin Prime Conjecture.

  Particular directions of the project can vary depending on the student's background and interests.

- **Topics in the model theory of fields** (Philip Dittmann) *This topic may lead to a dissertation in logic.*
Please contact the project supervisor for more details of this project

- **Fusion systems** (Charles Eaton)
Fusion in finite groups is about conjugacy (or fusion) of subgroups of a Sylow p-subgroup by elements of the group containing it. The classical result, by Burnside, is that if the Sylow p-subgroup is abelian, then two subgroups that are fused in the larger group must be fused within the normalizer of the Sylow p-subgroup. Study of patterns of fusion is fundamental in the classification of finite simple groups. A project would centre around the theory of fusion of finite groups. A subsequent dissertation would involve the modern theory of fusion systems, which are a categorical construction generalizing fusion in finite groups, with the possibility of contributing some new calculations.

  Fusion in finite groups is covered in Daniel Gorenstein's book Finite Groups. The theory of fusion systems is treated in David Craven's book (you've guessed it) "The theory of fusion systems".

- **Modular representation theory** (Charles Eaton)
The project would follow on from the Noncommutative Algebra course, specializing in the structure of modules of group algebras for finite groups touched on in that course. A dissertation would continue with basic algebras, which are minimal objects capturing the structure of the algebras being studied.

- **Group actions and invariant theory** (Florian Eisele)
If a group G acts on $\mathbb{C}^n$ we can construct an action of the same group on the polynomial ring $\mathbb{C}[x_1, \ldots, x_n]$. The invariants of G are the polynomials fixed under this action, and they form a ring $\mathbb{C}[x_1, \ldots, x_n]^G$ called an invariant ring. These invariant rings have a particularly nice structure theory when the group G is finite, which you will explore in this project. You will also describe some examples of invariant rings explicitly.
Reference: B. Sturmfels. *Algorithms in Invariant Theory.*

- **Path algebras and their representations** (Florian Eisele)
Similar to the notion of a group acting on a set, we can study actions of directed graphs Q, known as quivers in this context, on vector spaces like $\mathbb{C}^n$. Such an action is called a representation of Q. This leads to the definition of a path algebra $\mathbb{C}Q$, and modules over path algebras.

  The first aim of this project would be to understand these algebraic structures. The next goal would be to understand the statement and at least part of the proof of Gabriel's theorem, which states that a quiver Q has only finitely many indecomposable representations (the "building blocks" of arbitrary representations) if and only if the undirected version of Q is a Dynkin diagram (these diagrams pop up in various areas of mathematics). You could then work out the indecomposable representations explicitly in some examples.

In a subsequent dissertation you would look at one or more other families of finite-dimensional algebras, e.g. string algebras, gentle algebras, Brauer tree algebras, ... (see third reference for more possibilities).

References:

- D.J. Benson. *Representations and cohomology. I. Basic representation theory of finite groups and associative algebras.*
- M. Auslander, I. Reiten, S.O. Smalø. *Representation Theory of Artin Algebras*
- FD Atlas

- **A topic in representation theory** (Florian Eisele)
In addition to the two projects above, there are a number of topics in representation theory and algebra that would make for a good project. E.g. the representation theory of symmetric groups, Auslander-Reiten theory, tilting theory, etc. Interested students should get in touch with me directly.

- **O-minimality** (Gareth O. Jones) *This topic may lead to a dissertation in logic.*
O-minimal structures come from model theory, but have interesting connections with other parts of mathematics. One motivating idea is that they form a setting for tame geometry, avoiding, for example, space filling curves. This project will explore foundational results in o-minimality, from both the model-theoretic and geometric points of view. Some of this will be based on van den Dries's book 'Tame topology and o-minimal structures', and some on various papers. If there is time, we might also look at connections with some topics in number theory. This project relies on some of the ideas in the Model Theory module.

- **Topics in Set Theory** (Gareth O. Jones) *This topic may lead to a dissertation in logic.*
Please contact the project supervisor for more details of this project

- **Group representation theory** (Radha Kessar)
The project will be an introduction to the basic concepts in the representation theory of finite groups. The starting point will depend on your background, in particular on whether you have take the Non Commutative Algebra course and/or the Representation Theory course. As main reference we will use the book: A Course in Finite Group Representation Theory by Peter Webb. A subsequent dissertation could take one of several different directions, for example, further character theory, modular representation theory, or the representation theory of particular classes of groups such as the finite symmetric groups and the finite general linear groups.

