
□ **Ohood Althagafi (University of Manchester)**

Numerical Evaluation of H Function

In recent years, there has been significant advancement in the numerical computation of Mellin-Barnes integrals. This study focuses on employing Mellin-Barnes integrals for the representation of stable α Lévy distributions ($0 < \alpha \leq 2$), utilizing the H -Fox function. We evaluate existing methods for the precise and dependable calculation of this integral across various parameters and variable ranges. Our investigation encompasses both asymptotic series and contour integral techniques. Through numerical experiments, we identify the most effective strategy for specific parameters and variable conditions. Our findings for the optimal methods to be applied in each scenario, are presented.

□ **James Harborne (University of Nottingham)**

Stochastic Averaging for a Two-Strain Model of Infectious Disease Epidemiology

We consider an extension to the standard compartmental SIR model of infectious disease epidemiology first presented by Kermack and Mckendrick in 1927. Contrary to the standard SIR model with a single infected class, we consider the possibility of two competing strains of disease. We construct the stochastic model as a Continuous Time Markov Chain (CTMC), giving both the generator and the random time change representations. Under certain technical assumptions akin to those made in Quasi-Steady State Approximations (QSSA) literature, wherein we assume that the rates of infection and recovery of one strain is much faster than the other, we can use the stochastic averaging principle to obtain a reduced model. We then use this to prove a Functional Law of Large Numbers (FLLN), showing that the dynamics of our reduced stochastic model behave according to the solutions of a set of deterministic Ordinary Differential Equations (ODEs) in the large number limit. We also present numerical simulations of the solutions of the models.

□ **Nicholas Harbour (University of Nottingham)**

A Mathematical model for BMP4 induced differentiation therapy in combination with radiotherapy in glioblastoma.

Glioblastoma (GBM) is the most aggressive and most common primary brain tumour in adults and is uniformly fatal, with a poor median survival time of 15 months. Standard of care for GBM consist of radiotherapy either alone or following surgical resection, despite this, radio-resistance almost always occurs making recurrence inevitable. Failure of the current standard of care has been partly attributed to a special sub-population, the glioma stem cells (GSCs), which initiate and drive tumour growth. Treatment cannot be successful unless all GSCs are eliminated. However, GSCs are known to be highly resistant to radiotherapy, and complete surgical removal is usually impossible in GBM. Therefore, new treatments that specifically target the GSCs could have a potentially

large benefit. BMP4 has been shown to induce differentiation of GSCs towards a less malignant, astrocytic-like (ALCs) lineage. Furthermore, new delivery systems (nano particles) provide a potential mechanism by which BMP4 could be successfully administered to reverse the GSC state and reduce radio-resistance in a patient. We develop a data driven mechanistic mathematical model that accounts for the GSCs, tumour cells (TCs) and ALCs as well as their response to both radiotherapy and BMP4 induced differentiation therapy. Our model allows us to run in-silico experiments to investigate how varying several key parameters such as: the radiosensitivity of all cellular populations and the strength of BMP4 on differentiation rate, affect treatment outcome. Our model shows that treatments specifically targeting the GSCs are vital for prolonging survival in GBM and that a combination of both BMP4 therapy and radiotherapy can provide superior outcomes than either one individually.

□ **Humaira Hameed (University of Strathclyde)**

On semi-transitivity of (extended) Mycielski graphs.

An orientation of a graph is semi-transitive if it is acyclic, and for any directed path $v_0 \rightarrow v_1 \rightarrow \dots \rightarrow v_k$ either there is no arc between v_0 and v_k , or $v_i \rightarrow v_j$ is an arc for all $0 \leq i < j \leq k$. An undirected graph is semi-transitive if it admits a semi-transitive orientation. Semi-transitive graphs generalize several important classes of graphs, and they are precisely the class of word-representable graphs studied extensively in the literature.

The Mycielski graph of an undirected graph is a larger graph, constructed in a certain way, that maintains the property of being triangle-free but enlarges the chromatic number. These graphs are important as they allow to prove the existence of triangle-free graphs with arbitrarily large chromatic number. An extended Mycielski graph is a certain natural extension of Mycielski graphs.

In this talk, I will discuss a complete characterization of semi-transitive extended Mycielski graphs and comparability Mycielski graphs, as well as a conjectured complete characterization of semi-transitive Mycielski graphs. My results are a far-reaching extension of the result of Kitaev and Pyatkin on non-semi-transitive orientability of the Mycielski graph $\mu(C_5)$ of the cycle graph C_5 . Also, I will mention how to use a recent result of Kitaev and Sun to shorten the length of the original proof of non-semi-transitive orientability of $\mu(C_5)$ from 2 pages to a few lines.

□ **Anastasia Istratuca (University of Edinburgh, Maxwell Institute Graduate School)**

Multilevel Monte Carlo Methods for Chaotic Systems.

Co-authors: Abdul-Lateef Haji-Ali (*Heriot-Watt University, Maxwell Institute*) & Aretha Teckentrup (*University of Edinburgh, Maxwell Institute*)

We consider the computational efficiency of the Multilevel Monte Carlo (MLMC) algorithm applied to chaotic systems of the form $x'(t) = f(x(t))$, $t \in [0, T]$. Here, $f : \mathbb{R}^m \rightarrow \mathbb{R}^m$ is a Lipschitz function satisfying the dissipativity condition, but not the following contractivity condition:

$$\langle x - y, f(x) - f(y) \rangle \leq -\lambda \|x - y\|^2, \quad \forall x, y \in \mathbb{R}^m. \quad (1)$$

A direct application of MLMC to such systems is challenging due to the exponential increase of the variance of the level estimators with respect to the final time, T , and hence of the corresponding computational complexity. To alleviate this issue, Fang and Giles [W. Fang, M. Giles, Multilevel Monte Carlo method for ergodic SDEs without contractivity, *J. Math. Anal. Appl.* 476 (2019) 149–176] proposed the change of measure technique for the stochastic variant of the deterministic dynamical system, which recovers the contractivity of the path. Building on their work, our aim is to compute quantities of interest of the deterministic system with and without random coefficients, using its stochastic variant as a control variate. We apply our method to Lorenz63, a three-dimensional system modelling convection rolls in the atmosphere.

