$\aleph_0$-categoricity for semigroups

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Abstract

A countable structure M is $\aleph_0$-categorical if $\mathrm{Th}(M)$ has one countable model, up to isomorphism. Numerous authors have investigated the $\aleph_0$-categoricity of both relational and algebraic structures. Although the notion of $\aleph_0$-categoricity is primarily motivated by concerns of model theory, it has yielded significant results for both groups and rings. The Ryll-Nardzewski theorem tells us that a structure M is $\aleph_0$-categorical if and only if $\mathrm{Aut}(M)$ has only finitely many orbits on M^n for each $n \in \mathbb{N}$, that, is $\mathrm{Aut}(M)$ is an *oligomorphic* group. Thus $\aleph_0$-categoricity may be translated entirely into an algebraic context.

A *semigroup* is a set together with an associative binary operation. The aim of this talk is to present the beginnings of the exploration of $\aleph_0$-categorical semigroups. Our viewpoint is to break a semigroup S into constituents of 'simpler' type such as groups and sets and consider how categoricity of S passes to and from that of its components. We illustrate our approach by focussing on the case of *inverse semigroups*, where a semigroup is inverse if for each $a \in S$ there is a unique $a' \in S$ such that $a = aa'a$ and $a' = a'aa'$. Inverse semigroups share some of the properties of groups, yet are distinct enough to provide a good setting to illustrate our techniques, being determined, up to a point, by groups and semilattices.

No prior knowledge of semigroups will be assumed.

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