Feedback on the MATH36022 Exam (Numerical Analysis 2), May 2017.

Section A

Question **A1** was done well by most students. However, a couple of students simply wrote nothing at all, or did not attempt all parts of the question, and so scored low marks. Some students could not remember the Equioscillation Theorem and wrote down Weierstrass' Theorem (or some version of it). Please note that this question was just a special case (a particular choice of $f(x)$) of question 3 on Exercise Sheet 1.

Unfortunately, there was a mistake in question **A2**b). There was a missing x in the top line of the given expression for $r_3(x)$. Although this was queried during the exam, the invigilator(s) were not able to communicate this to the School of Mathematics. Several students noticed the mistake and corrected it themselves, and went on to complete the question correctly. Well done, if this was you! Note, the general expression for $r_n(x)$ was defined in class, a similar question with $r_2(x)$ was posed on the 2016 exam, and also discussed in the revision session. Since the x was missing, when the stated expression was written as a ratio of two polynomials, the resulting expression was a ratio of two linear polynomials. This could not be the [2/1] Padé approximant, as this has a numerator which is quadratic. The mark scheme for this question was adjusted for fairness. Part a) was unaffected, and two marks were available for that. Two marks were also awarded for writing the stated $r_3(x)$ as a ratio of two polynomials. The rest of the question was ignored, since it did not make sense to work with the given expression. As a result, this question was marked out of a total of four, and not nine marks (as stated on the paper). As a result, almost all students scored full marks on this question.

Question **A3** was done well by students who attempted it. However, several students simply did not answer it. Some students attempted to prove that the degree of precision was $2n-1$, but this was not asked for. Note that this question appeared on section B of the 2016 exam, and it was discussed in the revision session.

In question **A4**, a couple of students queried whether the interval of integration should have been $[-1,1]$, since we had previously considered that interval in a similar exercise (on the handout on 'Introduction to Gauss Quadrature'). The stated interval $[0,1]$ was correct. The idea was to apply the method in a new situation. In general, we do need to be able to approximate integrals on any interval, not just $[-1,1]$. Note that Legendre polynomials can be defined on any interval (by a shift), so if you remembered that the nodes needed to be the roots of Legendre polynomials, then you would have needed the polynomials corresponding to the interval $[0,1]$. However, the question asked you to use the method of undetermined coefficients, so you did not need to work out any Legendre polyomials. On the interval $[0,1]$ the equations that need to be solved to obtain a and b and hence the nodes x_1 and x_2 are a little more tricky to work with than the equations obtained for $[-1,1]$. This caused a few small errors, but on the whole, the method was correctly applied by most students.

Question **A5** was mostly done well. Several students were not able to explain which interpolating polynomial needed to be substituted for the integrand (did not specify the interpolation points). Quite a few students also assumed that $\ell = 2$, and reported orders of accuracy for that specific case, but the question asks about any value of ℓ. We discussed this material in the last two lectures of the course (which were not well attended).

Section B

Most students attempted question **B6**. Parts (a), (c) and (d) were mostly well done. However, several students were not able to complete (b), the main part of the question. It was much easier to work with the inner-product notation $\langle \cdot, \cdot \rangle_w$ than to write out integrals. So, for example, to prove the base case, using the fact that $\phi_0 = 1$, we have $\langle \phi_1, \phi_0 \rangle_w = \langle (x - \alpha_0)\phi_0, \phi_0 \rangle_w = \langle x\phi_0, \phi_0 \rangle_w - \alpha_0 \langle \phi_0, \phi_0 \rangle_w = 0$, using the definition of α_0. Many students also got in a mess by not being clear about what their inductive assumption was. The proof was covered in lectures.

In question **B7**, many students did not do part (a), which was worth seven of the twenty available marks. This was covered in lectures. The idea was to split the integral in the definition of $I(f)$ over the intervals and use the stated result for the error on each interval, then sum them up. Parts (b) and (c) were done better. Note that the Romberg scheme and the Euler-Maclaurin formula were explored on Exercise Sheet 5.

In question **B8**, parts (a), (c) and (d) were generally well done. However, some students confused exact values $y(x_n)$ with approximate values y_n in several places and lost some marks if the resulting expressions did not make sense. Some students also could not do the required Taylor series in part (b) and so left that part blank. The derivation of the order conditions was covered in lectures. Questions similar to parts (c) and (d) appeared on Exercise Sheet 8. There was a small mistake with the notation in (c). c_1 should have been c_2. However, this did not seem to cause any issues. Students who attempted this question did well on this part. Note that part (d) only asked for the polynomial that can be used to find the stability region, and did not ask for the actual stability region, so some students wrote more than was necessary.