Standard tableaux and Klyachko’s Theorem on Lie representations.

Marianne Johnson

4th October 2005

Let $\lambda$ be a partition of $n$. A standard tableau of shape $\lambda$ is a numbering of the Young diagram of $\lambda$ with the numbers from $\{1, \ldots, n\}$, such that the entries increase along each row and down each column (see example below).

I show that for all but two partitions $\lambda$ of $n > 6$ there exists a standard tableau of shape $\lambda$ with major index coprime to $n$. In conjunction with a deep result of Kraskiewicz and Weyman this provides a new purely combinatorial proof of Alexander Klyachko’s famous theorem on Lie representations of the general linear group.

\begin{center}
\begin{tabular}{c|c|c|c|c}
1 & 2 & 4 & 8 & 9 \\
3 & 5 & 11 & \\
6 & 10 & \\
7 & \\
\end{tabular}
\end{center}

The Young diagram of $\lambda = (5, 3, 2, 1)$  

A standard tableaux of shape $\lambda = (5, 3, 2, 1)$