- **Fusion Systems** (Radha Kessar)
One of the cornerstones of finite group theory are the Sylow theorems. Fusion systems provide an abstraction of Sylow theory which is applicable to mathematical objects other than finite groups, such as blocks of finite group algebras, certain types of topological spaces, and infinite groups. While the underlying ideas are classical, the theory of abstract fusion systems is rather recent and is a very active area of research. The project will involve learning the basic definitions and structure theory of fusion systems and connections to p-local finite group theory. A

subsequent dissertation could be about the topological (e.g. the theory of centric linking systems) or representation theoretic (block fusion systems) aspects of the subject or could delve deeper into the algebraic side of the picture (e.g. fusion systems on infinite groups, exotic systems).
References:
  - Markus Linckelmann, An introduction to fusion systems
  - David Craven, Fusion systems: Group Theory, Representation Theory and Topology
  - Michael Aschbacher and Bob Oliver, Fusion Systems (article in the Bulletin of the American Math. Society
  - Michael Aschbacher, Radha Kessar and Bob Oliver: Fusion systems in Algebra and Topology.

- **Differential Algebra** (Omar León-Sánchez) *This topic may lead to a dissertation in logic.*
  In this project we will look at rings equipped with an additive map satisfying the classical Leibniz rule from Calculus (from an algebraic point of view). We will pay close attention to the ring of differential polynomials which is the analogue of the classical polynomial ring and study its ideal structure. The elements of this ring are the algebraic formalism of differential equations.
  Reference: Differential Algebra and Algebraic Groups. E. Kolchin. Academic Press, 1973.
- **p-adic numbers** (Gabor Megyesi)
  Let p be a prime. One can define a metric on the integers by setting $d(m, n) = p^{-k}$, where p^k is the highest power of p dividing $m - n$. It can be also extended to the rationals. The completion of the rationals with respect to the usual metric gives the real numbers, the completion with respect to this new metric gives a different field, the field of p-adic numbers. The project would involve construction of p-adic numbers, their basic properties and applications to solving Diophantine equations.
  Reference: JWS Cassels, Local fields, CUP 1986.
- **Algebraic curves** (Gabor Megyesi)
  This project would involve the application of the Riemann-Roch theorem to study projective algebraic curves.
  Reference: K. Hulek; Elementary algebraic geometry, AMS 2003
- **Oppenheim's Conjecture** (Donald Robertson)
  Representing numbers by quadratic forms has a long history. Oppenheim conjectured in the 1950s that a quadratic form in at least three variables represents a dense set of real numbers if it is indefinite, degenerate, and not a multiple of a rational form. In the 1980s Margulis affirmed Oppenheim's conjecture using dynamics on the homogeneous space $\mathsf{SL}(3, R)/\mathsf{SL}(3, Z)$. Over the course of the project you could cover questions such as why the hypotheses are necessary, what the problem looks like in terms of dynamics on homogeneous spaces, and how ergodic theory can help to answer the dynamical question.
  References:
  - Ratner's Theorems on Unipotent Flows. D. Witte Morris University of Chicago Press, 2005.
  - Ergodic Theory with a view towards Number Theory. M. Einsiedler, T. Ward. Springer, 2011.

  - Discrete subgroups and ergodic theory. In Number theory, trace formulas and discrete groups (Oslo, 1987), Pages 377–398. G. A. Marguils Academic Press, 1989.

- **Finite simple groups** (Peter Rowley)
  Please contact the project supervisor for more details of this project

- **The Penrose tilings** (Yotam Smilansky)
  The Penrose tilings are a family of aperiodic tilings of the plane, known for their beauty, their forbidden five-fold symmetry and their intricate self-similar structure. Interestingly, these tilings may be constructed in a variety of ways. While Penrose originally constructed his tilings using matching rules on a given set of tiles, that is, rules on how two tiles can meet edge-to-edge, they can also be constructed from substitution rules on tiles, which decompose each tile into smaller tiles of the same original set of tiles used in the tiling, or obtained through a cut-and-project construction, in which a slice of a higher dimensional lattice is projected to the plane.

