MATH45061 COURSEWORK FEEDBACK: CURVILINEAR COORDINATES

In general the coursework was answered to a good standard. The most common error, if it can be called such, was including (and doing) too much working and not making the most efficient use of previous calculations!

1.) Everybody correctly computed the covariant base vectors and most correctly computed the contravariant base vectors as well. Many people used rather involved methods to compute the contravariant base vectors. The most efficient method is to recognise that the covariant base vectors are orthogonal, so all you need to do is scale by the square-length of each vector to obtain the contravariant vectors. The system is orthogonal (because all covariant, and hence contravariant, base vectors are mutually orthogonal), but not orthonormal because the base vectors do not have unit length. Note that if the covariant base vectors are not of unit length then the contravariant base vectors cannot be of unit length either! Some people did not answer this part of the question, which was worth 1 mark.

2.) Everybody correctly computed the metric tensor and had the right idea for the Christoffel symbols. There were a few tiny algebraic slips and/or wrong input (i.e. wrong contravariant base vectors) but almost everybody got this part totally correct, so well done.

3.) This question was well answered and the most common "error" was doing too much. Some people took the additional step of writing the expression in terms of components corresponding to unit base vectors in each coordinate direction, but this was not needed and was considerable extra algebra. Others left the expression in terms of dot products in the global Cartesian coordinate system, which is not an expression that is "fully" in paraboloidal coordinates.

4.) The simplest explanation is that the divergence of a vector is a scalar invariant and therefore the answer must be the same in all coordinate systems. In Cartesians $\boldsymbol{\nabla} \cdot \boldsymbol{r} = x_{I,I} = 3$, so the divergence of $\boldsymbol{r}$ must be 3 in any coordinate system. There were two marks for this part and you needed to mention the word invariance (or invariant) and do the calculation in Cartesians (or any other coordinate system) to get both marks.

You could also use the argument that

$$\boldsymbol{\nabla} \cdot \boldsymbol{r} = \boldsymbol{g}^j \boldsymbol{\cdot} \frac{\partial \boldsymbol{r}}{\partial \xi^j} = \boldsymbol{g}^j \boldsymbol{\cdot} \boldsymbol{g}_j = \delta^j_j = 3,$$

from the definitions of divergence, covariant base vectors and the Kronecker delta. This shows directly that the result must be true in all coordinate systems.

Having established the result and the expression for divergence found in question 3, you need simply to find $r^i = \boldsymbol{r} \cdot \boldsymbol{g}^i$ and substitute these into the formula found in q 3. Most people managed the appropriate algebra to establish the correct result.