

MATH45061 COURSEWORK:

CURVILINEAR COORDINATES

Please submit your work to reception in the Alan Turing Building by
1:00pm on Wednesday 18th October 2017.

The Cartesian components of the position vector \mathbf{r} in a general coordinate system are given by

$$x = uv \cos \phi, \quad y = uv \sin \phi, \quad z = \frac{1}{2}(u^2 - v^2),$$

where (u, v, ϕ) are known as paraboloidal coordinates.

We shall choose $\xi^1 = u$, $\xi^2 = v$ and $\xi^3 = \phi$.

- 1.) Find the covariant and contravariant base vectors corresponding to the paraboloidal coordinates. Is the coordinate system orthonormal, orthogonal or neither?
[6 marks]
- 2.) Find the corresponding metric tensor g_{ij} and all Christoffel symbols $\Gamma_{jk}^i = \mathbf{g}^i \cdot \mathbf{g}_{j,k}$.
You may assume the symmetry $\Gamma_{jk}^i = \Gamma_{kj}^i$.
[4 marks]
- 3.) Determine an expression for the divergence $\nabla \cdot \mathbf{F}$ in paraboloidal coordinates.
[5 marks]
- 4.) Explain why $\nabla \cdot \mathbf{r} = 3$ and confirm the result in paraboloidal coordinates by explicit calculation using the expression found above.
[5 marks].