  In this project we will investigate these construction methods, which are central in the field of aperiodic order. We will then take advantage of the multiple ways in which Penrose tilings can be represented to explore various dynamical, arithmetic, geometric and statistical properties of these tilings and their tile patterns.
  References:
  - Baake, M, Grimm, U. Aperiodic order. Volume 1, A mathematical invitation
  - Penrose tiling, Wikipedia page

- **Combinatorics on words** (Yotam Smilansky)
  An infinite word is a sequence of symbols taken from a finite alphabet, say a,b. The complexity of a word is measured by the growth rate of its language, that is, the number of distinct subwords of length n it contains, and is unbounded if and only if the word is aperiodic, namely if it is not eventually periodic. In this project we will study combinatorial, arithmetic and dynamical aspects of aperiodic words such as automatic sequences and fixed points of a primitive substitution, and explore illuminating examples including the Fibonacci word (more generally, Sturmian words) and the Thue-Morse sequence.
  References:
  - Pytheas Fogg, N.; Berthé, Valérie; Ferenczi, Sébastien; Mauduit, Christian; Siegel, A. (eds.). Substitutions in dynamics, arithmetics and combinatorics.

- **Existential Closed Structures** (Nikesh Solanki) *This topic may lead to a dissertation in logic.*
  Algebraically closed fields are hugely important to mathematics. They have the wonderful property that given any system of equations with coefficients from that field, if there exist solutions of that system some (larger) field then the solutions already exist in the field. As such, algebraically closed fields are what model theorists call an "existential closed" structures. In turns out that, other kinds of mathematical structures are also existential closed. This raises some interesting questions:
  - We know from this existential closedness condition of algebraically closed fields, many other wonderful properties of follow. Do equivalent properties follow for other kinds of existentially closed structures?

  - If we have a first order theory, all whose models are existentially closed, what can we say about the theory?

The purpose of this project is to address questions like these.
References:
  - Hodges, Wilfrid, "First-order Model Theory", The Stanford Encyclopedia of Philosophy (Summer 2005 Edition), Edward N. Zalta (ed.).
  - Marker, David (2002), Model theory: An introduction, Graduate Texts in Mathematics, 217, New York, NY: Springer-Verlag, ISBN 0-387-98760-6, Zbl 1003.03034

- **Algebraic Topology** (Peter Symonds)
Please contact the project supervisor for more details of this project
- **Invariant Theory** (Peter Symonds)
Please contact the project supervisor for more details of this project
- **Constraint solving, logic programming and problems in combinatorics** (David Stewart)
With Dr David Cushing, I have written a couple of papers using the constraint solving library in the logic programming language Prolog to solve some problems in pure mathematics. The one which attracted the most publicity was our calculation of the minimum number of tickets to win a prize of some sort in any draw in the UK National lottery. But there are many other invariants that can be calculated. This is a practical project in which you will learn how to write Prolog code and ideally use it to calculate some unknown combinatorial numbers by reference to the Handbook of Combinatorial Designs.
- **Introduction to inverse semigroups** (Nora Szakacs)
Invers semigroups are one of the most investigated class of semigroups, they are the mathematical abstractions of partial symmetries and crop up in various different areas of mathematics, mostly C*-algebras. This project in an introduction to the basics of inverse semigroups and will lead on to an MSc dissertation in a more specialized topic in inverse semigroup theory.
Reference: M. V. Lawson, Inverse semigroups (The theory of partial symmetries), World Scientific, 1998.
- **Model Theory of ordered algebraic structures** (Marcus Tressl) *This topic may lead to a dissertation in logic.*
Classical algebraic structures like groups, rings and fields are many times furnished with a natural order: Think of the ring of integers or the real field. Ordered algebraic structures can be thought of as algebraic structures equipped with an order or a partial order that is compatible with the algebraic operations (like addition and multiplication). The precise topic for the project will be decided after consultation with interested students and can be in the areas of ordered fields, lattice ordered groups, lattices or topology. The project may, but does not need to, invoke model theoretic methods. On the other hand, the project will prepare for a subsequent topic in the model theory of the structures studied in the project.
Prerequisites: Basic knowledge in commutative algebra or field theory. A first contact with partially ordered sets is desirable but not necessary. Students interested in a dissertation following this project are required to have some basic knowledge

of model theory as for example taught in the model theory course offered in the taught component.
Reference: Stuart A. Steinberg, Lattice-ordered Rings and Modules

