

MATH45061 COURSEWORK FEEDBACK: LIQUID CRYSTALS

The majority of students performed well on this coursework. The greatest problems were actually caused by the second question and, in particular, **proving** the results about objectivity, rather than just asserting that things were objective.

- 1.) There was some “fudging” of the divergence theorem in general coordinates in this question, which I was generally relaxed about because most people used the correct steps, but the details were a little vague. It is important to quote which theorems you are using if the results are at all non-trivial. Quite few people forgot to include the material derivative terms D/Dt in the balance equation (i.e. only used conservation), which means that technically the derivation was only valid for steady solutions.
- 2.) This was a mixed bag, probably the least well answered question. You can either go ahead and check that all the terms are objective or argue that this is a combination of objective quantities that must remain objective. It is not necessarily true (unless you prove it) that ANY combination of objective terms remains objective, however. In addition, in order to make the argument you must have shown (or quoted) that the constituent bits are objective. You also need to state what you are doing. Just writing a wall of mathematics is not sufficient and is often impossible to follow.

Some did not remember to take the transformation of the derivative when considering the objectivity of $A_{I,J}$:

$$A_{I,J}^* = \frac{\partial A_I^*}{\partial X_J^*}.$$

A few people interpreted the derivative to be in Lagrangian (rather than Eulerian) coordinates.

One area of confusion was M_{IJ} . You can assume that the alternating symbol e_{IJK} remains unchanged **because** we are only allowing orthogonal transformations (but you did need to say that!). You can also assume that it obeys tensor transformation laws because we are not allowing reflections; or you can argue based on invariance of vectors that arise from cross products.

- 3.) The most common mistakes here were simple sign errors. The key simplification is that $\mathbf{V} = 0 \Rightarrow \mathbf{D} = 0$, which leads to the loss of most terms in $\tilde{\mathbf{T}}$. Oh, a few people missed that they were supposed to define θ , here a picture helps or you can write in words the angle that is being measured.
- 4.) The standard assumption when $\theta \ll 1$ is that $\sin \theta \cos \theta \approx \theta$. You can then still separate the equation, but I did not penalise if people took the more extreme case and solved the pure diffusion equation. There is no need to write out all the possible cases for the separation constant; the only possibility for the spatial eigenfunctions is sinusoidal functions. Don't forget that the solution is a sum of all possible modes and that you need to say how you calculate the constants (via Fourier series). That said, most people correctly deduced that the slowest decaying mode is when $n = 1$. The $n = 0$ mode is trivial in this case because $\sin 0 = 0$.