Summary

This thesis is about cocycle equations over hyperbolic dynamical systems. The three chapters are inter-related but can be read independently. A more detailed description of our results can be found in the Introduction.

Chapters 1 and 2 are concerned with cocycles over hyperbolic flows. In Chapter 1 we discuss 'regularity theorems' for Hölder continuous cocycles taking values in either a compact Lie group or an abelian Lie group. In particular, we prove that any measurable solution to the cocycle equation (holding a.e. with respect to a Hölder equilibrium state) has a Hölder continuous version for which the cocycle equation holds everywhere. This generalises a well-known theorem due to Livšic.

In Chapter 2 we consider flows obtained from a hyperbolic flow by either a toral extension or a velocity change. A cocycle is associated to each flow so obtained. We link these cocycles to the first (Čech) cohomology group (realised as the Bruschlinsky group) of the underlying basic set. When the Bruschlinsky group has finite rank, we show how to use the results of Chapter 1 to construct open dense sets of mixing toral extensions and velocity changes. Similar results are proved for suspensions of shifts of finite type.

In Chapter 3 we consider Hölder continuous cocycles over hyperbolic diffeomorphisms. We discuss regularity theorems for cocycle equations that are 'twisted' with the action of an automorphism of the group. We shall mostly be interested in an automorphism of a compact connected Lie group with the following property: the eigenvalues of the derivative at the identity have modulus 1. (We say such an automorphism is 'unipotent'.) In this case, we prove that any measurable solution (again holding a.e. with respect to a Hölder equilibrium state) has a Hölder continuous version for which the cocycle equation holds everywhere. We also prove a C^r version. The unipotency hypothesis on the automorphism is optimal.