- **Model Theoretic Forcing** (Marcus Tressl) *This topic may lead to a dissertation in logic.*
  Forcing has originally been developed by Paul Cohen to show the independence of the axiom of choice and the continuum hypothesis from Zermelo-Fraenkel set theory. Abraham Robinson then realised that the method can be adapted to construct algebraic structures with special properties. The project will explain model theoretic forcing with a particular emphasize on Robinson forcing, which will then be the background for the subsequent dissertation on finitely generic models. For details, please contact the project supervisor.
  Prerequisites: This is for students who have already done a course in mathematical logic (containing a rigorous treatment of predicate logic). In addition, basic knowledge in commutative algebra is required. Students interested in a dissertation following this project are required to attend the model theory course offered in semester 1 of the taught component.
  Reference: W. Hodges, Building Models by Games
- **Non-Hausdorff Topology in Logic and Algebra** (Marcus Tressl) *This topic may lead to a dissertation in logic.*
  This is a broad topic requiring interests and some basic formation in the topics mentioned in the title. To see a fundamental connection of these topics, please have a look at Stone Duality for Boolean Algebras. Please get in touch with the project supervisor for further details of this project.
  Prerequisites: Basic knowledge in set theoretic topology and either some basic ring theory, or, basic knowledge in predicate logic (as for example taught in the model theory course offered in the taught component).
  References:
  (a) J. Goubault-Larrecq: Non-Hausdorff Topology and Domain Theory
  (b) M. Dickmann, N. Schwartz, M. Tressl: Spectral Spaces
- **Vector bundles, connections, curvature and characteristic classes.** (Ted Voronov)
  Vector bundles can be understood as families of vector spaces parameterized by points of a topological space (called base) and satisfying a certain regularity condition so that locally they look like a direct product of a piece of the base with some standard vector space (like $\mathbb{R}^k$ or $\mathbb{C}^k$). They are a particular case of more general "fiber bundles". Examples of fiber bundles and particularly vector bundles are abundant. Such are the set of all tangent vector for a given manifold (tangent bundle), the set of all normal vectors to a k-dimensional submanifold in $\mathbb{R}^n$ (normal bundle), or the famous Moebius strip. They provide standard language for physics, where physical fields are what are called "sections" of fiber bundles. Connection and curvature are central notions of differential geometry. They first appear for curves and surfaces in $\mathbb{R}^3$ and the language of fiber bundles is the most natural for them. Characteristic classes are topological invariants of vector bundles, which can be constructed using connections.
  Please contact the project supervisor for more details of this project.

- **Superalgebra, supermanifolds and their applications.** (Ted Voronov)
Supermanifolds are a powerful language of modern mathematical physics and differential geometry. Many simple notions of calculus such as derivatives can be extended to include formal variables that "commute up to a sign". A student of supermanifolds will quickly learn how knowing them helps him to understand classical subjects.
Please contact the project supervisor for more details of this project.

- **De Rham cohomology** (Ted Voronov)
De Rham theory (originated in the works of Poincaré and Cartan, in fact) is a most beautiful part of mathematics on the crossroads of topology, calculus and differential geometry. Typical questions leading to cohomology are "How one can distinguish $\mathbb{R}^1$ from $\mathbb{R}^2$?" and "Why some line integrals depend on path and some don't?". We can start from cohomology of forms and then travel deeper into topological invariants of simple and not so simple spaces.
Please contact the project supervisor for more details of this project.

- **Categories of Modules** (Rose Wagstaffe) *This topic may lead to a dissertation in logic.*
Given a ring R, the concept of a module over R generalises the idea of a vector space over a field. We can learn about a ring R by studying the category of modules over R. Category theory provides a convenient framework that allows us to observe and formalise relationships between different areas of mathematics and provides an abstract setting in which ideas can be generalised.
The main goal of the project is to explore the categorical structure and properties of the category of modules over a ring R. This will involve becoming familiar with category theoretic definitions and investigating particular examples.
Please feel free to email me (rose.wagstaffe@manchester.ac.uk) to discuss the project in more detail.
References:
F.W. Anderson, and K.R. Fuller, Rings and categories of modules.
S. Mac Lane, Categories for the Working Mathematician.
T. Leinster, Basic Category Theory. (Available at:
https://www.maths.ed.ac.uk/ tl/bct/)

- **Hyperbolic dynamical systems** (Charles Walkden)
A dynamical system consists of a phase space X (which may be an interval, a torus, a Cantor set, or a more complicated space) and a map $T : X \to X$. Dynamical systems is the study of how points in the phase space X behave as one iterates them under the action of T. A dynamical system is said to be hyperbolic if there is some exponential expansion or contraction in the system: two points that are close together may (locally) move apart or together exponentially fast (this is a particularly strong form of what is popularly called 'chaos'). Hyperbolic dynamical systems form a particularly tractable class of dynamical system. In this project you could survey some examples of hyperbolic systems (the doubling map, the cat map, Smale's horseshoe, etc) and study how they are particular examples of a more general construction.

