Polynomials on matrices, rational functions, and Berezinians

Hovhannes Khudaverdian (Manchester)

The ring of invariant polynomials on even $p|q \times p|q$ matrices has an infinite number of generators, $s_r := \mathrm{Tr} A^r$ (where $r = 1, 2, 3, \ldots$), if $q > 0$, with an infinite number of relations. Here Tr stands for supertrace, $\mathrm{Tr} A = \mathrm{tr} A_{00} - \mathrm{tr} A_{11}$. (If $q = 0$, then this ring is freely generated by p generators $s_1, \ldots s_p$, which are ordinary traces.) An adequate and transparent picture of this ring can be obtained by considering instead of it a ring of polynomial functions on fractions $P(z)/Q(z)$ where the numerator $P(z)$ is a polynomial of degree p and the denominator $Q(z)$ is a polynomial of degree q. Our considerations are inspired by the relations between Berezinians (superdeterminants) of linear operators on superspaces and rational functions.