□ **Vanessa Madu (Imperial College London)**

Physically Interpretable Error Metrics for Ocean Surface Current Models.

The ocean is a powerful system involving many complex processes, including extensive and elaborate circulation systems, wave dynamics, and influences from the atmosphere, lithosphere (land) and cryosphere (ice). This poster presents a case for the need for specialised phenomenon-centred statistical approaches for modelling ocean surface currents tailored to the ocean's dynamic and intricate nature. We highlight the potential benefits of phenomenon-centred model evaluation by demonstrating our proposed physically interpretable evaluation metrics for seven simple linear regression models that predict ocean drifter velocities from satellite geostrophic velocity and wind data, as well as position and time. We compare the results of using root mean square error (RMSE) and the error metrics proposed for these linear regression models as well as a spatially fixed current map before highlighting some of the insights gained by evaluating the simple models using this approach.

□ **Abigail Mellor (University of Manchester)**

An Assessment of Boundary Condition Modelling in Finance using Generalised Ornstein-Uhlenbeck Processes.

We investigate boundary condition modelling for stochastic processes with mean reversion and constant elasticity of variance (specifically, generalised Ornstein-Uhlenbeck processes), focusing on the implications for pricing financial derivatives; such dynamics are observable in various (popular) models such as the Cox-Ingersoll-Ross and Heston models. We highlight the shortcomings of commonly used boundary condition choices and demonstrate that reflection at the (lower) boundary is necessary in many cases due to the underlying behaviour of the process. We demonstrate that implementing a reflecting boundary condition ensures consistency between Monte Carlo simulations and partial difference equation approaches. Consistent numerical approaches are essential in practice due to the general lack of analytic results under these models. As a result of our proposed boundary treatment and careful numerical schemes, we can enhance the accuracy of pricing financial derivatives reliant upon an underlying governed by a generalised Ornstein-Uhlenbeck process, applicable primarily to interest rates/bonds and stochastic volatility.

□ **Hywel Normington (University of Strathclyde)**

A decoupled, convergent and fully linear algorithm for the Landau–Lifshitz–Gilbert equation with magnetoelastic effects

We consider the coupled system of the Landau–Lifshitz–Gilbert equation and Newton’s second law to describe magnetic processes in ferromagnetic materials including magnetoelastic effects in the small-strain regime [4,1]. For this nonlinear system of time-dependent partial differential equations, we present a decoupled integrator based on first-order finite elements in space and an implicit one-step method in time [3]. We prove unconditional convergence of the sequence of discrete approximations towards a weak solution of the system as the mesh size and the time-step size go to zero. Compared to previous numerical works on this problem [2], for our method, we prove a discrete energy law that mimics that of the continuous problem and, passing to the limit, yields an energy inequality satisfied by weak solutions. Moreover, we do not employ a nodal projection to impose the unit-length constraint on the discrete magnetisation, so that the stability of the method does not require weakly acute meshes. Furthermore, our integrator and its analysis hold for a more general setting, including body forces and traction, as well as a more general representation of the magnetostrain. Numerical experiments underpin the theory and showcase the applicability of the scheme for the simulation of the dynamical processes involving magnetoelastic materials at submicrometer length scales.

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□ **Mary Chriselda Antony Oliver (University of Cambridge)**

New Least-Squares Finite Element Method for the Oldroyd Fluid Problem

In this study, we explore the characteristics of a stationary Oldroyd viscoelastic fluid within two or three-dimensional domains. Our investigation focuses on the application of a novel least-squares finite element method to address the non-linearity inherent in the log conformation reformulation of the model problem. For solving the variational formulation, we undertake a comprehensive numerical analysis approach, including the linearization of the functional to demonstrate norm equivalence. The proposed methodology for solving the variational formulation is validated through numerical experiments using iterative fixed-point algorithm conducted on benchmark flow problems.

□ Yuwei Qi (University of Manchester)

Asymptotic Expansions for Valuing Perpetual American Put Options under the Heston Model

The valuation of American options under stochastic volatility has attracted considerable interest due to the complexity of calibrating real options with uncertain volatility. In this paper, we consider a free-boundary problem for pricing perpetual American put options under the Heston model and derive novel asymptotic expansions for both the option price and the optimal exercise boundary using perturbation techniques. We demonstrate the difficulty and inefficiency of obtaining accurate valuations for the fully elliptic partial differential equation problem with finite-difference PSOR methods. This leads us to simplify the problem by assuming small volatility of volatility, which usefully reduces the problem to be of parabolic type, thereby reducing the computational task considerably, and yet replicates the solution of the full problem well. This in turn leads to yet a further asymptotic and even simpler approach, by developing a quite straightforward series-solution approach, based on small but finite volatility of volatility and small displacements of the variance from its long-run mean (a critical region in parameter space). This approach too, when compared with the full benchmark solution, yields remarkably useful results, but at a tiny fraction of the computational cost.