- **Fractals and iterated function systems** (Charles Walkden)
Loosely speaking, a fractal is a subset of $\mathbf{R^n}$ that has structure at all scales: no

matter how much one 'zooms in', the set remains complicated. Well-known examples of fractals include the Middle Third Cantor Set, the Sierpinksi Gasket, the von Koch Curve, etc. One can attempt to quantify how complicated a fractal is in terms of its fractal dimension, which is often a non-integer. (In fact, there are various different definitions of fractal dimension: the most commonly occurring ones being Hausdorff dimension and box dimension.) Many fractals can be constructed as limit sets for iterated function schemes (IFSs), and one can derive a formula for the dimension of this limit set in terms of quantities that appear in the IFS.

- **Hyperbolic dynamics and hyperbolic geometry** (Charles Walkden)
  Imagine the surface of the Earth. Through any point on the Earth's surface and in any direction there is a unique geodesic; in fact, the geodesic is a great circle (a circle inscribed on the sphere with the same centre and radius). Now imagine, given a starting point and direction, moving along this geodesic at unit speed; this is the geodesic flow on (the unit tangent bundle of) a sphere. In the case of a sphere, you will always return to where you started and facing in the same direction after the same amount of time; dynamically this is not very interesting and is related to the fact the sphere has constant positive curvature. One can construct the geodesic flow on (the unit tangent bundle of) a surface of constant negative curvature which have much more interesting dynamical properties. This project studies such geodesic flows using techniques from hyperbolic geometry and ergodic theory.
- **Mapping class groups of surfaces** (Richard Webb)
  The mapping class group of a surface S is the group consisting of homeomorphisms from S to S modulo the isotopy relation. These groups occur naturally in topology, e.g. the braid groups, or when constructing 3-manifolds by gluing together boundary components (or more generally, surface bundles are determined by homomorphisms into the mapping class group). A deeper fact is that the mapping class group of a surface is the (orbifold) fundamental group of the moduli space of Riemann surfaces, so many different branches of mathematics, e.g. algebraic geometry, complex analysis, PDEs, encounter these groups.

  There are many different directions that this project and dissertation may take. So here are a few brief suggestions. The mapping class group can be fruitfully studied by its action on the Teichmueller space - so understanding properties of this action and the topology/geometry of Teichmueller space is interesting. Similarly, one can study the cohomological properties of mapping class groups, or, the coarse geometric properties of mapping class groups, or the properties of random walks. Another possible topic is the Nielsen-Thurston classification and dynamical properties of pseudo-Anosov maps. Having prior knowledge of fundamental groups and covering spaces would be helpful. For more details contact the supervisor.
  Reference: B. Farb, D. Margalit, A primer on mapping class groups
- **Outer automorphism groups of free groups** (Richard Webb)
  A free group is isomorphic to the fundamental group of some graph. In fact, this point of view gives beautiful proofs of algebraic results concerning the free group e.g. subgroups of free groups are free. In this project and dissertation, we will go one step beyond and study the Culler Vogtmann Outer Space CV_n. This is the space of (marked, metric) graphs (without leaves) whose fundamental group

is isomorphic to a fixed free group F_n. There are different ways of defining CV_n, but the important point is that it is symmetric in a natural way, and its group of symmetries is naturally isomorphic to the outer automorphism group of F_n i.e. $Out(F_n)$. The space CV_n can be used to understand the cohomological properties of $Out(F_n)$, and the interested student may choose to write about the sharpest and most recent results on this. There are many topics one can cover, so alternatively, one can also choose to study the general Whitehead's algorithm, or study deeper algebraic results of $Out(F_n)$ (such as SQ-universality) which rely on group actions on the free factor complex of F_n and some geometric group theory. Having prior knowledge of fundamental groups and covering spaces would be helpful. For more details contact the supervisor.
Reference: M. Culler, K. Vogtmann, Moduli of graphs and automorphisms of free groups